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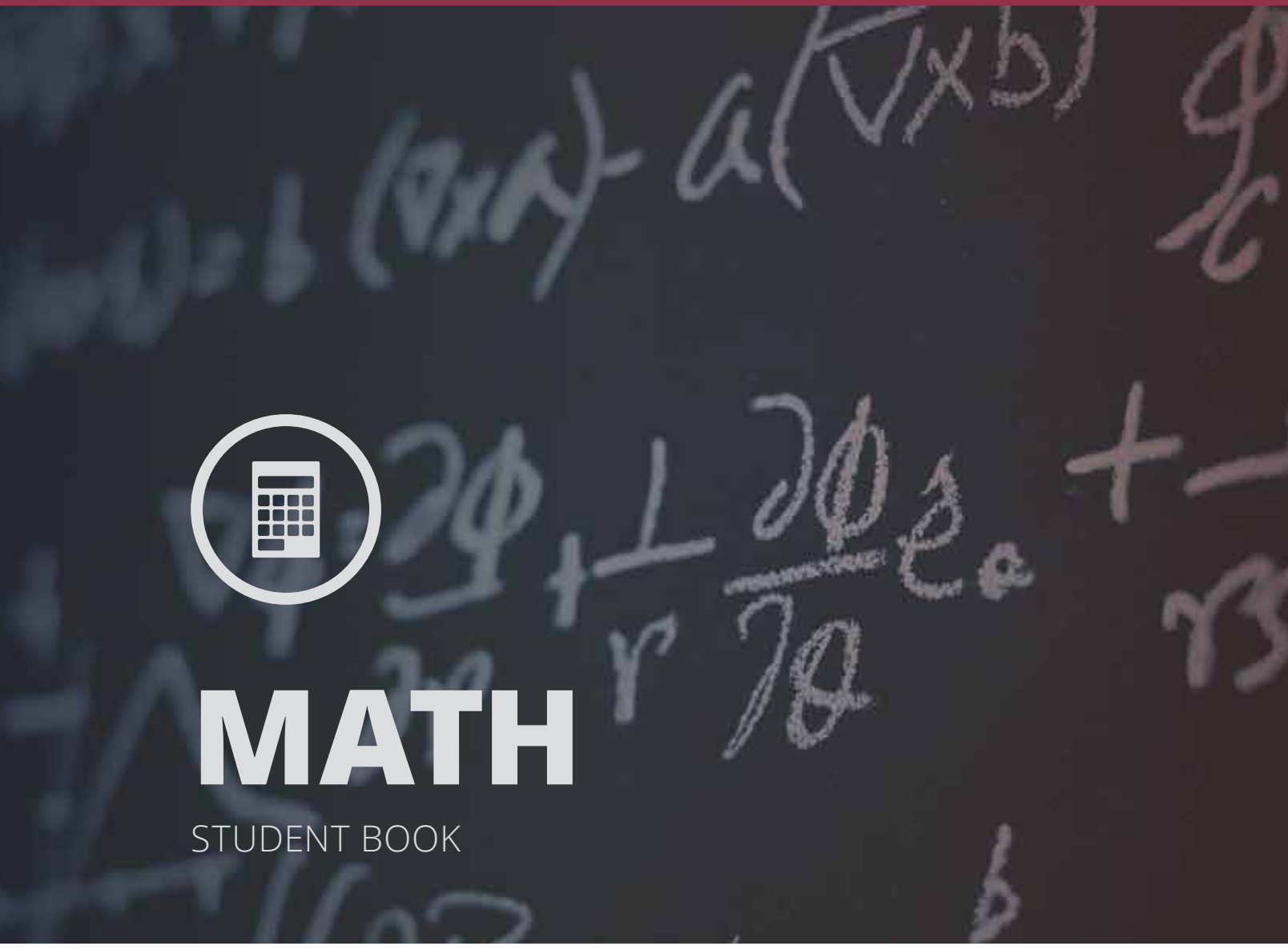

MATH

STUDENT BOOK

6th Grade | Unit 5

Alpha Omega
PUBLICATIONS

MATH 605

Fraction Operations

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Author:
Glynlyon Staff

Editor:
Alan Christopherson, M.S.

MEDIA CREDITS:
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804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

Fraction Operations

Introduction

In this unit, you will learn how to add, subtract, multiply, and divide with fractions and mixed numbers. You will also apply these skills to solve real-life problems. In addition, you will study the U.S. Customary System of Units and how to estimate, measure, and convert with customary units.

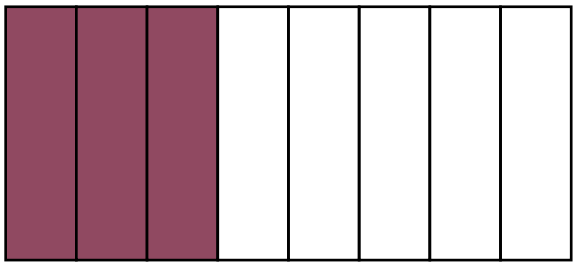
Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Add and subtract fractions with like and unlike denominators.
- Add and subtract mixed numbers.
- Multiply and divide fractions.
- Multiply and divide mixed numbers.
- Estimate and measure with customary units.
- Convert customary units.

1. ADDING AND SUBTRACTING FRACTIONS

Do you remember how to represent a fraction using a rectangle? The denominator of the fraction is the number of pieces that the rectangle is evenly divided into. And, the numerator is the number of pieces that are shaded. For example, to model the fraction $\frac{3}{8}$ we would evenly divide a rectangle into eight pieces and then shade three of them in.



In this lesson, we'll continue exploring fractions and how to add and subtract fractions that have **like denominators**. We'll also use a rectangular model to represent an addition or subtraction problem.

Objectives

Review these objectives. When you have completed this section, you should be able to:

- Add and subtract fractions with like denominators.
- Add and subtract fraction with unlike denominators.
- Add and subtract mixed numbers.
- Rename mixed numbers.
- Subtract with mixed numbers.

Vocabulary

arithmetic sequence. A set of numbers where the difference is the same between any two consecutive terms.

least common denominator. Smallest number that all the given denominators divide into evenly.

like denominators. Denominators that are the same number.

term. A member (or number) of a sequence.

unlike denominators. Denominators that are different numbers.

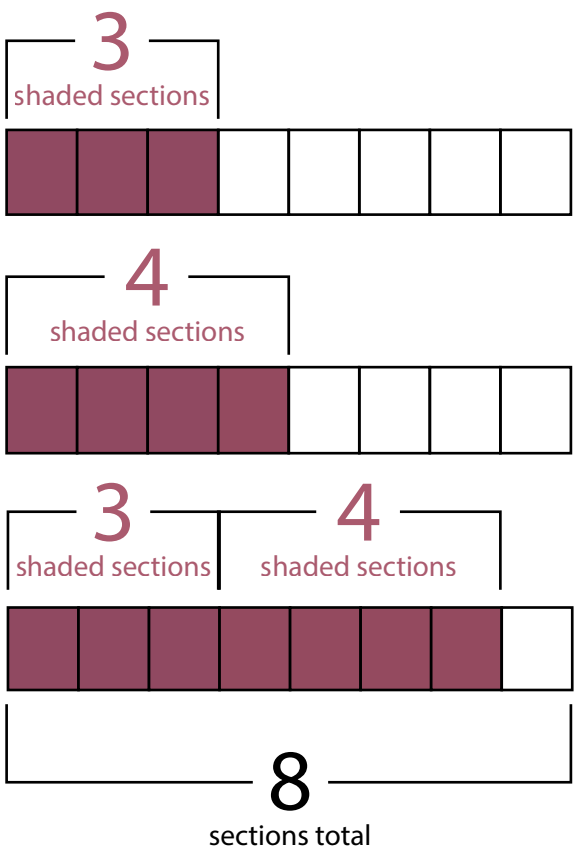
Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

ADDING FRACTIONS WITH LIKE DENOMINATORS

Let's represent the addition problem $\frac{3}{8} + \frac{4}{8}$. We can use a rectangular model.

Since the addends have like denominators, or denominators that are the same number, the pieces from each rectangle will be the same size. We can add them together simply by combining the rectangles. When we combine the rectangles, in the same way as we did with mixed numbers, we end up with seven shaded pieces of a rectangle that has been evenly divided into eight pieces. So, the sum of $\frac{3}{8}$ and $\frac{4}{8}$ is $\frac{7}{8}$.

In short, to add fractions with like denominators, add the numerators and keep the denominator the same.



Example:

Find the sum.

$$\frac{1}{6} + \frac{2}{6}$$

Solution:

The denominators are the same, so we can just add the numerators and keep the denominator the same.

$$\frac{1+2}{6} + \frac{3}{6}$$

Notice that $\frac{3}{6}$ can be reduced. The GCF of 3 and 6 is 3, so divide the numerator and denominator by 3.

$$\frac{3 \div 3}{6 \div 6} + \frac{1}{2}$$

So, the sum of $\frac{1}{6}$ and $\frac{2}{6}$ is $\frac{1}{2}$.

Keep in mind...

Always reduce the sum to lowest terms.

Example:

What is $\frac{4}{7} + \frac{3}{7}$?

Solution:

Add the numerators and keep the denominator the same.

$$\frac{4+3}{7} = \frac{7}{7}$$

The fraction $\frac{7}{7}$ is an improper fraction. Rewrite it as a mixed number. Remember that anytime the numerator and denominator are the same number, the value of the fraction is 1. So, the answer is 1.

Think about it!

Remember that the fraction line means division. When you divide a number by itself, you always get 1.

Example:

Add.

$$\frac{3}{4} + \frac{3}{4}$$

Solution:

Add the numerators and keep the denominator the same.

$$\frac{3+3}{4} = \frac{6}{4}$$

The fraction $\frac{6}{4}$ is an improper fraction. It can be rewritten as a mixed number. So, 4 goes into 6 one time, with a remainder of 2. Therefore, $\frac{6}{4}$ can be rewritten as $1\frac{2}{4}$. Then $\frac{2}{4}$ can be reduced to $\frac{1}{2}$ by dividing the numerator and denominator each by 2. So, in lowest terms, the sum is $1\frac{1}{2}$.

This might help!

To convert an improper fraction to a mixed number, divide the numerator by the denominator. The quotient is the whole number; the remainder is the numerator; and, the denominator stays the same.

SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

Subtracting fractions with like denominators works in the same way. Subtract the numerators and keep the denominator the same.

Example:

Find the difference between $\frac{7}{11}$ and $\frac{2}{11}$.

Solution:

Finding the difference means to subtract.

$$\frac{7 - 2}{11} = \frac{5}{11}$$

Be careful!

When adding and subtracting fractions with like denominators, keep the denominator the same. Do not add or subtract the denominators!

Example:

Mrs. Shoe made an apple pie. She cut the pie into eight pieces and ate one of them, so there is $\frac{7}{8}$ of the pie left. If her children eat five pieces of the pie after dinner, how much is left?

Solution:

Subtract $\frac{5}{8}$ from $\frac{7}{8}$ to find out how much of the pie is left.

$$\frac{7 - 5}{8} = \frac{2}{8}$$

The fraction $\frac{2}{8}$ can be reduced by dividing the numerator and denominator both by 2.

$$\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$

One-fourth of the pie is left.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ Fractions with like denominators have the denominators that are the same number.
- ✓ To add fractions with like denominators, add the numerators and keep the denominator the same.
- ✓ To subtract fractions with like denominators, subtract the numerators and keep the denominator the same.
- ✓ Represent a sum or difference as a mixed number or a fraction in lowest terms.

Write the correct answer on the blank.

1.1 Fractions that have the same denominators are said to have _____ denominators.

Circle the correct letter and answer.

1.2 Find the sum in lowest terms. $\frac{3}{16} + \frac{11}{16}$

a. $\frac{7}{8}$ b. $\frac{14}{16}$ c. $\frac{14}{32}$ d. $\frac{7}{16}$

1.3 Add. Express your answer in lowest terms. $\frac{6}{20} + \frac{14}{20}$

a. $\frac{20}{20}$ b. $\frac{1}{2}$ c. 1 d. $\frac{20}{40}$

1.4 What is $\frac{1}{9} + \frac{2}{9}$?

a. $\frac{1}{3}$ b. $\frac{3}{18}$ c. $\frac{1}{6}$ d. 1

1.5 A recipe calls for $\frac{1}{6}$ of a cup of white sugar and $\frac{3}{6}$ of a cup of brown sugar. What is the total amount of sugar in the recipe?

a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{4}{6}$ d. $\frac{4}{12}$

1.6 Find the difference in lowest terms. $\frac{7}{18} - \frac{3}{18}$

a. $\frac{4}{18}$ b. $\frac{10}{18}$ c. $\frac{4}{0}$ d. $\frac{2}{9}$

1.7 Subtract. Express your answer in lowest terms.

$\frac{15}{19} - \frac{7}{19}$

a. $\frac{8}{19}$ b. $\frac{4}{9}$ c. $\frac{8}{0}$ d. $1\frac{3}{19}$

1.8 What is $\frac{16}{24} - \frac{6}{24}$?

a. $\frac{10}{0}$ b. $\frac{10}{12}$ c. $\frac{11}{12}$ d. $\frac{5}{12}$

1.9 The space between each rail on a deck is $\frac{3}{4}$ of a foot. If the carpenter wants there to be $\frac{1}{2}$ of a foot less space between each rail, what will the new spacing be?

a. $\frac{7}{12}$ b. $\frac{3}{12}$ c. $\frac{1}{4}$ d. $\frac{1}{8}$

Express your answer in lowest terms.

1.10 $\frac{11}{12} + \frac{11}{12} =$ ___

1.11 $\frac{21}{25} - \frac{7}{25} =$ ___

FRACTIONS WITH UNLIKE DENOMINATORS

Adding and subtracting fractions with like denominators is simple. Just add the numerators and keep the denominators the same. Also, you may need to reduce the sum or difference to lowest terms, or convert it to a mixed number.

In this lesson, we'll begin adding and subtracting fractions with unlike (or different) denominators.

LEAST COMMON DENOMINATOR

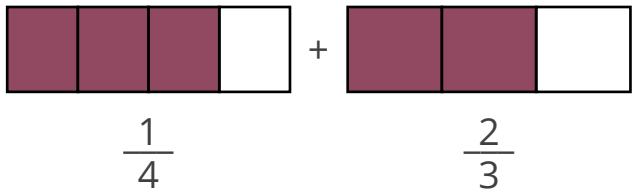
Adding fractions with like denominators is simple because the fractions are already expressed in the same terms. On a rectangular model, that means that the shaded pieces that we're adding together are all the same size. For example, when adding the fractions $\frac{1}{5}$ and $\frac{2}{5}$, we simply add the shaded pieces together to get $\frac{3}{5}$. This works because all the shaded pieces are the same size.

When fractions have different denominators,

though, adding them becomes a little trickier. On a rectangular model, the shaded pieces will be different sizes because the rectangles are each divided into a different number of pieces. In the models on the right, adding $\frac{1}{4}$ and $\frac{2}{3}$ is impossible right now because the shaded pieces are different sizes.

Did you know?

The least common denominator, or LCD, of two fractions is the smallest number that each denominator divides into evenly. The LCD of $\frac{1}{4}$ and $\frac{2}{3}$ is 12 because 12 is the smallest number that 3 and 4 both divide evenly into.



Remember that to compare fractions with unlike denominators, you can express each fraction in the same terms using their least common denominator. Once each fraction is written in the same terms, they can be compared by their numerators. The same is true for adding fractions! Once each fraction is written in the same terms, they can easily be added together.

So, the sum of $\frac{1}{4}$ and $\frac{2}{3}$ is the same as $\frac{3}{12} + \frac{8}{12}$, or $\frac{11}{12}$.

Example:

Add.

$\frac{1}{6} + \frac{1}{2}$

Solution:

First, find the least common denominator of these two fractions. One way to find the LCD is to list some of the multiples of 6 and 2 and identify their smallest common factor.

6: **6**, 12, 18, 24, ...

2: 2, 4, **6**, 8, 10, ...

The LCD is 6. Notice that $\frac{1}{6}$ is already written in terms of the LCD. Rewrite in terms of the LCD.

$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

Now, add the fractions together.

$\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$

The final step is rewrite $\frac{4}{6}$ as a fraction in lowest terms. Divide both the numerator and denominator by 2. So, the sum of $\frac{1}{6}$ and $\frac{1}{2}$ is $\frac{2}{3}$.

Step by Step

To add fractions with unlike denominators, first find the LCD of the fractions. Then, rewrite each fraction using the LCD. Finally, add the fractions and reduce the sum to lowest terms.

Example:

What is $\frac{5}{6} + \frac{4}{9}$?

Solution:

Find the LCD of the two fractions.

6: 6, 12, **18**, 24, ... 9: 9, **18**, 27, 36, ...

Rewrite each fraction using the LCD.

$\frac{5 \times 3}{6 \times 3} = \frac{15}{18}$ $\frac{4 \times 2}{9 \times 2} = \frac{8}{18}$

Add the fractions together.

$\frac{15}{18} + \frac{8}{18} = \frac{23}{18}$

Rewrite the improper fraction as a mixed number. So, 18 goes in 23 one time, with a remainder of 5.

$\frac{23}{18} = 1\frac{5}{18}$

Keep in mind...

Improper fractions should be converted to mixed numbers.

SUBTRACTING FRACTIONS

Subtracting fractions with unlike denominators works in the same way as adding fractions with unlike denominators. Rewrite each fraction using the least common denominator. Then, subtract the fractions and express the answer in lowest terms.

Example:

Find the difference betwen $\frac{2}{5}$ and $\frac{1}{12}$.

Solution:

Find the LCD of the two fractions. Another way to find the LCD is to write the prime factorization of each number. Then, multiply the common prime factors by the remaining factors.

$5 = 5$

$12 = 2 \times 2 \times 3$

In this case, there are no common factors, so multiply all the prime factors together.

$LCD = 5 \times 2 \times 2 \times 3 = 60$

Rewrite each fraction using the LCD.

$$\frac{2 \times 12}{5 \times 12} = \frac{24}{60}$$

$$\frac{1 \times 5}{12 \times 5} = \frac{5}{60}$$

Subtract the fractions.

$$\frac{24}{60} - \frac{5}{60} = \frac{19}{60}$$

There are no common factors between the numerator and denominator, so $\frac{19}{60}$ is already written in lowest terms.

Be careful...

When fractions have unlike denominators, you can't just subtract the numerators. You must rewrite each fraction using their LCD before subtracting.

Example:

Jada picked three-fourths of a pound of berries. Three-tenths of a pound were strawberries and the rest were blueberries. How many pounds of blueberries did she pick?

Solution:

To find the amount of blueberries, subtract the amount of strawberries from the total amount of berries.

$$\frac{3}{4} - \frac{3}{10}$$

Find the LCD of the fractions.

$$4 = 2 \times 2$$
$$10 = 2 \times 5$$
$$\text{LCD} = 2 \times 2 \times 5 = 20$$

Rewrite each fraction using the LCD.

$$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$
$$\frac{3 \times 2}{10 \times 2} = \frac{6}{20}$$

Subtract the fractions.

$$\frac{15}{20} - \frac{6}{20} = \frac{9}{20}$$

Jada picked $\frac{9}{20}$ of a pound of blueberries.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ To add or subtract fractions with unlike denominators, rewrite each fraction using their LCD before adding or subtracting.
- ✓ The LCD can be found by listing the multiples of each denominator or by using their prime factorizations.
- ✓ A sum or difference should be expressed in lowest terms or as a mixed number.

SELF TEST 1: ADDING AND SUBTRACTING FRACTIONS

Answer true or false (each answer, 4 points).

1.01 _____ The least common denominator of $\frac{3}{4}$ and $\frac{1}{6}$ is 24.

1.02 _____ The first term in an arithmetic sequence is $1\frac{1}{5}$, and the second term is $2\frac{3}{5}$. The difference between each pair of consecutive terms in the sequence is $1\frac{2}{5}$.

Circle the correct letter and answer (each answer, 8 points).

1.03 Add. Express your answer in lowest terms.

$\frac{7}{8} + \frac{3}{8}$

a. $\frac{5}{8}$ b. $\frac{10}{16}$ c. $1\frac{1}{2}$ d. $1\frac{1}{4}$

1.04 Subtract. Express your answer in lowest terms.

$\frac{11}{12} - \frac{5}{12}$

a. $\frac{6}{0}$ b. $\frac{1}{2}$ c. $1\frac{1}{3}$ d. $\frac{6}{12}$

1.05 Jenna and her mom are addressing envelopes for Jenna's wedding invitations. So far, Jenna has addressed $\frac{2}{9}$ of the envelopes, and her mom has also addressed $\frac{2}{9}$. What portion of the envelopes have they addressed so far?

a. $\frac{4}{9}$ b. $\frac{4}{18}$ c. $\frac{0}{9}$ d. $\frac{1}{2}$

1.06 Melissa lives $\frac{5}{6}$ of a mile from the bus stop. Pete lives $\frac{2}{3}$ of a mile from the bus stop. How much closer to the bus stop does Pete live than Melissa?

a. $1\frac{1}{2}$ miles b. 1 mile c. $\frac{1}{6}$ of a mile d. $\frac{7}{9}$ of a mile

1.07 Find the sum. Express your answer in lowest terms.

$\frac{3}{8} + \frac{5}{12}$

a. $\frac{2}{5}$ b. $\frac{8}{20}$ c. $\frac{1}{3}$ d. $\frac{19}{24}$

1.08 Find the difference. Express your answer in lowest terms.

$22\frac{1}{3} - 6\frac{3}{4}$

a. $16\frac{1}{2}$ b. $15\frac{5}{12}$ c. $16\frac{5}{12}$ d. $15\frac{7}{12}$

1.09 Angie bought four cherry pies from the bake sale and brought them to work to share. If $1\frac{5}{6}$ of the pies have been eaten, how many are left?

a. $2\frac{5}{6}$ b. $3\frac{5}{6}$ c. $2\frac{1}{6}$ d. $3\frac{1}{6}$

1.010 The difference between each pair of consecutive terms in this arithmetic sequence is $1\frac{1}{4}$. Find the next term in the sequence.

$2\frac{3}{4}$, 4, $5\frac{1}{4}$, $6\frac{1}{2}$, ...

a. $6\frac{3}{4}$ b. $7\frac{3}{4}$ c. $7\frac{1}{3}$ d. $6\frac{1}{3}$

1.011 Brandon is landscaping his backyard. He bought $2\frac{7}{8}$ tons of decorative rock and $1\frac{2}{5}$ tons of boulders. How many total tons of rock did he buy?

a. $4\frac{11}{40}$ b. $3\frac{11}{40}$ c. $3\frac{9}{13}$ d. $3\frac{9}{40}$

Write the correct answers on each blank, and express your answer in lowest terms (each answer, 5 points).

1.012 $\frac{12}{13} - \frac{5}{13} =$ _____

1.013 $\frac{6}{7} + \frac{2}{3} =$ _____

1.014 $8\frac{3}{10} + 7\frac{2}{15} =$ _____

1.015 $10\frac{2}{5} + 3\frac{4}{5} =$ _____

80/100 **SCORE** _____ **TEACHER** _____ _____
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