



MATH

STUDENT BOOK

▶ **6th Grade | Unit 6**

MATH 606

Ratio, Proportion, and Percent

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Ratio, Proportion, and Percent

Introduction

In this unit, you will thoroughly investigate ratios, proportions, and percents. You will learn how to compare quantities using a ratio or rate. You will also learn about equivalent ratios and use a proportion to find a missing value. In addition, you will study percents and learn how to represent a percent as both a fraction and a decimal. You will apply these skills to circles, scale drawings, and circle graphs.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAK. When you have finished this LIFEPAK, you should be able to:

- Compare quantities with ratios.
- Solve problems using a unit rate.
- Find the circumference of a circle.
- Solve proportions.
- Apply proportions to scale drawings.
- Convert decimals to percent and find the percent of a number.
- Interpret circle graphs.

1. RATIOS

Did you know that we can describe a piano keyboard using math? A full-sized keyboard has 36 black keys and 52 white keys. So, the *ratio* of black to white keys is 36 to 52. And, there are a total of 88 keys on a keyboard, so the ratio of black keys to total keys is 36 to 88. The ratio of

white keys to total keys is 52 to 88. There are lots of ratios we can use to describe this one keyboard!

In this lesson, we'll learn more about ratios and how they can be used to solve problems.



Objectives

Review these objectives. When you have completed this section, you should be able to:

- Use a ratio to compare two quantities.
- Express a ratio in lowest terms.
- Use a ratio table to solve a problem.
- Find the circumference of a circle.
- Understand that the ratio of circumference to diameter is pi.
- Determine a unit rate.
- Compare rates.
- Solve problems using a unit rate.

Vocabulary

circumference. The distance around the outside of a circle.

diameter. The distance across a circle through the center.

equivalent ratios. Two ratios that show the same comparison.

formula. An expression that uses variables to state a rule.

pi. The ratio of the circumference of a circle to its diameter; approximately 3.14.

radius. The distance from the center of a circle to any point

rate. A type of ratio that compares two different kinds of quantities or numbers.

ratio. A comparison of two quantities or numbers.

ratio table. A table that shows the relationship between two quantities.

unit rate. A rate with a denominator of 1; a rate that shows an amount of something compared to 1 of something else.

Note: All vocabulary words in this LIFEPAC appear in **boldface** print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.

RATIOS

A **ratio** compares two quantities or numbers. Above, we compared the number of black keys to the number of white keys on a piano keyboard. We also compared the number of black and white keys to the total number of keys on a keyboard. Ratios can be expressed in three different ways—using the word "to," a colon, or the fraction line. Let's look again at the piano keyboard ratios and the different ways we can represent them.

black keys to white keys

$$36 \text{ to } 52 \quad \frac{36}{52} \quad 36:52$$

black keys to total keys

$$36 \text{ to } 88 \quad \frac{36}{88} \quad 36:88$$

white keys to total keys

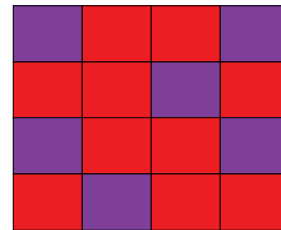
$$52 \text{ to } 88 \quad \frac{52}{88} \quad 52:88$$

Example:

Write a ratio that compares the number of red squares to purple squares.

Solution:

There are 10 red squares and 6 purple squares. So, the ratio of red to purple is 10 to 6. The ratio can also be written as 10:6 or $\frac{10}{6}$.



Be careful!

Notice that we were asked to give the ratio of red to purple squares, not purple to red. The order of the numbers is important!

Example:

There are 14 girls and 16 boys in Jenny's class. What is the ratio of boys to girls? What is the ratio of girls to students in the class?

Solution:

There are 16 boys and 14 girls, so the ratio of boys to girls is 16 to 14.

There are $14 + 16$, or 30 students in Jenny's class. So, the ratio of girls to students is 14 to 30.

Did you know?

The ratio 16 to 14 can also be written as 16:14 or $\frac{16}{14}$. The ratio 14 to 30 can also be written as 14:30 or $\frac{14}{30}$.

SIMPLIFYING RATIOS

You may be wondering how a ratio is different from a fraction. Remember that a ratio compares two quantities, such as 16 boys to 14 girls, or 14 girls to 30 total students. A fraction, however, always tells you how many parts of the whole you have. The denominator of a

fraction is the number of parts in the whole, and the numerator is the number of parts that you have. Also, an improper fraction can be written as a mixed number. But, because a ratio always compares two quantities, a ratio is never written as a mixed number.

Even though ratios and fractions are different, they have some similarities. Just like fractions, ratios can be simplified, or written in lowest terms. Common factors between the numbers in a ratio can be canceled out. For example, in

the previous example, the ratio of boys to girls was 16:14. Each number has a factor of 2 that can be divided out. So, another way to express the ratio is 8:7. For every 8 boys in Jenny's class, there are 7 girls.

Example:

Rewrite the ratio of 36 black keys to 88 total keys in simplest form.

Solution:

36 and 88 have a common factor of 4. So, divide each part of the ratio by 4. In simplest form, 36:88 is 9:22.

RATIO TABLES

Many problems can be solved using ratios. A **ratio table** is a helpful way to record the relationship between two quantities.

Let's look at an application of ratios and use a ratio table to help us solve the problem.

Example:

At Jefferson Middle School, the ratio of students to teachers is 15:1. If there are 180 students, how many teachers are there?

Solution:

A student to teacher ratio of 15:1 means that for every 15 students, there is 1 teacher. Look at the table below.

Students	15×2	30×2	60×3	180
Teachers	1×2	2×2	4×3	12

The top row shows the number of students and the bottom row shows the number of teachers. The first column shows the ratio: 15 students to 1 teacher.

Now, if there is one teacher for fifteen students, then there are two teachers for thirty students. The second column shows each quantity multiplied by two.

As long as we multiply or divide each quantity by the same number, the table will be accurate. So if there are 60 students (30×2) then there are 4 teachers (2×2).

But remember what our goal is. We want to find the number of teachers there are for one hundred and eighty students. We're using the table to show the relationship between the quantities but also to get to one hundred and eighty. If we multiply sixty by three, we'll get one hundred and eighty. We also have to multiply four by three.

So, for one hundred and eighty students, there are twelve teachers.

Example:

A person weighing 150 pounds on Earth weighs 25 pounds on the moon. If a person weighs 120 pounds on Earth, how much does she weigh on the moon?

Solution:

Let's use a ratio table again to help us show the relationship between what a person weighs on Earth and on the moon.

Earth weight	$150 \div 5$	30×2	60×2	120
Moon weight	$25 \div 5$	5×2	10×2	20

The top row shows Earth weight and the bottom row shows moon weight, and the first column shows the given ratio: 150 to 25.

Remember that we can only multiply and divide to put new quantities in the ratio table. Since 150 and 25 are larger numbers, let's start by dividing each quantity.

150 divided by 5 is 30 and 25 divided by 5 is 5. So the second column shows 30 and 5.

Then we multiply 30 and 5 by 2 to get 60 and 10.

Multiplying by 2 again gives us 120 and 20.

So a person who weighs 120 pounds on Earth weighs 20 pounds on the moon.

Here are some tips for using ratio tables:

- Only multiply or divide to enter new quantities into the ratio table.
- Try dividing if you need to get a smaller quantity.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ A ratio compares two quantities.
- ✓ Ratios can be expressed using the word "to," a colon, or the fraction line.
- ✓ Although different than fractions, ratios can be simplified in the same way as fractions.
- ✓ A ratio table shows the relationship between two quantities and can be used to solve problems.



Match the following items.

- 1.1 _____ two ratios that show the same comparison
 _____ a comparison of two quantities or numbers
 _____ a table that shows the relationship between two quantities
- a. equivalent ratios
 b. ratio table
 c. ratio

Use the word **MISSISSIPPI** to match each ratio in the following exercise.

- 1.2 _____ the ratio of S's to P's 4:2
 _____ the ratio of I's to M's 4:4
 _____ the ratio of S's to I's 11:1
 _____ the ratio of P's to letters 2:11
 _____ the ratio of letters to M's 4:1

Answer true or false.

- 1.3 _____ 3:5, 3 to 5, and $\frac{5}{3}$ all represent the same ratio.

Circle each correct answer.

- 1.4 Shiloh's basketball team won 9 games this season and lost 5 games. Write the win-loss ratio in simplest form.
 a. 5:9 b. 9:5 c. 5:1 d. 1:5
- 1.5 Shiloh's basketball team won 9 games this season and lost 5 games. Write the ratio of wins to total games in simplest form.
 a. $\frac{5}{9}$ b. $\frac{9}{5}$ c. $\frac{5}{14}$ d. $\frac{9}{14}$
- 1.6 Misty has 10 shirts and 4 pairs of pants. Write the ratio of her shirts to pants in simplest form.
 a. 10 to 4 b. 2 to 5 c. 5 to 2 d. 4 to 10
- 1.7 Write the ratio of dogs to cats in simplest form.
 12 dogs, 16 cats
 a. 4 to 3 b. 3 to 4 c. 6 to 8 d. 8 to 6
- 1.8 Write the ratio of bikes to cars in simplest form.
 35 bikes, 14 cars
 a. $\frac{5}{2}$ b. $\frac{2}{5}$ c. $\frac{35}{14}$ d. $\frac{7}{3}$

Use Figure 1 for questions 1.9 – 1.11.

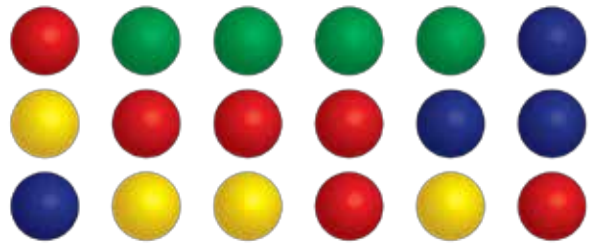


Figure 1

1.9 What is the ratio of red gumballs to yellow gumballs in lowest terms?

- a. 2:3
- b. 3:2
- c. 2:1
- d. 1:2

1.10 What is the ratio of green gumballs to blue gumballs in lowest terms?

- a. 4:4
- b. 2:2
- c. 1:1
- d. 3:2

1.11 What is the ratio of yellow gumballs to total gumballs in lowest terms?

- a. $\frac{4}{18}$
- b. $\frac{4}{14}$
- c. $\frac{2}{7}$
- d. $\frac{2}{9}$

1.12 The ratio of roses to carnations is 7 to 5. If there are 28 roses, how many carnations are there? Complete the ratio table to find the number of carnations.

- a. 10
- b. 20
- c. 30
- d. 40

Roses	7	14	28
Carnations	5		

1.13 The ratio of adults to children is 16 to 10. If there are 40 adults, how many children are there? Complete the ratio table to find the number of children.

- a. 25
- b. 20
- c. 15
- d. 30

Adults	16	8	40
Children	10		

GEOMETRY: CIRCUMFERENCE

Do you know what circumference is? How about diameter? Radius? All three of these terms represent measurements on a circle. Take a look.

In this lesson, we'll explore how to find the circumference of a circle. We'll also learn what ratios have to do with geometry!

RATIO OF CIRCUMFERENCE: DIAMETER

Circles have a special property: the ratio of the circumference of a circle to its diameter is always the same. Remember that a ratio is a comparison of two numbers as a quotient. So, the quotient of the circumference and the diameter (or circumference \div diameter) is the same value in every circle! Let's look at two circles that you are probably very familiar with—the penny and dime.

Every type of coin has the exact same measurements. The penny always has a diameter of $\frac{3}{4}$ of an inch and a circumference of $2\frac{5}{14}$ inches.

And, the dime always has a diameter of $\frac{141}{200}$ of an inch and a circumference of $2\frac{151}{700}$ inches.

Now, let's try dividing the circumference by the diameter for both the penny and the dime. We'll have to remember what we've learned about dividing with fractions to help us!

Penny:

Divide the circumference by the diameter.

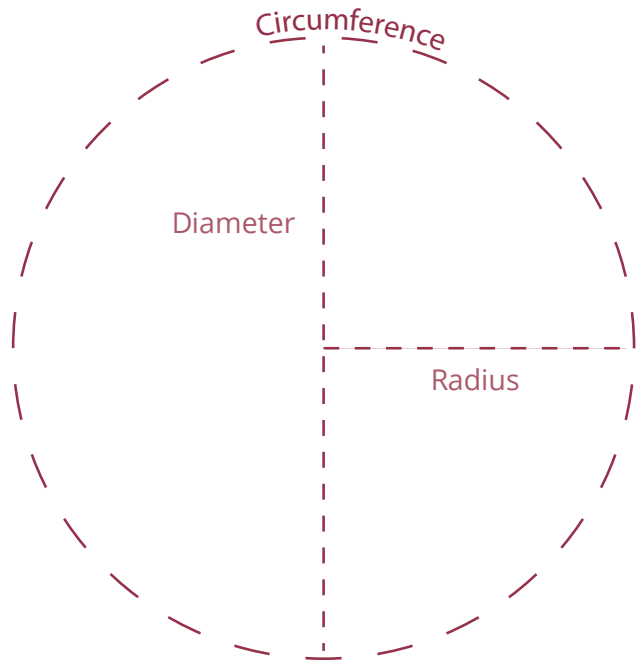
$$2\frac{5}{14} \div \frac{3}{4}$$

Rewrite the mixed number as an improper fraction.

$$\frac{33}{14} \div \frac{3}{4}$$

Multiply by the reciprocal of the divisor.

$$\frac{33}{14} \times \frac{4}{3}$$



$$C = 2\frac{5}{14} \text{ in.}$$

$$d = \frac{3}{4} \text{ in.}$$



$$C = 2\frac{151}{700} \text{ in.}$$

$$d = \frac{141}{200} \text{ in.}$$

Cancel common factors between the numerator and denominator.

$$\frac{33^{\cancel{11}}}{14^{\cancel{7}}} \times \frac{\cancel{2}^2 4}{\cancel{1}_3}$$

Simplify.

$$\frac{22}{7}$$

The ratio of the circumference of a penny to its diameter is $\frac{22}{7}$.

Dime:

Divide the circumference by the diameter.

$$2 \frac{151}{700} \div \frac{141}{200}$$

Rewrite the mixed number as an improper fraction.

$$\frac{1551}{700} \div \frac{141}{200}$$

Multiply by the reciprocal of the divisor.

$$\frac{1551}{700} \times \frac{200}{141}$$

Cancel common factors between the numerator and denominator.

$$\frac{\cancel{1551}^{\cancel{11}}}{700^{\cancel{7}}} \times \frac{\cancel{200}^2}{\cancel{141}_1}$$

Simplify.

$$\frac{22}{7}$$

SOLVING FOR CIRCUMFERENCE

Since the ratio between a circle's circumference and diameter is always Pi, if we know the diameter of a circle, we can find its circumference. To find the circumference of a circle, simply multiply its diameter by Pi. If the diameter is

Keep in mind...

To divide with mixed numbers and fractions, begin by expressing each mixed number as an improper fraction. Then, multiply the dividend by the reciprocal of the divisor. Cancel any common factors between the numerator and denominator so that the quotient is written in simplest form.

The ratio of the circumference of a dime to its diameter is also $\frac{22}{7}$! Since this ratio is the same for every circle, it has its own name: Pi (pronounced "pie"). The fraction $\frac{22}{7}$ or the decimal 3.14 are used to represent Pi.

Vocabulary

The rule for finding the circumference of a circle can be written as a formula, which uses variables and symbols. The formula for finding the circumference of a circle is $C = \pi \times d$, which means that the circumference equals Pi times the diameter.

given as a fraction, use $\frac{22}{7}$ for Pi. If the diameter is given as a decimal number, use 3.14 for Pi.

Here are a couple of examples.

SELF TEST 1: RATIOS

Answer true or false (each answer, 5 points).

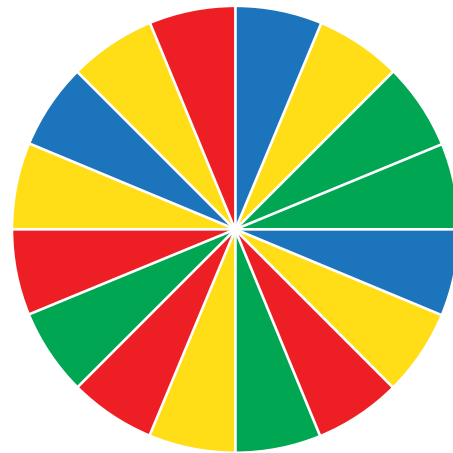
- 1.01** _____ In lowest terms, the ratio 10:4 is written as 5:1.
- 1.02** _____ The circumference of a circle is found by multiplying Pi by the diameter.

Circle each correct answer (each answer, 6 points).

- 1.03** The distance from the center of a circle to any point on the circle is called the _____.
 a. radius b. circumference c. diameter
- 1.04** All of the following represent the same ratio except _____.
 a. 4:3 b. $\frac{4}{3}$ c. 3 to 4

Use this pie chart to answer questions 1.05 and 1.06.

- 1.05** What is the ratio of yellow sections to blue sections, in lowest terms?
 a. 5 to 3 b. 3 to 4
 c. 3 to 5 d. 1 to 2
- 1.06** What is the ratio of red sections to total sections, in lowest terms?
 a. $\frac{3}{1}$ b. $\frac{1}{3}$
 c. $\frac{4}{1}$ d. $\frac{1}{4}$
- 1.07** If the radius of a circle is 14 ft, then the diameter of the circle is _____.
 a. 7 ft b. 28 ft
 c. 42 ft d. 87.92 ft



- 1.08** Find the circumference of a circle that has a diameter of 9 mm.
 a. $28\frac{2}{7}$ mm b. $27\frac{2}{7}$ mm c. $56\frac{4}{7}$ mm d. $14\frac{1}{7}$ mm
- 1.09** The radius of a circle is 11 ft. Find the circumference of the circle.
 a. 17.27 ft b. 34.54 ft c. 69.08 ft d. 25.14 ft
- 1.010** Express the following rate as a unit rate.
 \$5.75 for 5 attempts
 a. \$1.25 per attempt b. \$1.75 per attempt
 c. \$0.75 per attempt d. \$1.15 per attempt

1.011 Given the chart on the right, which student types the most words per minute?

- a. Ben
- b. Kevin
- c. Lucy
- d. Reese

Student	Typed words	Minutes
Ben	240	5
Kevin	270	6
Lucy	156	3
Reese	50	1

1.012 A train travels 150 miles in 3 hours. At this same rate, how many miles will it travel in 8 hours?

- a. 400 miles
- b. 450 miles
- c. 500 miles
- d. 350 miles

Fill in each blank with the correct answer (each answer, 10 points).

1.013 In a crowd, the ratio of men to women is 5 to 6. If there are 90 men, how many women are there? Use the ratio table to help you find the number of women.

Men	5	10	30	90
Women	6	12		

_____ women

1.014 Estimate the circumference of a circle that has a diameter of 13 yards.

_____ yd

1.015 Express the following rate as a unit rate.

63 jumping jacks in 3 minutes

_____ jumping jacks per minute

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80	100						



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