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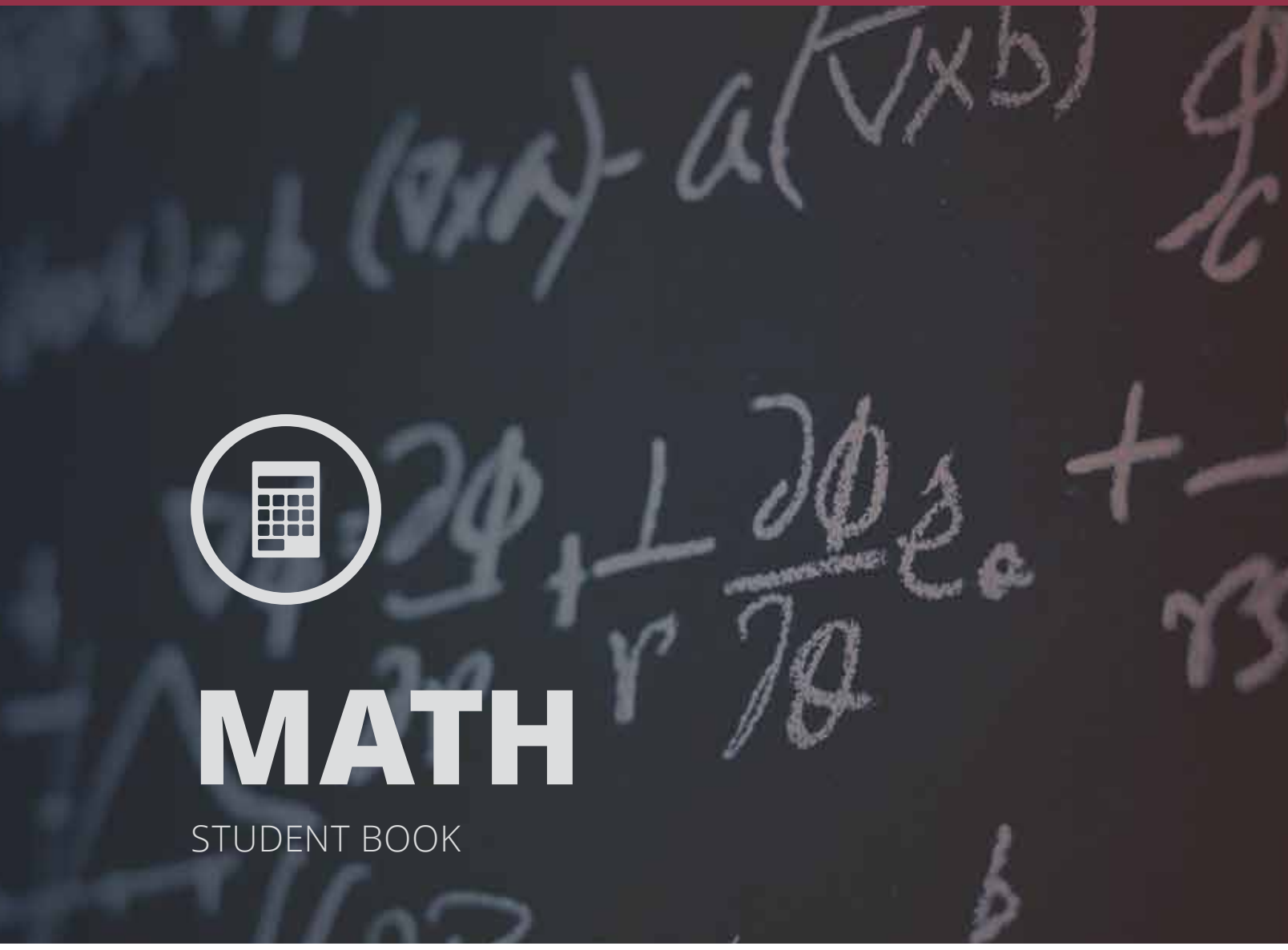

MATH

STUDENT BOOK

6th Grade | Unit 10

Alpha Omega
PUBLICATIONS

MATH 610

Equations and Functions

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Equations and Functions

Introduction

In this workbook, you will be introduced to the topic of equations and functions. You will begin by exploring equations with one variable and substituting solution choices into the equation. You will learn to solve one-step equations for each of the four basic operations (addition, subtraction, multiplication, and division) by using inverse operations. You will also learn to solve inequalities and graph their solutions on a number line. For both equations and inequalities, you will learn to translate between words and mathematical language.

Next, you will explore equations with two variables, called functions. You will find that each input of a function has one output. You will also represent functions using an input/output table. You will be able to solve for each of the three parts of a function (input value, output value, and function rule), given two of the three parts. Finally, you will graph functions in the coordinate plane and begin to see the relationship between input and output values in a function.

As you move on in mathematics, you will use equations, inequalities, and functions frequently. This unit will be an introduction to an important area of your mathematical future.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Determine if a given value is a solution of a one- or two-step equation.
- Translate and write one- and two-step equations and inequalities.
- Solve one-step addition, subtraction, multiplication, and division equations using inverse operations.
- Determine if a given value is a solution of a one- or two-step inequality.
- Graph inequality statements.
- Given two of the following: the function rule, an output value, and an input value; find the third.
- Graph functions on a coordinate plane.

1. EQUATIONS

$4 + 3x = 34$

Can you find the *solution* for this *equation*? What value of x will make the equation true? Is the value of x equal to 7? Given the following choices: 7, 11, 6, and 10, which number is the correct value of x?

In this unit, you will learn more about equations and solve problems like the one above. In this lesson, you will explore equations and one way to find their solutions.

Objectives

Review these objectives. When you have completed this section, you should be able to:

- Determine if a given value is a solution of a one- or two-step equation.
- Translate and write one- and two-step equations.
- Solve one-step addition equations using inverse operations.
- Solve one-step subtraction equations using inverse operations.

Vocabulary

commutative property. A property of the real numbers that states that the order in which numbers are added or multiplied does not change the value.

equation. A mathematical statement that shows two expressions are equal using an equal sign.

inverse operations. Opposite operations that undo one another.

order of operations. A system for simplifying expressions that ensures that there is only one right answer.

solution. A value of the variable that makes an algebraic sentence true.

substitute. To replace a variable in a mathematical expression with an actual value.

variable. A letter used to represent an unknown number or quantity.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

An equation is formed when two expressions are equal to each other. If 3 times some number is 15, then $3n = 15$. The **variable** in the equation is n because it represents the value of the unknown number. The variable can be any letter of the alphabet.

The **solution** to an equation is the value of the variable that makes the equation true. In the equation $3n = 15$, if n is 5 ($3 \times 5 = 15$), the equation is true, so 5 is the solution.

Many equations can be solved using mental math. You probably knew the solution to the equation above as soon as you saw it. Mental math is a useful tool, but in this unit we will explore other methods to solve equations, especially when mental math is not the fastest way to solve the equation.

One method we can use to solve an equation is to **substitute** a value for the variable, and see if the equation is true. This may not seem like an efficient way to solve a problem, but if we are given a list of choices for the solution, it can be very efficient.

Let's try one together.

Keep in mind...

If a number is shown next to a variable, it indicates multiplication. $3n$ is 3 multiplied by n.

Test Tip

Tests are often multiple choice, and are often timed. So, if you are asked to solve an equation, you are given a list of choices for the solution. Substituting each choice into the equation is one strategy to quickly solve the problem.

Example:

Given the following choices: 7, 5, 10, and 9, what is the solution to the equation $12n = 108$?

Solution:

We can substitute each solution choice for the variable n into the equation and see which one makes the equation true.

$n = 7$	$n = 5$	$n = 10$	$n = 9$
$12 \times 7 = 84$	$12 \times 5 = 60$	$12 \times 10 = 120$	$12 \times 9 = 108$
$84 \neq 108$	$60 \neq 108$	$120 \neq 108$	$108 = 108$

If n is 9, the equation is true, so the solution is 9.

Think about it!

Sometimes we can narrow down the choices for the solution if we look at the results of a solution choice. Did you notice that when n was 7, the result was too low (84, compared to 108)? Since we are multiplying, this means that a smaller number would give a smaller result, so 5 could be eliminated as a solution choice because it is less than 7. The solution must be larger than 7.

Example:

Given the following choices: 37, 42, 48, and 65, what is the solution to the equation $x + 13 = 61$?

Solution:

We will substitute each solution choice for the variable x into the equation and see which one makes the equation true.

$x = 37$	$x = 42$	$x = 48$	$x = 65$
$37 + 13 = 50$	$42 + 13 = 55$	$48 + 13 = 61$	$165 + 13 = 78$
$50 \neq 61$	$55 \neq 61$	$61 = 61$	$78 \neq 61$

If x is 48, the equation is true, so the solution is 48.

Think about it!

As you substitute the choices for the variable, think about the numbers: 65 could not have been the solution because it's already larger than the sum of 61, and then 13 is added to it! Sometimes, if we stop and examine the numbers, choices can be eliminated without even trying them in the equation because it becomes obvious that they don't make sense.

Example:

Given the following choices: 8, 12, 16, and 24, what is the solution to the equation $48 \div s = 3$?

Solution:

We will substitute each solution choice for the variable s into the equation and see which one makes the equation true.

$s = 8$	$s = 12$	$s = 16$	$s = 24$
$48 + 8 = 6$	$48 \div 12 = 4$	$48 \div 16 = 3$	$48 \div 24 = 2$
$6 \neq 3$	$4 \neq 3$	$3 = 3$	$2 \neq 3$

If s is 16, the equation is true, so the solution is 16.

Example:

Let's take a look at the equation at the start of the lesson: $4 + 3x = 34$. This problem is a little more complicated because it involves two steps: addition and multiplication.

Given the following choices: 7, 11, 6, and 10, what is the solution to the equation $4 + 3x = 34$?

Solution:

We will substitute each solution choice for the variable x into the equation and see which one makes the equation true.

$x = 7$	$x = 11$	$x = 6$	$x = 10$
$4 + 3(7) = 34$	$4 + 3(11) = 34$	$4 + 3(6) = 34$	$4 + 3(10) = 34$
$4 + 21 = 34$	$4 + 33 = 34$	$4 + 18 = 34$	$4 + 30 = 34$
$25 \neq 34$	$37 \neq 34$	$22 \neq 34$	$34 = 34$

If x is 10, the equation is true, so the solution is 10.

As you worked through the choices, did you notice that 7 gave a result that was too small ($25 < 34$) and 11 gave a result that was too large ($37 > 34$)? This meant the solution had to be greater than 7, but less than 11, leaving 10 as the only choice.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ The solution to an equation is the value of the variable that makes the equation true.
- ✓ Substituting values for the variable, especially if a list of choices is given, is one strategy to solve an equation.

Match the following items.

1.1

_____ a mathematical statement that shows two expressions are equal using an equal sign

_____ a system for simplifying expressions that ensures that there is only one right answer

_____ a value of the variable that makes an algebraic sentence true

_____ to replace a variable in a mathematical expression with an actual value

_____ a letter used to represent an unknown number or quantity

a. substitute
b. equation
c. solution
d. variable
e. order of operations

Circle the letter of each correct answer.

1.2 What is the solution to the equation $4x = 44$??
a. $x = 4$ b. $x = 10$ c. $x = 11$ d. $x = 40$

1.3 What is the solution to the equation $x + 16 = 64$?
a. $x = 4$ b. $x = 32$ c. $x = 36$ d. $x = 48$

1.4 For which equation would $x = 5$ be a solution?
a. $x + 4 = 54$ b. $45 \div x = 9$ c. $x - 7 = 12$ d. $8x = 85$

1.5 For which equation would $x = 12$ be a solution?
a. $x + 4 = 12$ b. $48 \div x = 12$ c. $12 - x = 4$ d. $12x = 144$

1.6 What is the solution to the equation $3x - 8 = 22$?
a. $x = 10$ b. $x = 8$ c. $x = 6$ d. $x = 0$

1.7 What is the solution to the equation $5 + 9x = 50$?
a. $x = 1$ b. $x = 4$ c. $x = 5$ d. $x = 6$

1.8 For which equation would $x = 4$ be a solution?
a. $2x + 7 = 22$ b. $6x \div 8 = 3$ c. $8 - 3x = 20$ d. $2x + 8 = 4$

1.9 For which equation would $x = 12$ not be a solution?
a. $5 + 4x = 53$ b. $9x - 7 = 101$ c. $x + 4 = 10$ d. $96 \div x = 8$

WRITING EQUATIONS

Bob earns $400 in one week of work. He works five days and earns the same amount of money each day, plus a $50 bonus on Friday. How much does he earn each day?

The first step in solving this problem is to translate the words into an equation. Then the equation can be solved. In this lesson we will learn the first step in solving these kinds of problems: translating them into equations.

Math is a language that can be translated like any other language. You already know how to translate the language of words into a mathematical expression. In this lesson, you'll learn to translate from words to a mathematical *equation*.

The first part of mathematical language is numbers. You know how to translate between numbers and words:

Words	Number
three	3
forty-seven	47
one thousand eight	1,008

You have learned several words and phrases that indicate each of the four operations. Here is a reminder of those words and phrases:

Addition	Subtraction	multiplication	division	equal
plus	minus	times	divided by	is
sum	difference	product	quotient	equal to
increased by	decreased by	multiplied by	share	equals
added to	subtracted from	each		equivalent

We can refer to theses words and phrases to translate words into mathematical statements. For example, the phrase, "The sum of three and five," means 3 + 5.

An equation is different from an expression because it is a mathematical statement where two expressions are equal. The symbol we use to indicate equality is the equal sign (=). Here are words and phrases that indicate "equal":

Each of the following statements translates to 3 + 5 = 8:

The sum of 3 and five is eight.
The sum of three and five is equal to eight.
The sum of three and five equals eight.
3 added to five is equivalent ot eight.

We use variables in an equation when we need to solve for an unknown quantity. In words, the variable is the amount we want to solve for. We can use any letter to represent a variable in an equation.

Here are a few examples:

words	equation
The sum of **what number** and three is eight?	$x + 3 = 8$
What number subtracted from 12 is 4?	$12 - z = 4$
The product of 6 and 7 is **what number**?	$6 \times 7 = p$

So, to translate written or verbal language into mathematical equations, we need to translate numbers, symbols, and variables, just as we did with mathematical expressions. Let's try some examples.

Be careful!

Subtraction is not commutative, meaning that the order of the numbers changes the expression. "What number subtracted from 12" does not translate as $z - 12$. The unknown number is subtracted *from* 12: $12 - z$.

Example:

Translate the following mathematical equation into written language:

$5x + 3 = 13$

Solution:

In this equation, x is an unknown number multiplied by 5. We can translate each part of the equation to form one sentence:

$5x$	Five times a number ...
$+$	plus ...
3	three ...
$=$	is ...
13	thirteen.

Five times a number, plus three, is thirteen.

Example:

Translate the following statement into a mathematical equation:

Six less than the product of a number and seven is eight.

Solution:

We can translate each part of the statement, and then form the equation:

Six less ...	- 6
than the product ...	×
of a number ...	n
and seven ...	7
is	=
eight ...	8

Combining each part, we get:

six less than ...	- 6
the product of a number and seven ...	$7n$
is ...	=
eight.	8

So, the statement: Six less than the product of a number and seven is eight, translates to the equation $7n - 6 = 8$.

Example:

Translate the following mathematical equation into words.

$6 + \frac{d}{4} = 9$

Solution:

In this equation, d is an unknown number divided by 4. We can translate each part of the equation to form one sentence:

6	six ...
+	plus ...
$\frac{d}{4}$	a number divided by four ...
=	is ...
9	nine

Six, plus a number divided by four, is nine.

Did you know?

The placement, or absence, of a comma can change the translation of an equation:

Six, plus a number divided by four, is nine: $6 + \frac{d}{4} = 9$

Six plus a number, divided by four, is nine: $\frac{6 + d}{4} = 9$

With no comma(s), the meaning is unclear. Six plus a number divided by four, is nine.

Example:

Let's take a look at the problem at the start of the lesson. Bob earns $400 in one week of work. He works five days and earns the same amount of money each day, plus a $50 bonus on Friday. How much does he earn each day? Write an equation to represent the situation.

Solution:

We need to find the amount of money Bob earns each day, so this is the unknown number. Let's call it m.

The problem describes how much Bob earns in a week, and this is $400, so the right side of the equation will be: = 400.

We can translate each part of the rest of the problem:

He works five days ...	5
same amount each day ...	$5m$
plus $50	$5m + 50$

$5m + 50$ represents how much Bob earns in a week, and we know that that amount is $400. So, the equation is $5m + 50 = 400$.

SELF TEST 1: EQUATIONS

Circle the correct letter and answer (each answer, 6 points).

1.01 Translate the following statement into a mathematical equation:
Six, plus four times a number, is eighteen.

a. $(6 + 4)n = 18$ b. $4n + 18 = 6$ c. $6 + 4n = 18$ d. $6 + 4 + n = 18$

1.02 What is the solution to the equation $\frac{x}{3} + 5 = 8$?

a. $x = 1$ b. $x = 6$ c. $x = 7$ d. $x = 9$

1.03 What is the solution to the equation $2x + 4 = 24$?

a. $x = 1$ b. $x = 3$ c. $x = 6$ d. $x = 10$

1.04 For which equation would $x = 4$ be a solution?

a. $4x + 7 = 23$ b. $4x \div 8 = 8$ c. $8 - 5x = 1$ d. $7 + 3x = 18$

1.05 Which equation is the correct translation of the following statement?
Four less than a number is nine.

a. $4 - x = 9$ b. $9 - 4 = x$ c. $9 - x = 4$ d. $x - 4 = 9$

1.06 Which statement is the clearest translation of $4j - 9 = 1$?

a. A number, times four minus nine, is one.
b. A number times, four minus, nine is one.
c. A number times four, minus nine, is one.
d. A number times four minus nine is one.

1.07 What should be done to solve the following equation?
$c - 7 = 0$

a. Add 7.
b. Subtract 0 from both sides.
c. Add 7 to both sides.
d. Subtract 7 from both sides.

1.08 What is the solution to the equation $x + 5.7 = 6.1$?

a. $x = 0.4$ b. $x = 0.6$ c. $x = 1.4$ d. $x = 11.8$

1.09 For which equation would $x = 1$ be a solution?

a. $x - 3 = 4$ b. $6 + x = 4$ c. $x + 8 = 10$ d. $x - 1 = 0$

1.010 For which equation is 0 *not* a solution?

a. $x + 9 = 9$ b. $6x + 4 = 4$ c. $3x = 0$ d. $x - 5 = 4$

1.011 What is the solution to the equation $2\frac{2}{5} + r = 5\frac{2}{5}$?

a. $r = 7\frac{4}{5}$ b. $r = 3\frac{3}{5}$ c. $r = 3\frac{2}{5}$ d. $r = 3$

1.012 Marisa has 37 jelly beans. She eats some and has 16 left. How many jelly beans did she eat?

a. 53 b. 37 c. 21 d. 16

1.013 What number added to four, is twelve?

a. 4 b. 8 c. 12 d. 16

1.014 What is the inverse operation for addition?

a. multiplication b. subtraction c. division d. addition

1.015 A number divided by 4, minus 3 is 0. What is the number?

a. 0 b. 4 c. 8 d. 12

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