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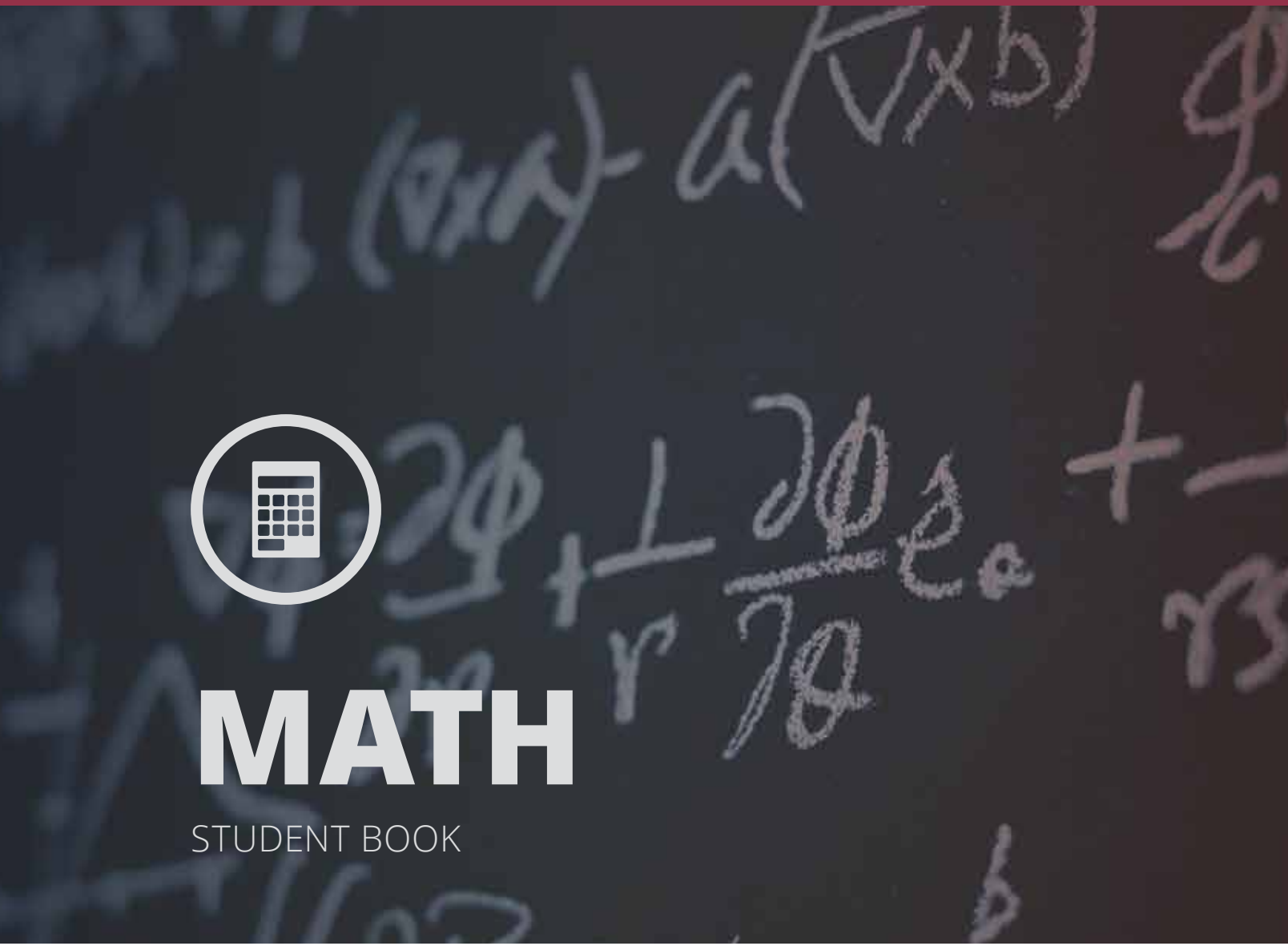

MATH

STUDENT BOOK

7th Grade | Unit 4

Alpha Omega
PUBLICATIONS

Math 704

Patterns and Equations

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Patterns and Equations

Introduction

In this unit, students will start exploring the world of algebra. They will use variables to represent unknown numbers and learn how to set up and solve equations and inequalities in order to find the value of those unknown numbers. They will also learn how to represent solutions on a graph. In addition, they will explore sequences and functions and how equations can be used to represent them.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Use variables to represent unknown numbers.
- Translate between word phrases or sentences and mathematical expressions, equations, or inequalities.
- Evaluate expressions and formulas for specific values.
- Identify arithmetic and geometric sequences and the equations that can be used to describe them.
- Identify functions and function rules.
- Solve equations using the four operations.
- Solve inequalities and graph the solution sets on a number line.

1. Variable Expressions

Working with Variables and Expressions

In Ondi and Carlton's health class, they have to keep track of how many calories they eat each day for a week. Ondi forgot to record everything she ate yesterday, so now she has to go back and try to remember. Unfortunately, she's having a hard time!

Objectives

- Use a variable to represent an unknown number.
- Translate a word phrase into a mathematical expression.

Vocabulary

constant—a number; a term containing no variables
expression—a single term; multiple terms connected by an addition or subtraction sign
term—a number, a variable, or the product of a number and variable(s)
variable—a letter used to represent an unknown number

Terms and Expressions

Sometimes in math, there are situations in which you have to work with unknown numbers or numbers whose values can change. For example, in the cartoon, Ondi couldn't remember how many cookies she ate. But her total number of daily calories depended on the number of cookies she ate. So Carlton recommended that Ondi use the letter c to represent the number of cookies. That letter is called a *variable*. A variable is a letter that is used to represent an unknown number.

There are a few other vocabulary words you should look at before you continue working with variables. From the situation above, $60 \cdot c$ is called a *term*. A term can be made up of one number, also called a *constant*, or one variable. It can also be the product of a number and variables, like the term $60 \cdot c$. This could also be expressed as $60c$.

Key point! When a number and a variable, or multiple variables, are written next to each other, it means that they are multiplied together. So 12mn is the same as 12 · m · n.

Example:

- Translate the following phrase to a mathematical expression.
- a number increased by nine

Solution:

- Use the variable *n* to represent "a number." "Increased by nine" translates to addition, or + 9. So the phrase can be written as *n* + 9.
- Here are some more examples of terms.

Examples:

- 4*x*
- -9
- *y*
- 12*mn*

Make note! Notice in the terms that have both a number and a variable that the number comes first. Also, multiple variables are usually written in alphabetical order.

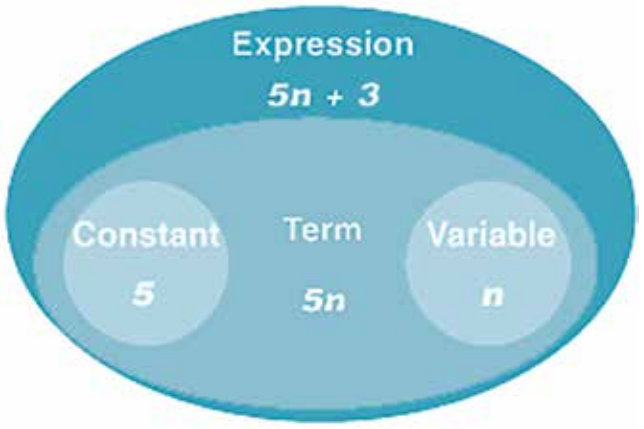
An *expression* is formed when multiple terms are connected by an addition or subtraction sign. One term by itself can also be called an expression. So actually, constants, variables, and terms are all considered expressions! Here are some examples of expressions.

Examples:

- 5
- -*n*
- 3*a*
- 12*b* + 3
- 8*g* - 2*h*

You can use a Venn diagram to show the relationship between constants, variables, terms, and expressions. Remember that in a Venn diagram, a smaller circle inside a larger circle means that every part of the smaller circle is also part of the larger circle. So every constant or variable is also considered a term. And every term is also considered an expression.

Vocabulary! A constant is any term that is just a number. A variable is any term that is just a letter.



Vocabulary! A constant is any term that is just a number. A variable is any term that is just a letter.

Translating Terms and Expressions

Did you know that math has its own language? That might sound strange, but if you think about it, math uses numbers, symbols, and variables to mean specific things, just like other languages. Being able to communicate using mathematical language is important, just like it is important to be able to communicate using verbal language. That means that you often

need to translate between word phrases and mathematical expressions in order to solve problems. Take a look at the three parts of mathematical language and ways to translate them.

The first part of mathematical language is numbers. You've been working with numbers and translating written numbers into numerals for a long time. For example, if you saw the phrase "thirty-seven," you would know that it could also be represented as "37."

The second part of mathematical language is symbols. Symbols are used to represent things like the four basic operations. Often, key words can help you in the translation of symbols. For example, you are already familiar with the terms *sum*, *difference*, *product*, and *quotient*. These are key words that tell you what operation to use. There are many other key words that can help you, too. Take a look at Figure 1, which lists words and phrases that indicate the different operations.

The final part of mathematical language is variables. Earlier in this lesson, you learned that variables represent unknown numbers. When translating, a variable is used to represent any part of the problem or phrase that you don't know the exact value of. For example, the phrase "a number" is often used to represent the unknown part and can be translated as the variable.

This might help! It really doesn't matter which letter you use as the variable. However, it is usually helpful to use a letter that matches the situation. Remember Carlton in the cartoon? He used the letter *c* to represent the unknown number of cookies that Ondi had eaten.

Look at some examples.

Example:

- Translate the following phrase to a mathematical expression.
- five less than twice a number

This might help! The words *from* and *than* tell you that you need to switch the order of the terms in the expression. For example, even though the word *five* comes first in the word phrase, the word *than* means that 5 will come after $2n$ in the expression. Since subtraction is not commutative, this will make a big difference!

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)
sum	difference	product	quotient
increased by	decreased by	times	divided by
more than	less than	of	per
more	less	multiplied by	
added to	subtracted from	by	
plus	minus		
greater than	fewer than		

Figure 1 | Words and Phrases that indicate operations

Solution:

- Use the variable *n* to represent "a number." Twice something means to multiply by two, so twice a number is $2 \cdot n$, or $2n$. "Five less than" translates to subtraction, or - 5. So the phrase can be rewritten as $2n - 5$.

Example:

- Translate the following phrase to a mathematical expression.
- six more than half a number

This might help! The word *half* tells you that you need divide the tem by 2. Set up a fraction with the term as the numerator and the number 2 as the denominator. If the question asked for a third, you would put a 3 in the denominator. A fourth would be a 4 in the denominator, etc.

Solution:

- Use the variable *n* to represent "a number." Half of something means to divide by two, so half a number is $n \div 2$ or $\frac{n}{2}$. "Six more than" translates to addition, or +6. So the phrase can be rewritten as $\frac{n}{2} + 6$.
- Sometimes, you'll need to translate within an actual problem situation, like Carlton did. Here are a few hints that can help:
  - "Total" means to add all the parts together.
  - Splitting something up means to divide it into equal parts.
  - Any part that has an unknown value should be represented by a variable.

Example:

- Three friends share an apartment. They also share all the monthly costs of the apartment, including the rent and utilities. Write an expression that represents what each friend pays each month.

Solution:

- Start by choosing a variable to represent the value that is unknown, the total monthly cost. You can use any letter you want, but for this example, use *c* to represent the cost.
- The cost, *c*, is divided three ways each month. So every month, each of the three friends pays $c \div 3$.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Variables, constants, and terms are all types of expressions.
- Numbers and variables written right next to other variables are multiplied together.
- Mathematical language has three parts: numbers, symbols, and variables.
- Word phrases can be translated into mathematical phrases.

Complete the following activities.

1.1 Every term is _____ .

☐ constant ☐ variable ☐ expression ☐ term

1.2 The expression -7*y* is a _____ .

☐ constant ☐ variable ☐ expression ☐ term

1.3 All of the following represent the same expression *except* _____.

☐ -8*ab* ☐ -8(*a*)(*b*) ☐ 8 - *ab* ☐ -8 · *a* · *b*

1.4 Which of the following expressions represents the phrase "the quotient of a number and -5"?

☐ *n* - 5 ☐ *n* ÷ -5 ☐ -5*n* ☐ *n* + (-5)

1.5 If the cost of a raffle ticket is $2, which of the following expressions could be used to represent the cost of *m* tickets?

☐ 2*m* ☐ 2 + *m* ☐ *m* - 2 ☐ *m* ÷ 2

1.6 The cost of a raffle ticket is $2. There is also a one-time $5 fee to be part of the raffle. Which of the following expressions could be used to represent the total cost of being in the raffle and buying *m* tickets?

☐ 10*m* ☐ 5*m* + 2 ☐ 7 + *m* ☐ 2*m* + 5

1.7 Which of the following expressions represents the phrase "19 less than a number"?

☐ *n* - 19 ☐ 19 - *n* ☐ *n* ÷ 19 ☐ *n* + 19

1.8 Which of the following expressions represents the phrase "six added to five times a number"?

☐ 5 + 6*x* ☐ 6 + 5*x* ☐ 11*x* ☐ 6 + 5 + *x*

1.9 Which of the following phrases could represent the expression *k* - 4?

☐ a number subtracted from four ☐ a number decreased by four

☐ four less a number ☐ a number less than four

1.10 Dylan is six years older than his sister. If his sister's age is represented by the variable *s*, which of the following expressions represents Dylan's age?

☐ 6*s* ☐ *s* - 6 ☐ *s* + 6 ☐ 6 - *s*

Write an expression for each situation below.

1.11 If a giant candy bar costs $2, what is the cost of buying *c* candy bars?

1.12 A movie theater charges $4 for a drink and $6 for pail of popcorn. What is the total charge for *d* drinks and p popcorns?

1.13 The county fair charges $10 admission and $2 per ride. What is the cost for one admission and *r* rides?

1.14 The county fair charges $10 admission and $2 per ride. What is the cost for *a* admissions and *r* rides?

1.15 A concession stand starts the day with *h* hamburgers. How many hamburgers do they have left at the end of the day if they sold 58 hamburgers?

Translating Word Sentences

In this lesson, you'll be translating word sentences into the language of math. In order to do that, you'll have to use what you know about translating phrases.

Objectives

- Translate between word sentences and mathematical equations.
- Write an equation to represent a word problem.

Vocabulary

equation—a mathematical statement that shows two expressions are equal using an equal sign

Translating sentences is almost exactly the same as translating phrases. But there is one more symbol that you have to use: the equal sign! A word phrase translates to an expression. A word sentence translates to an *equation*. An equation is a complete thought. It tells you that two expressions are equal to each other. So an equation is two expressions joined by an equal sign.

Vocabulary! Remember that a variable, or letter, represents the value that is unknown.

Do you remember the three parts of mathematical language? They are numbers, symbols, and variables. The equal sign is a symbol. To determine which symbols to use in an expression or equation, look for key words. Key words can also help you determine when to use the equal sign. Figure 2 will remind you of the key words that indicate the different operations. Notice that the last column shows words that indicate the equal sign.

Vocabulary! Remember that a variable, or letter, represents the value that is unknown.

Now look at the difference between phrases and sentences. Remember that a phrase translates to an expression while a sentence is a complete thought and translates to an equation.

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)	Equality (=)
sum	difference	product	quotient	equals
increased by	decreased by	times	divided by	is
more than	less than	of	per	the result is
more	less	multiplied by		yields
added to	subtracted from	by		is equal to
plus	minus			the solution is
greater than	fewer than			

Figure 2 | Words and Phrases that indicate operations

Phrase: a number decreased by eleven

Translation: n - 11

Sentence: A number decreased by eleven is equal to fifteen.

Translation: n - 11 = 15

Phrase: half of a number

Translation: $\frac{1}{2}n$

Sentence: Half of a number equals negative twelve.

Translation: $\frac{1}{2}n = -12$

Can you see the difference between a phrase and a sentence, or an expression and an equation? A phrase, or expression, doesn't tell you much. It's only half of a thought. But when you complete the thought, you have so much more information. In fact, given a sentence, or equation, you can actually find the unknown number! That's something you'll work on later. First, you need to be able to translate sentences and equations. Here is another example.

Key point! An equation gives you two expressions that are equal to each other. That's why equations are so important. With one known expression, you have enough information to find the value of an unknown part in the other expression!

Example:

- Translate the following sentence to an equation.
- Nine fewer than a number is equal to negative five.

Solution:

- Remember that "fewer *than*" means you need to switch "nine" and "a number" in the first part of the equation. So nine fewer than a number is written as n - 9.
- So this sentence translates to n - 9 = -5.

You can also translate an equation to a word sentence. For any given equation, there may be many possible translations. For example, the equation -4 + n = 7 could be translated as "the sum of negative four and a number is equal to seven." It could also be translated as "if negative four is added to a number, the result is seven." Can you think of any other variations for this sentence?

Example:

- Translate the following sentence to a mathematical equation.
- six more than half a number is thirteen

Solution:

- Use the variable n to represent "a number." Half of something means to divide by two, so half a number is $n \div 2$ or $\frac{n}{2}$. "Six more than" translates to addition, or +6. The word *is* means *equals*. So the sentence can be rewritten as $\frac{n}{2} + 6 = 13$.

Self Test 1: Variable Expressions

Complete the following activities (5 points, each numbered activity).

1.01 _____ states a rule using variables.

- ☐ A variable
- ☐ An expression
- ☐ A formula
- ☐ A term

1.02 _____ terms are also expressions.

- ☐ No
- ☐ Some
- ☐ All

1.03 Which of the following expressions could be used to represent "three less than half a number"?

- ☐ $3 - \frac{1}{2}n$
- ☐ $\frac{1}{2}n - 3$
- ☐ $3n - \frac{1}{2}$
- ☐ $\frac{1}{2} - 3n$

1.04 Which of the following phrases translates to the expression $y + 7$?

- ☐ a number decreased by seven
- ☐ seven more than a number
- ☐ the product of a number and seven
- ☐ a number divided by seven

1.05 Four friends are going to split the cost of a pizza. If p stands for the cost of the pizza, which of the following expressions could represent what each friend will pay?

- ☐ $4p$
- ☐ $p - 4$
- ☐ p
- ☐ $\frac{p}{4}$

1.06 The product of a number and negative three is equal to twenty-seven. Which of the following could represent this statement?

- ☐ $-3n + 27$
- ☐ $-3(27) = n$
- ☐ $-3n = 27$
- ☐ $-3 = 27n$

1.07 Skye ran r laps today at soccer practice. Nadine ran five fewer laps than Skye. If Nadine ran eight laps at practice, which of the following equations could be used to find how many laps Skye ran?

- ☐ $r - 5 = 8$
- ☐ $r + 5 = 8$
- ☐ $r = 8 - 5$
- ☐ $-5r = 8$

1.08 Which of the following statements would *not* translate to the equation $8 \div x = -4$?

- ☐ If eight is divided by a number, the result is negative four.
- ☐ Eight divided by a number is equal to negative four.
- ☐ The quotient of eight and a number equals negative four.
- ☐ The solution of a number divided by eight is negative four.

1.09 If $a = -9$ and $b = -4$, what is the value of $a + b$?

☐ -5 ☐ 5 ☐ -13 ☐ 13

1.010 Simplify $2x^2 - y$ for $x = 3$ and $y = -2$.

☐ 20 ☐ 14 ☐ 16 ☐ 10

1.011 The cost for children under twelve at a certain buffet restaurant is a base charge of a dollar plus twenty-five cents for each year of the child's age. Represent this cost as an expression, using y to stand for the child's age in years. Then use the expression to find the cost for a child who is nine.

☐ $1 + 0.25y$; \$2.25 ☐ $0.25 + y$; \$9.25

☐ $1 + 0.25y$; \$3.25 ☐ $0.25 + y$; \$8.75

1.012 Tommy and his family drove home from his grandparent's house in three and a half hours. If they drove at an average rate of 48 miles per hour, which of the following formulas could be used to find how far away Tommy lives from his grandparents?

☐ $d = rt$ ☐ $t = \frac{d}{r}$ ☐ $r = \frac{d}{t}$

1.013 Tommy and his family drove home from his grandparent's house in three and a half hours. If they drove at an average rate of 48 miles per hour, about how far away does Tommy live from his grandparents?

☐ 144 miles ☐ 192 miles ☐ 168 miles ☐ 180 miles

1.014 Hillary opened a savings account with \$1,500. How much simple interest will she earn if she leaves the money in the account for two years at an interest rate of 0.025? Use the formula $i = prt$.

☐ \$750.00 ☐ \$37.50 ☐ \$375.00 ☐ \$75.00

1.015 Convert 15°C to Fahrenheit. Use the formula $F = \frac{9}{5}C + 32$.

☐ 59°F ☐ 27°F ☐ 85°F ☐ 33°F

1.016 A student brought 2 dozen cookies to share with the students in his class. Write an expression to show how many cookies each student should get if there are *s* students.

1.017 Solve the expression $6w - 5z$ for $w = 4$ and $z = -2$.

1.018 A runner completes a 5-mile race in 40 minutes. What was the runner's average speed in miles per hour? ($40 \text{ min.} = \frac{2}{3}\text{hr.}$)

1.019 A credit card offers no interest financing if the entire balance is paid in full within 12 months. If the balance is not paid in full at the end of 12 months, the customer is charged 18% interest on the original balance. What will be the amount of interest charged on a $500 purchase that is not paid in full at the end of the 12-month period? (18% = 0.18) Use the formula $i = prt$

1.020 Convert $-40°$ F to Celsius. Use the formula $C = \frac{5}{9}(F - 32)$.

80/100 **SCORE** _____ **TEACHER** _____ _____
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