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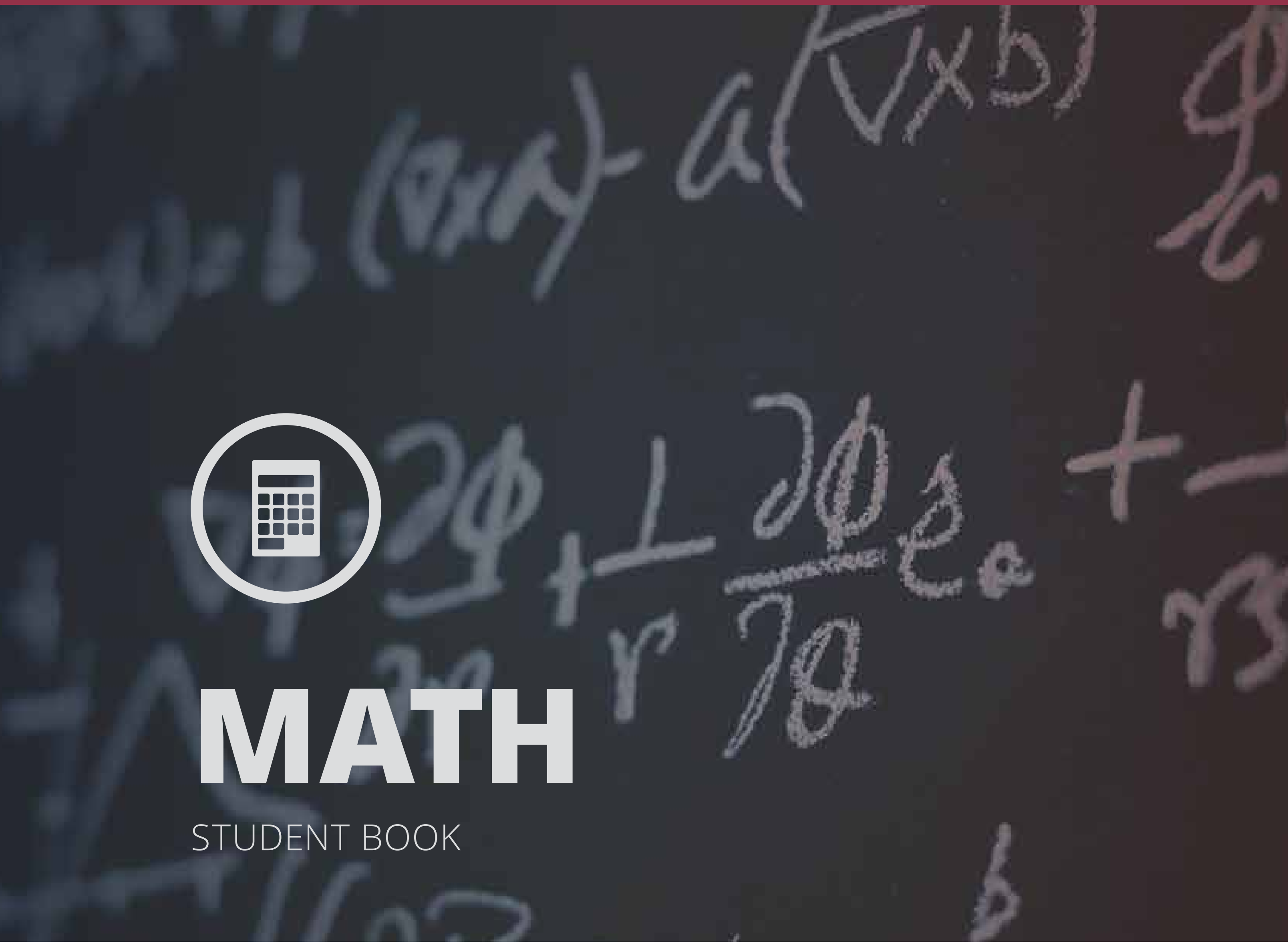

MATH

STUDENT BOOK

7th Grade | Unit 6

Alpha Omega
PUBLICATIONS

Math 706
Probability and Graphing

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Probability and Graphing

Introduction

In this unit, students will be introduced to basic probability. They will determine theoretical and experimental probability and learn that experimental probability approaches theoretical probability as the number of trials increases. Students will determine the probability for compound events and find sample space using a tree diagram and a table. Students will learn about the counting principle and apply it to finding the probability of compound events. They will also learn the difference between independent and dependent events.

Students will be introduced to the coordinate plane and use it to graph linear functions. They will plot ordered pairs and find the location of points in the coordinate plane. Students will graph linear equations and determine the slope of a line using the slope formula. They will also learn about direct variation functions and their characteristics.

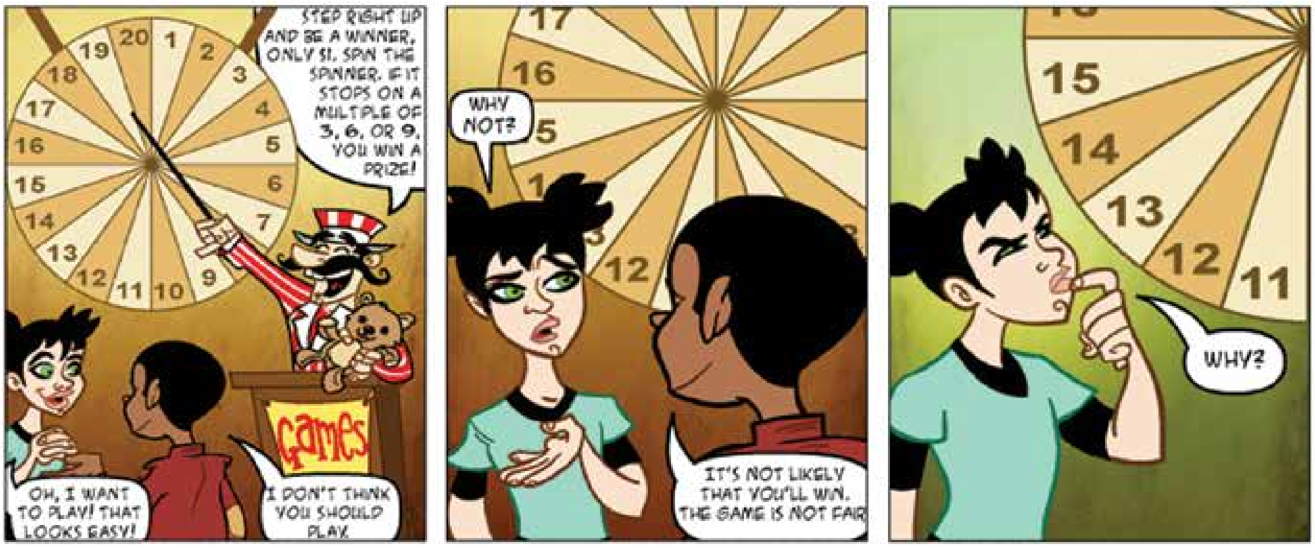
Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Determine the theoretical and experimental probability of an event.
- Determine the sample space for an experiment.
- Determine if events are independent or dependent.
- Determine the probability of independent and dependent events.
- Plot ordered pairs on a rectangular coordinate system.
- Use a table to graph a linear equation.
- Determine the slope of a linear function, including direct variation.
- Determine if a function is a direct variation.
- Graph direct variations.

1. Probability

Theoretical Probability



How likely is it that Ondi will win the game? What are her chances, and how can you figure that out? In this lesson, you will learn how to find the likelihood that *events* will occur. The measure of this likelihood is called *probability*.

Objectives

- Determine the theoretical probability of an event.

Vocabulary

complementary events—two disjoint events of which one or the other must occur
disjoint events—events that have no outcomes in common
event—a specific outcome or group of outcomes
experiment—any activity that has two or more outcomes
favorable outcome—outcome for a specific event
outcome—any possible result of an experiment
probability—the measure of the likelihood of an event
theoretical probability—a ratio representing the likelihood of an event

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In the area of mathematics known as probability, the carnival game that Ondi wanted to play is called an *experiment*. There are 20 possible *outcomes*, or results of the experiment because the spinner could stop on any number. The event, or specific outcome, that Ondi would need to win is any multiple of 3 from 1 and 20. The outcomes for this event are called *favorable outcomes*.

Probability is a measure of how likely an event is to occur. If all outcomes are equally likely, the probability (*P*) of the event is expressed as a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

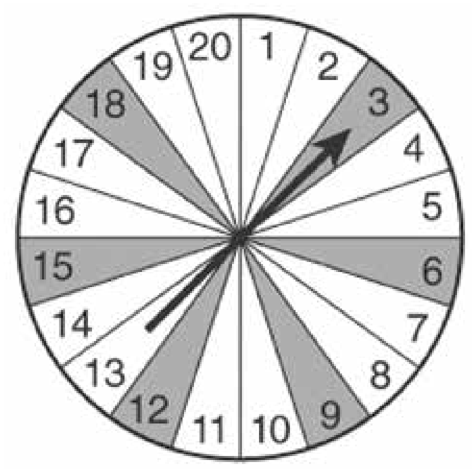
For Ondi, the event is spinning a multiple of 3 on the wheel. Take a look at the probability of Ondi winning the carnival game.

Example:

- What is the probability of spinning a multiple of 3 on a spinner with 20 equally spaced sections numbered from 1 to 20?

Solution:

- You need to find the number of favorable outcomes compared to the total number of outcomes.
- You know there are 20 spaces on the spinner, so the total number of outcomes is 20.
- To find the number of favorable outcomes, look at the multiples of 3 from 1 to 20:
  3, 6, 9, 12, 15, 18
- There are 6 multiples of 3, so there are 6 favorable outcomes.
- Shading in the multiples of 3 on the wheel makes this easier to see.



Reminder! Percents, decimals, and fractions can each express the same ratio:

$$\frac{1}{4} = \frac{25}{100} = 25\% = 0.25$$

Now compare the number of favorable outcomes to the total number of outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{multiple of 3}) = \frac{6}{20}$$

Since probability is a ratio, it can be written as a fraction, decimal, or percent. When you express the probability of an event as a ratio, it is called the *theoretical probability*.

Reminder! To change a fraction to a percent, rewrite it with a denominator of 100. To change a percent to a decimal, move the decimal point two places to the left.

$\frac{6 \div 2}{20 \div 2} = \frac{3}{10}$ Divide by a common factor of 2.

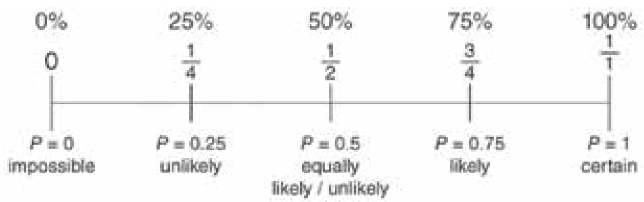
$\frac{3 \cdot 10}{10 \cdot 10} = \frac{30}{100}$ Multiply by 10 to get a denominator of 100.

$\frac{30}{100} = 30\%$ Convert the fraction to a percent.

$30\% = 0.3$ Move the decimal point two places to the left.

So there is a $\frac{3}{10}$, 30%, or 0.3 chance of spinning a multiple of 3 on the wheel.

Probability will always be a number from 0 to 1. The closer the probability is to 1, the more likely it is that the event will occur. You can see this relationship on a number line.

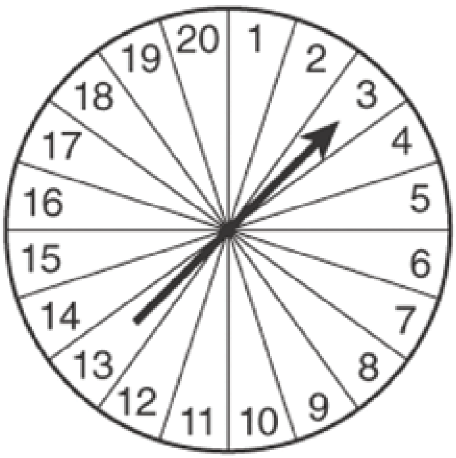


Events that have less than a 50% probability are less likely to occur. Events that have more than a 50% probability are more likely to occur.

As Carlton said, it is *unlikely* that Ondi would win the game. She has well under a 50% chance of winning.

You can also describe everyday events in terms of likelihood. For example, it is very unlikely that it will snow on a warm summer day. Or suppose you've attended school 48 out of the last 50 school days. Based on your previous attendance, it is very likely that you will be at school on the next school day.

Take a look at a couple of examples using the carnival spinning wheel. For each event, look at the number of favorable outcomes compared to the number of total outcomes.



What is the probability that you would spin the number 30?

There are 20 total outcomes, because there are 20 sections on the wheel. However, you can see from looking at the wheel that there is no space labeled 30, so there are 0 favorable outcomes. You can express this probability using a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(30) = \frac{0}{20}$$

Make note! In probability, parentheses do not indicate multiplication. The contents of the parentheses are the event. You are not multiplying P by 30.

So the probability is 0 out of 20, or 0%. In other words, the outcome is impossible.

What is the probability that you would spin a number less than 21?

Again, there are 20 total outcomes, because there are 20 sections on the wheel. There are 20 favorable outcomes because all of the outcomes are less than 21. You can express this probability using a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(< 21) = \frac{20}{20}$$

So the probability is 20 out of 20, or 1, or 100%. In other words, the outcome is certain.

Here's another example.

Example:

- There are 2 blue marbles, 1 red marble, and 9 green marbles in a bag. What is the probability of drawing a green marble from the bag?

Solution:

- Find the number of favorable outcomes and the total number of outcomes and compare them to find the probability of the event.
- There are 9 green marbles, so there are 9 favorable outcomes.
- To find the total number of outcomes, you need to find out how many marbles are in the bag:
  - 2 blue + 1 red + 9 green = 12 marbles
- Since there are 12 marbles in the bag, there are 12 total outcomes.
- Now compare the outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{green}) = \frac{9}{12}$$

Keep in mind! Any proportion can be solved by cross multiplying:

$$\frac{3}{4} = \frac{x}{100}$$

$$300 = 4x$$

$$\frac{300}{4} = x$$

$$75 = x$$

- You can simplify the fraction and change it to a percent or a decimal.

$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$	Divide by a common factor of 3.
$\frac{3 \cdot 25}{4 \cdot 25} = \frac{75}{100}$	Multiply by 25 to get a denominator of 100.
75% = 0.75	Move the decimal point two places to the left.

So there is a $\frac{3}{4}$, 0.75, or 75% chance of drawing a green marble from the bag.

When you have two events of which one or the other *must* occur, the events are called *complementary events*, and their probabilities will always add up to 1:

- $P(\text{event}) + P(\textit{not}\ \text{event}) = 1$

Try an example involving complementary events.

Example:

- If you roll a regular 6-sided number cube, what is the probability that you will roll a 4? What is the probability that you won't roll a 4?

Solution:

- Compare the number of favorable outcomes to the total number of outcomes for each experiment. Write the probability as a fraction this time.

- There is only one favorable outcome for each of the 6 numbers on the cube, so there is one favorable outcome for rolling a 4. There are 6 sides to the number cube, so there are a total of 6 outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(4) = \frac{1}{6}$$

- The favorable outcomes for *not* rolling a 4 are 1, 2, 3, 5, and 6. So there are 5 favorable outcomes out of 6 total outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\textit{not } 4) = \frac{5}{6}$$

- Notice that the probability of rolling a 4 and the probability of not rolling a 4 add up to 1:

$$P(4) + P(\textit{not } 4)$$

$$= \frac{1}{6} + \frac{5}{6}$$

$$= 1$$

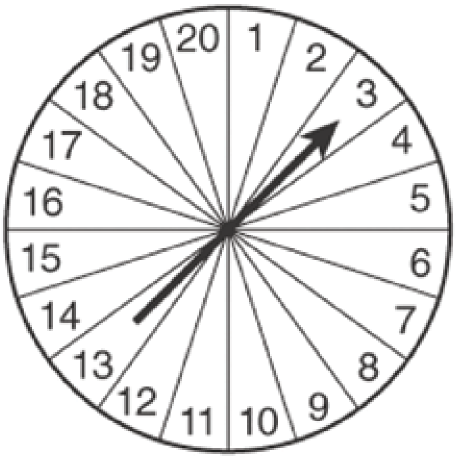
- They add up to 1 because both events together account for all of the outcomes. You could say that all rolls of the number cube are either 4 or not 4.
- Events that have no outcomes in common are called *disjoint events*, and the probability that either event will occur is the sum of the probabilities of the events:

$$P(\text{event 1 } \textit{or} \text{ event 2}) = P(\text{event 1}) + P(\text{event 2})$$

- Try an example involving disjoint events.

Example:

- If you spin the carnival spinning wheel, what is the probability that you will spin a multiple of 5 or a multiple of 7?



Solution:

- Compare the number of favorable outcomes to the total outcomes for each event. This will give you the probability for each event. If there are no outcomes in common, you can add the probabilities of the events.
- The favorable outcomes for a multiple of 5 are 5, 10, 15, and 20. So there are 4 favorable outcomes.
- You know there are 20 total outcomes because there are 20 spaces on the wheel.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

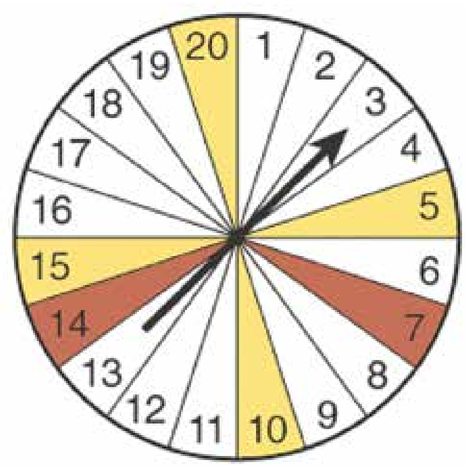
$$P(\text{multiple of 5}) = \frac{4}{20}$$

- The favorable outcomes for a multiple of 7 are 7 and 14. So there are 2 favorable outcomes out of 20 total outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{multiple of 7}) = \frac{2}{20}$$

- If the wheel is colored, you can see the favorable outcomes more easily.
- yellow: multiples of 5
- orange: multiples of 7



- There are no outcomes in common, so you can add the probabilities.
- *P*(event 1 or event 2) = *P*(event 1) + *P*(event 2)
- *P*(multiple of 5 *or* multiple of 7) = *P*(multiple of 5) + *P*(multiple of 7)
- *P*(multiple of 5 *or* multiple of 7) = $\frac{4}{20} + \frac{2}{20}$
- *P*(multiple of 5 *or* multiple of 7) = $\frac{6}{20}$
- This was the same probability as the game Ondi wanted to play, and you found that 6 out of 20 was 30%. So there is a 30% chance of spinning a multiple of 5 or 7.

Example:

- A side game at the fair requires the game operator to guess the month of the guest's birth within 2 months. If the game operator is off by more than two months, the guest wins a prize. What is the probability that the game operator will randomly guess a person's birth month within two months of the correct month?

Solution:

- There are 12 possible months to choose from, so there are 12 possible outcomes. Of these possible outcomes, the game operator must either guess the correct month, or one of the two months on either side of the correct month. For example, if the guest was born in July, the game operator could guess July (the correct month), August, September (the 2 months after July), June, or May (the 2 months before July). This allows 5 possible favorable outcomes.
- Compare the number of favorable outcomes with the number of possible outcomes and form a ratio.

$$\frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{5}{12}$$

$$P(\text{birth month within 2 months}) = \frac{5}{12}$$

or $41.\overline{6}\%$ or $0.41\overline{6}$

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Theoretical probability is expressed as a ratio and can be written as a fraction, decimal, or percent.
- To find the theoretical probability of an event, compare the number of favorable outcomes to the total number of outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Complete the following activities.

1.1 Select all that apply. If there are 8 chocolate chip cookies out of 20 total cookies in a jar, what is the probability that you will randomly choose a chocolate chip cookie?

☐ $\frac{2}{5}$ ☐ 40% ☐ 0.8 ☐ 0.4

1.2 Select all that apply. In a carnival game, there is a 45% probability of winning a prize. Which of the following is true?

☐ The probability that you won't win is $\frac{11}{20}$.
☐ The probability that you will win is 4.5.
☐ The probability that you won't win is 0.55.
☐ The probability that you will win is $\frac{9}{20}$.

1.3 In a raffle, Scott buys 10 tickets and his friend Tom buys 6 tickets. If there are 80 tickets sold, what is the probability that Scott *or Tom will win?*

☐ $\frac{1}{8}$ ☐ $\frac{1}{5}$ ☐ $\frac{3}{40}$ ☐ $\frac{4}{5}$

1.4 A jar contains 100 coins. If it is very *unlikely* that you will randomly choose a quarter out of the jar, how many quarters could be in the jar?

☐ 50 ☐ 35 ☐ 85 ☐ 10

1.5 At a school, there are 526 students and 263 are girls. About how likely is it that a randomly chosen student will be a boy?

☐ unlikely
☐ somewhat likely
☐ very likely
☐ equally as likely as unlikely

1.6 What is the probability of rolling a 2 or not rolling a 2 using a regular 6-sided number cube?

☐ $\frac{1}{6}$ ☐ 33.3% ☐ $\frac{5}{6}$ ☐ 100%

1.7 Chris has 2 pairs of black socks, 4 pairs of red socks, and 18 pairs of white socks in a dresser drawer. If he reaches in his drawer without looking, what is the probability that he will choose a pair of white socks?

☐ 75% ☐ 33.3% ☐ 25% ☐ 50%

1.8 There are 6 red marbles, 4 blue marbles, and 15 green marbles in a jar. If you reach in and randomly draw one, what is the probability that you will choose a red marble?

☐ 66.7% ☐ 40% ☐ 30% ☐ 24%

1.9 There are 12 boys and 13 girls in a class. If the teacher randomly chooses a student's name out of a hat, what is the probability it will be a girl?

☐ 48% ☐ 50% ☐ 52% ☐ 92%

1.10 You are one of 50 people with an entry into a random drawing for a new bicycle. What is the probability that you will win the drawing?

1.11 You must roll an even number on a standard 6-sided game die to win the game. What is the probability that you will win the game on your next roll?

1.12 What is the probability of a football player correctly guessing whether the coin toss will be heads or tails?

1.13 What is the probability of rolling either an even number or a 5 on a standard 6-side game die?

1.14 A deck of cards has 52 cards (13 cards in each of 4 suits: clubs, diamonds, spades, and hearts). What is the probability of drawing a card with a diamond on it?

Self Test 1: Probability

Complete the following activities (5 points, each numbered activity).

1.01 Suppose you are asked to pick 3 numbers from 1 to 20 to win a prize. What is the probability that one of the numbers you will pick is the winning number?

- ☐ 5%
- ☐ 10%
- ☐ 15%
- ☐ 20%

1.02 At a carnival game, there is a 38% probability of winning a prize. What is the probability of not winning a prize?

- ☐ 38%
- ☐ 50%
- ☐ 62%
- ☐ 76%

1.03 Alice and Finn roll two number cubes. Which of the following rules will make the game fair?

- ☐ Alice wins if a total of 5 is rolled. Finn wins if a total of 9 is rolled.
- ☐ Alice wins if a total of 3 is rolled. Finn wins if a total of 10 is rolled.
- ☐ Alice wins if a total of 7 is rolled. Finn wins if a total of 8 is rolled.
- ☐ Alice wins if a total of 4 is rolled. Finn wins if a total of 11 is rolled.

1.04 You have 2 spreads, 5 meats, and 2 kinds of bread. How many different sandwiches can you make using one of each type of ingredient?

- ☐ 9
- ☐ 12
- ☐ 20
- ☐ 40

1.05 There are 6 red marbles, 8 blue marbles, and 11 green marbles in a bag. What is the probability that you will randomly draw either a red or a blue marble?

- ☐ 24%
- ☐ 56%
- ☐ 32%
- ☐ 10%

1.06 What is the experimental probability of drawing a red marble, given the following results?

Marble Color	Blue	Green	Red
Times Drawn	6	6	8

- ☐ $\frac{2}{5}$
- ☐ $\frac{4}{5}$
- ☐ $\frac{2}{3}$
- ☐ $\frac{7}{10}$

1.07 A coin is flipped 40 times, and it lands on heads 16 times. Based on the experimental probability, how many heads would you predict for 200 flips of the coin?

- ☐ 90
- ☐ 80
- ☐ 56
- ☐ 112

1.08 Select all that apply. A spinner is divided into 4 equal sections. Which of the following are true?

- ☐ The spinner will land on each section of the spinner an equal number of times.
- ☐ The theoretical probability is 25% for each section.
- ☐ If the spinner is spun 64 times, you would predict it to land on each section 16 times.
- ☐ The experimental probability is $\frac{1}{4}$ for each section.

1.09 Select all that apply. There are 3 red marbles, 5 green marbles, and 2 blue marbles in a bag. Which of the following are true?

- ☐ The probability of randomly drawing either a red marble or a green marble is 80%.
- ☐ The probability of randomly drawing a red marble and then a green marble is $\frac{1}{6}$.
- ☐ The probability of not drawing a blue marble is 20%.
- ☐ The probability of drawing a green marble is the same as the probability of drawing either a red or a blue marble.

1.010 Select all that apply. A coin is flipped and a number cube is rolled. Which of the following are true?

- ☐ The sample space has 12 different outcomes.
- ☐ The sample space has 8 different outcomes.
- ☐ Each result is equally likely.
- ☐ Heads and an even number are very likely.

1.011 Three coins are flipped. What is the probability that there will be at least two tails?

☐ $\frac{3}{8}$ ☐ $\frac{1}{2}$ ☐ $\frac{1}{8}$ ☐ $\frac{1}{9}$

1.012 Suppose there are 21 students in your class. If the teacher draws 2 names at random, what is the probability that you and your best friend will be chosen?

☐ $\frac{2}{21}$ ☐ $\frac{1}{20}$ ☐ $\frac{1}{105}$ ☐ $\frac{1}{210}$

1.013 Which of the following are dependent events?

- ☐ rolling a number cube and then flipping a coin
- ☐ spinning a spinner and then rolling a number cube
- ☐ drawing a marble from a bag, not replacing it, and then drawing a second marble
- ☐ choosing a number from a hat, replacing it, and then choosing another number

1.014 What is the probability of rolling an odd number and spinning "B," given the sample space below?

	1	2	3	4	5	6
A	A1	A2	A3	A4	A5	A6
B	B1	B2	B3	B4	B5	B6
C	C1	C2	C3	C4	C5	C6
D	D1	D2	D3	D4	D5	D6

☐ $\frac{1}{2}$ ☐ $\frac{1}{8}$

☐ $\frac{1}{6}$ ☐ $\frac{1}{12}$

1.015 A number cube is rolled and a coin is flipped. Predict how many times you would get heads and a number less than 3 in 240 trials.

☐ 60 ☐ 40 ☐ 30 ☐ 20

1.016 What is the probability of rolling doubles on a pair of standard 6-sided dice?

1.017 What is the probability of rolling doubles three times in a row on a pair of standard 6-sided dice?

1.018 You have 8 tickets out of the 150 tickets in a drawing. What is the probability that one of your tickets will be drawn first?

1.019 A baseball player's batting average is 0.333, which means he gets a hit 33% of the time, or $\frac{1}{3}$ of the time. Based on this information, how many hits would you expect the player to have in a series if he has 12 at-bats in the series?

1.020 How many meal combinations are possible that contain one appetizer, one entrée, and one dessert from a menu with 4 appetizers, 5 entrées, and 3 desserts?

80/100 **SCORE** ________ **TEACHER** ________ ________
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