



MATH

STUDENT BOOK

▶ **7th Grade | Unit 9**

Math 709

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Measurement and Area

Introduction

In this unit, students will explore different ways to measure and describe plane figures. They will learn how to find the perimeter and area of parallelograms, triangles, trapezoids, circles, and composite figures. Students will also look at how changing all the dimensions of a plane figure by the same factor affects its area. They will complete the unit by studying squares and square roots and use these inverse operations to apply the Pythagorean Theorem.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAAC. When you have finished this LIFEPAAC, you should be able to:

- Find the perimeter or circumference of a plane figure.
- Use the perimeter, circumference, or area of a plane figure to find a missing length.
- Find the area of parallelograms, triangles, trapezoids, circles, and composite figures.
- Determine the area of a figure after its dimensions have changed.
- Calculate squares and square roots.
- Use the Pythagorean Theorem to find a missing side length of a right triangle and to solve application problems

1. Perimeter

PERIMETER



Is Carlton right? Does it really matter if Ondi specifies what 18 is referring to? Yes! She could be talking about 18 miles or 18 feet. There is a big difference between those two distances. Distances must be labeled with

a unit of measure in order to give them meaning.

In this lesson, you'll be finding the distance around the outside of a *polygon*. Like Ondi, you'll want to be sure to label your answers.

Objectives

- Calculate the perimeter of a polygon.
- Use the perimeter of a polygon to find a missing side length.

Vocabulary

dimensions—the measurements of an object (e.g., length, width, or height)

perimeter—the distance around the outside of a plane figure

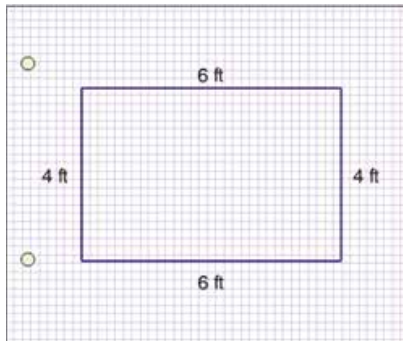
polygon—a closed figure made up of line segments

The distance around the outside of a polygon is called the *perimeter*. Since a polygon is made up of line segments, finding its perimeter is very simple. Just add up the lengths of *all* the sides of the polygon. Take a look.

Keep in mind! The distance around the outside of any plane figure (or two-dimensional figure) is called the perimeter. In this lesson, though, you'll just be looking at the perimeter of polygons.

Natasha is going to paint a mural on her bedroom wall. She plans to tape off the section of wall she is going to use. If the mural will be a rectangle with *dimensions* of 6 feet by 4 feet, how much tape will she need? Remember that the dimensions of a figure are the measurements that tell you its size. In this case, the dimensions represent the length and width of the rectangle.

Draw a picture of the rectangle and label the appropriate sides. Since rectangles have opposite sides of the same length, you already know the lengths of all four sides.



To find the amount of tape Natasha will need, you need to find the perimeter of the rectangle. So add up the lengths of the four sides:

- $4\text{ ft} + 6\text{ ft} + 4\text{ ft} + 6\text{ ft} = 20\text{ ft}$
- So Natasha will need 20 feet of tape.

Because opposite sides of a rectangle are congruent, the perimeter of a rectangle will always be equal to the sum the two lengths and the two widths. A formula can be used to describe this relationship. The variable P is used to represent the perimeter, l represents the length, and w represents the width. So the perimeter of a rectangle can be found using the formula $P = 2l + 2w$. Try solving Natasha's problem using this formula:

- $P = 2l + 2w$
- $P = 2(6\text{ ft}) + 2(4\text{ ft})$
- $P = 12\text{ ft} + 8\text{ ft}$
- $P = 20\text{ ft}$

John is planting a garden that is 3 yards wide and 5 yards long. He plans to put a fence around his garden to keep animals out. How many yards of fence will John need?

Remember that opposite sides of a rectangle are congruent, so you can use the formula

- $P = 2l + 2w$ where l is the length, or 5 yards, and w is the width, or 3 yards.
- $$P = 2(5\text{ yards}) + 2(3\text{ yards})$$
- $$P = 10\text{ yards} + 6\text{ yards}$$
- $$P = 16\text{ yards}$$

John will need 16 yards of fence for his garden.

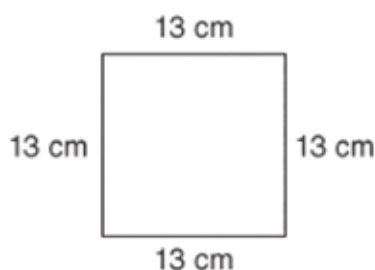
Keep in mind! Because perimeter represents a distance, it is always measured in units of length, or linear units. The most common standard linear units are the inch, foot, yard, and mile. The most common SI linear units are the millimeter, centimeter, meter, and kilometer.

Remember to label the perimeter with the correct unit of measure so that the amount has meaning (Remember Ondi's problem!). It's also important to use an appropriate unit of measure. For example, Natasha used feet to describe the dimensions of her rectangle. She probably could have used inches or yards as well. But would it have made sense to use miles? No! Miles are far too large of a unit to describe this situation.

Always consider the situation when determining what unit of measure to use.

Example:

- ▶ Each side of a square has a length of 13 centimeters. What is the perimeter of the square?



Solution:

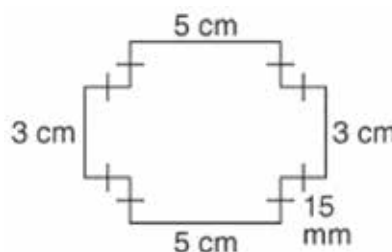
- ▶ Find the sum of the lengths of the four sides:
- ▶ $13\text{ cm} + 13\text{ cm} + 13\text{ cm} + 13\text{ cm} = 52\text{ cm}$
- ▶ The perimeter of this square is 52 centimeters.
- ▶ Notice in the previous example that because the figure was a square, you could have just multiplied the side length by 4. This alternative method for finding the perimeter of a square can also be described using a formula. If s represents the length of a side, then the perimeter of a square is equal to 4 multiplied by s . So the formula is $P = 4s$. The same idea can be applied to any polygon in which all the sides have the same length. To find the perimeter of a regular polygon, multiply the length of each side by the number of sides in the polygon.

Vocabulary! A regular polygon is a polygon that has congruent sides and angles.

In the next example, notice that some of the side lengths are given in a different unit of measure. Before calculating the perimeter, all the side lengths must be expressed in the same unit. You can use a proportion to convert units of measurement.

Example:

- ▶ What is the perimeter of the polygon in *centimeters*?



This might help! Remember that a tick mark on a side is used to show that the sides have the same length. On this figure, every side that has one tick mark has a length of 15 mm.

Solution:

- ▶ Express 15 millimeters in centimeters using a proportion:

$$\frac{15\text{ mm}}{x} = \frac{10\text{ mm}}{1\text{ cm}} \quad \text{Set up the proportion.}$$

$$15\text{ mm} \cdot \text{cm} = 10x\text{ mm} \quad \text{Cross multiply.}$$

$$1.5\text{ cm} = x \quad \text{Divide both sides of the equation by 10 mm.}$$

- ▶ So 1.5 centimeters is equivalent to 15 millimeters. To find the perimeter, find the sum of the 12 sides of the figure. Notice that eight of the sides have the same length of 1.5 centimeters. Remember

that multiplication can be used to represent repeated addition.

- $P = 3 \text{ cm} + 1.5 \text{ cm} + 1.5 \text{ cm} + 5 \text{ cm} + 1.5 \text{ cm} + 1.5 \text{ cm} + 3 \text{ cm} + 1.5 \text{ cm} + 1.5 \text{ cm} + 5 \text{ cm} + 1.5 \text{ cm} + 1.5 \text{ cm}$
 - $P = 2(3 \text{ cm}) + 2(5 \text{ cm}) + 8(1.5 \text{ cm})$
 - $P = 6 \text{ cm} + 10 \text{ cm} + 12 \text{ cm}$
 - $P = 28 \text{ cm}$
- ▶ The perimeter of this figure is 28 centimeters.

Let's Review

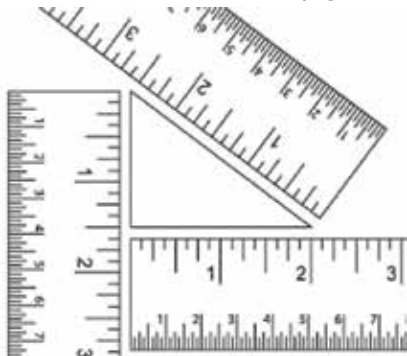
Before going on to the practice problems, make sure you understand the main points of this lesson:

- Perimeter is the distance around the outside of a polygon.
- Perimeter can be found by adding up the lengths of each side of a polygon or by using a formula.
- All side lengths must be expressed in the same unit of measure before the perimeter is calculated.
- A length has no meaning unless it is labeled with the correct unit of measure.



Complete the following activities.

- 1.1 The perimeter of a regular hexagon could be found using the formula $P = 6s$.
- True
 False
- 1.2 The most appropriate unit of measure to describe the distance around the outside of a small picture frame would be the _____.
- inch yard mile
- 1.3 The most appropriate unit of measure to describe the perimeter of a city would be the _____.
- millimeter centimeter meter kilometer
- 1.4 Use the rulers to help you estimate the perimeter of this triangle.

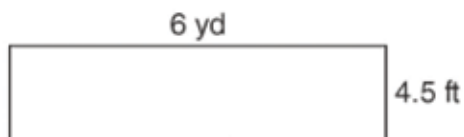


- 6 in.
 3 in.
 7 in.
 6.5 in.

- 1.5 What is the perimeter of a regular pentagon in which all sides have a length of 7 feet?

49 ft 28 ft 35 ft 42 ft

- 1.6 Hans is building a fence to put around his rectangular garden. How many *yards* of fencing will Hans need to enclose the garden on all four sides?

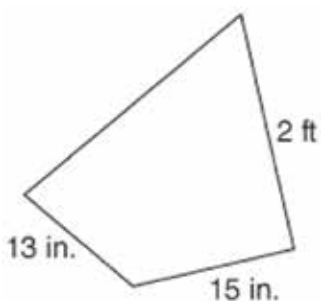


10.5 yd 7.5 yd
 21 yd 15 yd

- 1.7 Two sides of a triangle measure 18 meters and 11 meters. If the perimeter of the triangle is 37 meters, what is the length of the third side?

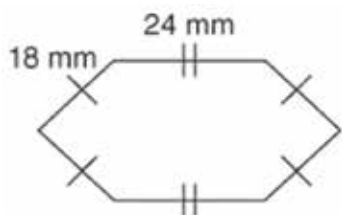
12 m 8 m 66 m 30 m

- 1.8 If the perimeter of this quadrilateral is 79 inches, what is the measure of the missing side length in inches?



77 in
 52 in.
 28 in.
 27 in.

- 1.9 Which of the following expressions could be used to find the perimeter of this figure?



$P = 4(18 \text{ mm}) + 2(24 \text{ mm})$
 $P = 18 \text{ mm} + 24 \text{ mm}$
 $P = 2(18 \text{ mm}) + 4(24 \text{ mm})$
 $P = (18 \text{ mm})(24 \text{ mm})$

- 1.10 The perimeter of a square is 64 millimeters. What is the length of each side?

8 mm 12 mm 16 mm 32 mm



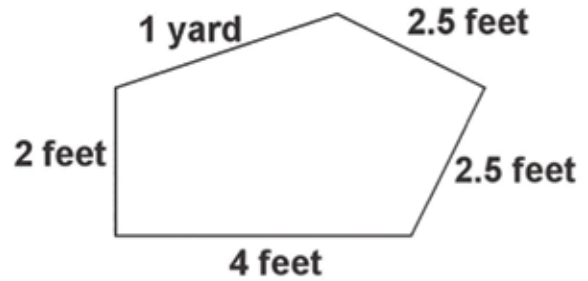
Complete the following activities.

1.11 What is the perimeter of a rectangle with a length of 7 cm and a width of 13 cm?

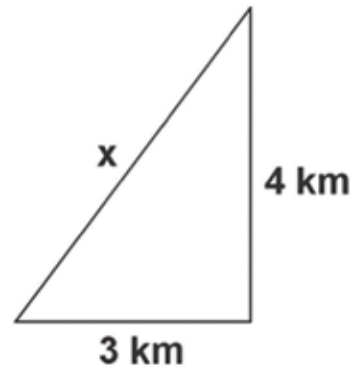
1.12 A pentagon has a perimeter of 65 inches. If each side is the same length, how long is each side?

1.13 A baseball player runs around a square formed by bases that are 90 feet apart. What is the perimeter of the square formed by the bases?

1.14 Find the perimeter.



1.15 Find length of the missing side if the perimeter is 12 km.



CIRCUMFERENCE

Do you remember how simple finding the perimeter of a polygon is? You just add up the lengths of all the sides of the polygon. What about a circle? How do you find the perimeter of a shape that has no sides?

One method is to use a string to physically measure the distance around the circle. In this lesson, you'll look at another method that can be used to calculate the perimeter of a circle.

Objectives

- Calculate the circumference of a circle.
- Use the circumference of a circle to find the radius or diameter.

Vocabulary

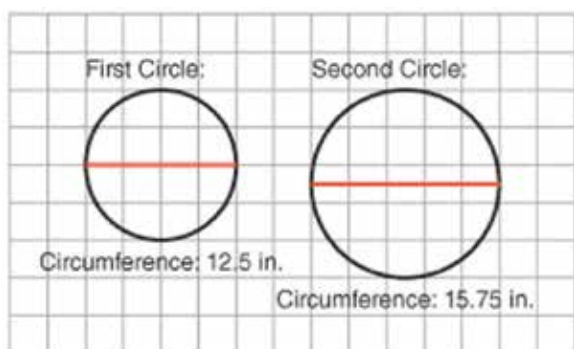
circumference—the distance around the outside of a circle

diameter—the distance across a circle through the center

pi—the ratio of the circumference of a circle to its diameter; approximately 3.14

radius—the distance from the center of a circle to any point on the circle

The perimeter of (or distance around) a circle has a special name. It's called the *circumference*. In the activity above, you saw that the first circle had a circumference of about 12.5 inches. And the second circle had a circumference of about 15.75 inches. Take a look at these circles again. This time, look at them on a grid.



Each square on the grid represents 1 inch. So the *diameter* of the first circle is 4 inches, and the diameter of the second circle is 5 inches. See if there is a relationship between the circumference of a circle

and its diameter. Try dividing each circle's circumference by its diameter.

- first circle:

$$\frac{\text{circumference}}{\text{diameter}} = \frac{12.5 \text{ in.}}{4 \text{ in.}} = 3.125$$

- second circle:

$$\frac{\text{circumference}}{\text{diameter}} = \frac{15.75 \text{ in.}}{5 \text{ in.}} = 3.15$$

Think about it! Why aren't the ratios exactly 3.141592653, or even 3.14? Because physical measurements are never perfect. They are estimates. Since the ratios for the circles were so close to 3.14, though, they were very good estimates.

For both circles, the ratio of the circumference to the diameter was just a little over 3. That is not a coincidence! Every circle shows this same relationship! The ratio of the circumference of a circle to its diameter is always the number

3.141592653.... This number has a special name; it's called *pi* (pronounced like the word *pie*). The symbol for pi is π . Because this number is irrational (never ends or repeats), it is usually rounded to 3.14 for calculations.

RULE: The ratio of the circumference of a circle to its diameter always equals pi.

Since the ratio of the circumference of any circle to its diameter is pi, all you have to know is a circle's diameter in order to find its circumference. Take a look:

$$\frac{\text{circumference (C)}}{\text{diameter (d)}} = \pi$$

The ratio of the circumference to the diameter is equal to pi.

$$\frac{C}{d} = \pi$$

Write the ratio using symbols.

$$d \cdot \frac{C}{d} = \pi \cdot d$$

To get the circumference by itself, multiply both sides by the diameter.

$$C = \pi d$$

Simplify.

So the circumference of a circle can be found by multiplying pi (or 3.14) by its diameter. The formula is $C = \pi d$, or $C = 3.14d$. Try an example.

Example:

- ▶ What is the circumference of a circle that has a diameter of 9 centimeters? Use 3.14 for pi.

Solution:

- ▶ Use the formula $C = \pi d$ to find the circumference. Substitute 9 centimeters in for d :
 - $C = \pi d$
 - $C = (3.14)(9 \text{ cm})$
 - $C = 28.26 \text{ cm}$
- ▶ The circumference of the circle is 28.26 centimeters.

Keep in mind! Make sure you label the circumference of a circle. Distances have no meaning if they don't specify the unit of measurement that was used.

You can also find the circumference of a circle if you know the *radius*. Remember that the radius of a circle represents the distance from the center of the circle to any point on the circle. The diameter is two times the radius of a circle. For example, if the radius of a circle is 6 feet, the diameter is 12 feet. So there's just one extra step to finding the circumference if you know the radius. You have to first multiply the radius by 2 in order to get the value of the diameter. Then you can multiply by pi, or 3.14. This relationship can be described using the formula $C = 2 \pi r$.

Example:

- ▶ The radius of a bicycle wheel is 11 inches. What is the circumference of the wheel?

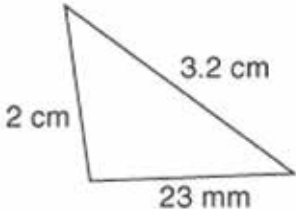
Solution:

- ▶ Use the formula $C = 2 \pi r$ to find the circumference. Substitute 11 inches in for r :
 - $C = 2 \pi r$
 - $C = 2(3.14)(11 \text{ in.})$
 - $C = 69.08 \text{ in.}$
- ▶ The circumference of the wheel is 69.08 inches.

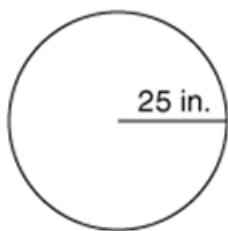
The diameter or radius of a circle can be used to find its circumference. Is the reverse true, too? Can the circumference of a circle be used to find its diameter or radius? Yes! You'll have to use what you know about solving equations to help you. Here's an example.

Self Test 1: Perimeter

Complete the following activities (5 points, each numbered activity).

- 1.01** The perimeter of a rectangle can be found using the formula $P = 4s$.
 True
 False
- 1.02** The ratio of the _____ of a circle to its _____ is approximately 3.14.
 radius circumference diameter
- 1.03** If the diameter of a circle is 12 feet, then the radius of the circle is _____ feet.
- 1.04** A man is going to fence in his backyard. Which of the following units of measure would be the most appropriate unit to describe the amount of fencing he'll need?
 inch foot mile
- 1.05** The width of a rectangle is 8 inches and the length is 5 inches. What is the perimeter of the rectangle?
 13 in. 26 in. 40 in. 21 in.
- 1.06** What is the perimeter of the following figure in centimeters?
 28.2 cm
 5.7 cm
 7.5 cm
 57 cm
- 
- 1.07** The perimeter of a quadrilateral is 38 yards. If three of the sides measure 5.8 yards, 7 yards, and 11.2 yards, what is the length of the fourth side?
 24 yd 62 yd 20.3 yd 14 yd
- 1.08** What is the circumference of a circle that has a diameter of 11 millimeters?
 34.54 mm 69.08 mm 17.27 mm 22 mm

1.09 What is the circumference of the following circle?

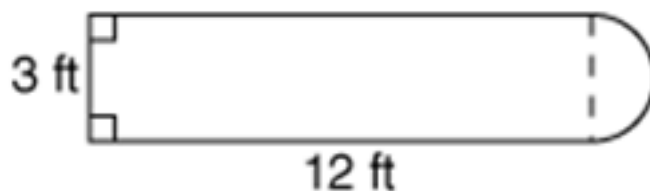


- 157 in.
 78.5 in.
 39.25 in.
 50 in.

1.010 The circumference of a circle is 56.52 meters. What is the radius of the circle?

- 28.26 m 9 m 18 m 4.5 m

1.011 The perimeter of the following figure consists of _____ line segment(s) and one semicircle.



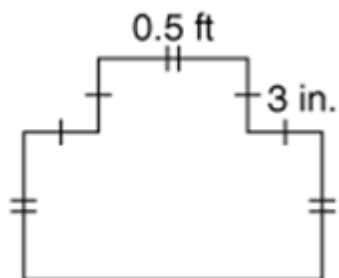
1.012 What is the circumference of the semicircle in the figure used in the previous problem?

- 9.42 ft 37.68 ft 4.71 ft 18.84 ft

1.013 What is the total perimeter of the figure used in the previous problems?

- 31.71 ft 34.71 ft 36.42 ft 39.42 ft

1.014 What is the length of the bottom edge of this figure in inches? (Hint: Add the measurements of the line segments that make up the same length as the bottom edge.)



- 6 in.
 9 in.
 12 in.
 15 in.

1.015 What is the total perimeter of the figure in the previous problem in inches?

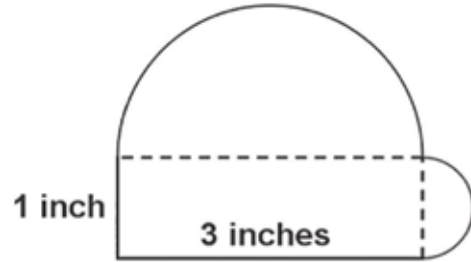
- 31.5 in. 45 in. 28.5 in. 42 in.

1.016 A square has a perimeter of 36 cm.
What is the length of a side?

1.017 A circle has a radius of 5 inches.
What is its circumference?

1.018 What is the circumference of a
semicircle with a radius of 3 meters?

Use the figure below (a rectangle with a semicircle on each of two sides) to answer the following questions.



1.019 What is the circumference of the
large semicircle?

1.020 What is the perimeter of the figure
above?

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