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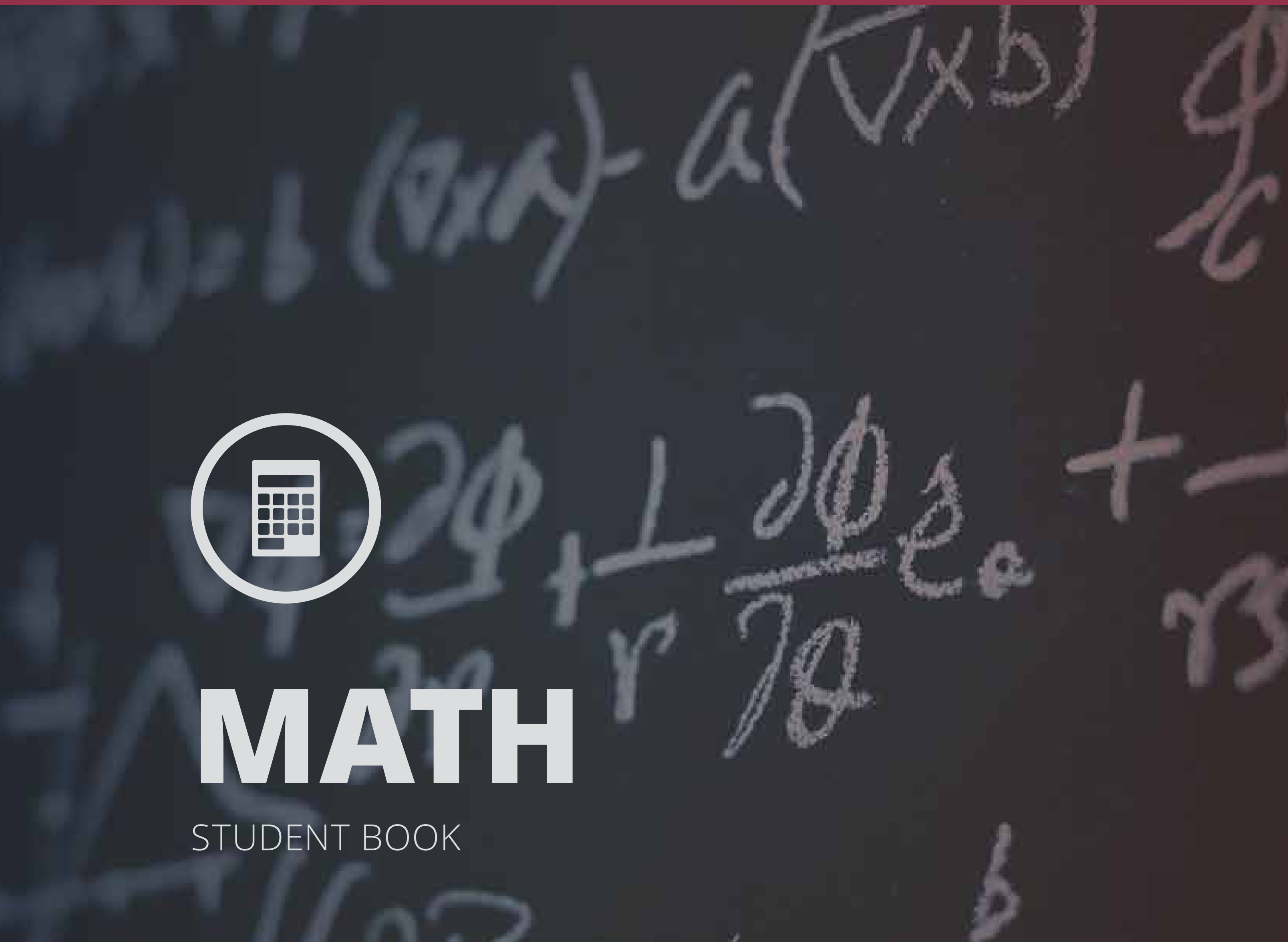

MATH

STUDENT BOOK

8th Grade | Unit 2

Alpha Omega
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Math 802

Modeling Problems in Integers

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Modeling Problems in Integers

Introduction

In this unit, functions are represented in a variety of ways. Students are asked to interpret graphical models in practical situations. Variable, expression, and equation are defined. Students solve one-step and two-step equations with whole numbers first. After algebra tiles and the number line are used to establish the "rules" for adding, subtracting, multiplying, and dividing integers, students solve equations in the integers. The order of operations is revisited in this unit. Students evaluate expressions using signed numbers. Finally, students are guided through the process for using an equation.

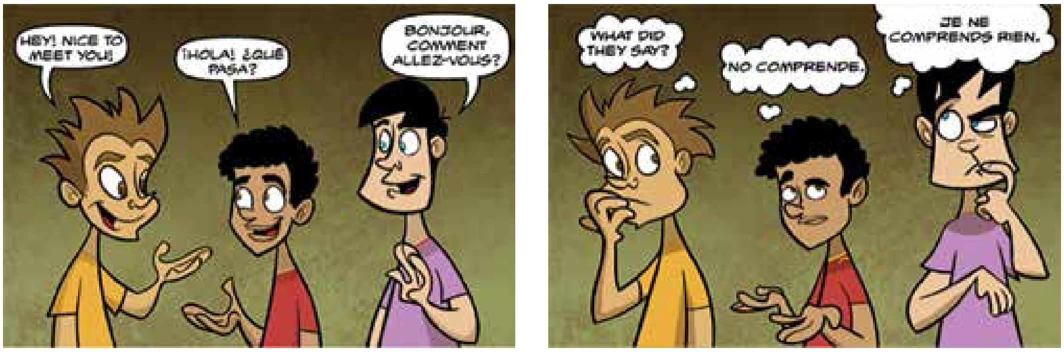
Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Perform operations of integers.
- Solve one-step and two-step equations, with real numbers and integers.
- Translate contextual situations into one-step and two-step equations before solving them.
- Identify relations and functions in their many forms, including ordered pairs, mapping diagrams, t-charts, and graphing.
- Identify domains, ranges, independent variables, dependent variables, and inputs and outputs.
- Graph functions and read the graphs of functions.

1. Equations with Real Numbers

Translating Expressions and Equations



Objectives

- Translate written statements into math symbols, expressions, and equations.
- Represent a simple word problem as an equation.

Vocabulary

equation—a mathematical statement that shows two expressions are equal; it has an equal sign

Translating Expressions

Have you ever read a word problem and had to reread it a couple of times to try to figure out what it is asking? Sometimes it feels like it was written in a foreign language. In a way, it was. It was written in mathematical language. Similar to foreign languages, the mathematics language also has rules of usage and conventions. In order to understand it, all you have to do is be able to translate it into a math problem full of numbers, symbols, and variables.

To be able to read mathematical statements, you must become familiar with how to give them meaning. You can learn to translate from phrases and sentences written in English to those written in math. You will learn the language of math, just as you learned English. You started simply by learning sounds of letters. Then, you learned to use letters to make words.

Numbers are the first part of the mathematical language that you learned. You know how to translate from a number written as a word in English to its mathematical symbol, the numeral. The word "five" is the numeral "5" and you understand what it means in both languages.

Figure 1 is a table of math symbols. In the columns are some of the words that translate from English into that math symbol. You may be able to think of more words that you have used previously to represent these.

+	-	×, ·, ( )	÷, —	=
add	subtract	multiply	divide	is
addition	subtraction	multiplied by	quotient	equals
sum	difference	product	divided into	equal to
more than	minus	times	per	is equal to
more	from	of		
plus	less than	by		
added to	less			

Figure 1| Math Symbols

Now when we see the these words, we can translate them into the correct mathematical symbol. Let's move on to putting some words together to form expressions.

English Clause	Mathematical Expression
the sum of sixteen and seven	16 + 7 *or* 7 + 16
the quotient of fifty-five and eleven	$55 \div 11$ *or* $\frac{55}{11}$
thirteen multiplied by six	13×6 *or* $13 \cdot 6$ *or* 13(6) *or* 6×13 *or* $6 \cdot 13$ *or* 6(13)
twenty less than ninety	90 - 20

Keep in mind! There is a difference between "less" and "less than." Words like "than" and "from" mean you need to switch the order of the numbers in the expression.

Remember that addition and multiplication are commutative, which means that it doesn't matter what order we write the numbers in. Subtraction and division are not commutative, so we have to be careful to write the numbers in the correct order. "Twenty less than ninety" cannot be written as 20 - 90.

Variables

Sometimes in math, we need to represent an unknown quantity. Consider the statement, "Somebody is three years old." We do not know who somebody is, so it is an unknown. In math, we might say, "seventeen added to a number." In this case, we know seventeen means 17, but "a number" could mean any number. Since we are unsure of the number, we need a variable to represent it in the expression.

Look at the following table to see how we use variables in math expressions.

English Clause	Mathematical Expression
fifteen added to a number	$15 + x$ *or* $x + 15$
a number divided by nine	$n \div 9$ *or* $\frac{n}{9}$
four subtracted from a number	$y - 4$
the product of a number and nine	$9k$

You can see that there are different letters in the expressions. There is no rule that says what letter you have to use, but you may find it easier to use a letter that makes sense for the problem. People often use the first letter of the name of the variable. For example, if you are finding the cost of something, you may want to use the variable *c*.

You might think that there is a mistake in the expression 9*k*. Product means to multiply, but there is not a multiplication symbol between nine and the variable *k*. This is actually a convention in math. *When a number and a variable, or two variables, are written next to each other with no operation sign between them, it means to multiply.*

Another convention that is important to note is that when a number and variable are written next to each other as a product, we always write the number first. For example, “a number times ten” would be written as 10*x*. “The product of a number and seven” would be written as 7*x*.

Writing Equations

In the English language, we find sentences to be more useful than clauses. Similarly, in the language of mathematics, we find *equations* to be more useful than expressions. Equations are similar to sentences because they allow us to show a complete thought.

The sum of a number and three equals five is a complete thought, which we can write mathematically as $x + 3 = 5$. We now know enough about our unknown number to find a value. An equation gives us information for each side of an equal sign. Look at the Figure 2 below for more examples of translating equations.

Let’s look at more examples of translating expressions and equations. Be sure to look carefully for the difference between the *expressions* and the *equations*.

Examples:

- The quotient of a number and nine is four.
  - Translation: $\frac{x}{9} = 4$
- Sixteen more than a number.
  - Translation: $y + 16$
- A number multiplied by six is equal to 72.
  - Translation: $6n = 72$
- There are 27 people in the room, and they line up into several rows, each row having the same number of people. Write an expression to represent the number of people that would be in each row.
  - Translation: $27 \div r$
- Sally’s neighbor asks her to babysit. She gets paid $5 an hour. Write an expression that represents how much money Sally would make for babysitting *h* hours.
  - Translation: $5h$
- Seven friends went to the movies last weekend. Each paid the same ticket price. Their total admission cost was $49. Write an equation to represent the cost *c* of each person.
  - Translation: $7c = 49$
- The Language Club is going to a French play. Tickets cost thirteen dollars each. It will cost forty-five dollars for bus rental. Write an expression that will represent the total cost for *s* students to go to the play.
  - Translation: $13s + 45$

English Sentence	Mathematical Equation
The product of seven and a number is 42.	$7x = 42$
Six more than twice a number equals 20.	$6 + 2x = 20$ *or* $2x + 6 = 20$
Five times a number less twelve is three.	$5y - 12 = 3$
The quotient of four times a number and seven is four.	$\frac{4x}{7} = 4$ *or* $4x \div 7 = 4$

Figure 2 | Translating Equations

In English, you moved from learning the alphabet to eventually writing essays. Our goal in math is to be able to solve problems. As you learn the language of mathematics you will be able to move from solving simple to more difficult word problems. Don't expect to skip the simple stuff and jump right into the tough stuff.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- You can translate English expressions that involve quantities into mathematical statements.
- A mathematical expression can be a constant, a variable, or terms combined using mathematical operations.
- An equation is a mathematical statement that says two expressions have the same value.

Complete the following activities.

1.1 Match the mathematical expression with its translation.

________	xy	the product of two numbers is six
________	$\frac{x}{y}$	six times a number equals y
________	$x - y$	the sum of x and y
________	$x + y$	the sum of two numbers is six
________	$y - x$	x divided by y
________	$y \div x$	y divided by x
________	$x + y = 6$	x subtracted from y
________	$xy = 6$	x minus y
________	$6x = y$	six less than a number is y
________	$y = x - 6$	the product of two numbers

1.2 Which of the following expressions represents "the product of a number and eight"?

☐ $8x$ ☐ $x + 8$ ☐ $x \div 8$ ☐ $8 - x$

1.3 Which expression would represent the cost of one CD, if the total cost for three of them is $36?

☐ 3(36) ☐ 3 + 36 ☐ 36 - 3 ☐ $\frac{36}{3}$

1.4 All of the following represent the same concept *except* _____.

- ☐ 47 take away 28
- ☐ 47 less than 28
- ☐ 47 minus 28
- ☐ 47 subtract 28

1.5 The expression $\frac{x}{8}$ could be translated as _____.

- ☐ a number and 8
- ☐ *x* divided by 8
- ☐ the product of *x* and 8
- ☐ 8 divided by *x*

1.6 Each of the following correctly match an English phrase with a mathematical expression *except* _____.

- ☐ a number times seven, 7*x*
- ☐ the sum of a number and seven, 7 + *x*
- ☐ a number subtracted from seven, 7 - *x*
- ☐ a number divided by 7, $\frac{7}{x}$

1.7 It costs twelve dollars to get in to the fair. Tickets for rides cost extra and are *d* dollars each. If Tyra buys 20 tickets, how much would it cost her for the fair?

- ☐ \$12 - 20*d*
- ☐ \$12*d* + 20
- ☐ \$12 + 20*d*
- ☐ 20*d* - \$12

1.8 In order for an expression to be an equation, it must have _____.

- ☐ an equal sign
- ☐ more than one variable
- ☐ a variable
- ☐ an addition sign

1.9 *xy* = 20 can be read as _____.

- ☐ the product of two numbers is twenty
- ☐ the quotient of two numbers is twenty
- ☐ the sum of two numbers is twenty
- ☐ the difference of two numbers is twenty

1.10 Ike is five years older than his brother. If Ike's age is represented by *x*, which of the following would represent his brother's age?

- ☐ *x* + 5
- ☐ 5*x*
- ☐ 5 - *x*
- ☐ *x* - 5

Solving One-Step Equations

You and two of your friends have $55 saved to buy concert tickets for your favorite band. You heard that tickets cost $18 a piece. Will you have enough money for all of you to attend the concert?

This lesson will introduce you to ways to solve word problems by translating them into math problems.

Objectives

- Translate and solve one-step equations in context.
- Identify the inverse operation needed to solve a one-step equation.
- Identify the property of equality used to solve a one-step equation.

Vocabulary

inverse operations—opposite operations that undo one another

property of equality—what happens to one side of an equation must also happen to the other side of the equation

solution—a value or values of the variable that make an algebraic sentence true

solve—find the solution(s) to an equation

Translating Word Problems to Equations

You can translate the word problem in the introduction into an equation. The total cost of the tickets is what we need to find. We can use a variable to represent this unknown. Because you are buying three tickets at the same price, you can find the total cost by adding 18 + 18 + 18. It can also be found by multiplying $3 \cdot 18$. So, if we use the variable c to represent the cost, then $c = 3 \cdot 18$, or $c = 54$. Since the total cost is $54, then yes, you have enough money!

Granted, you probably could have *solved* this problem without writing an equation. So, why did we?

It takes practice and skill building to get comfortable with problem solving. Some problems do not have answers that are obvious or arrived at easily. Try to compare it to reading. Your teacher did not give you a 200-page novel to read in second grade. As you went through school, teachers asked you to read more-challenging books. In math, you will start out with fairly simple problems, and they will get more difficult as you move on.

Some of the problems in this lesson may seem too simple and you won't want to bother with writing out the work. Try to keep in mind that what you learn now will prepare you to tackle the more difficult problems that you will face later on. There is a system for problem solving using an equation that you will need to practice.

Let's go back to the problem about the concert tickets. What if the problem said that you spent $81 on concert tickets for yourself and two of your friends? Your friends are paying you back and you need to tell each one how much they owe you. How much did each ticket cost?

We can write an equation: total cost = number of tickets · price of a ticket. Putting in the known values gives us $81 = 3 · *p*. We can simplify this equation to $81 = 3*p* or 3*p* = $81. The variable *p* represents the unknown, the price of a ticket.

Reminder! Two things touching, like the 3 and *p*, mean multiplication.

This problem differs from the previous problem because the variable is not all by itself on one side of the equation. We don't have *p* equal to something. We actually need to solve for the value of *p*.

To solve an equation means to find the value of the unknown, the variable. In order to do this, we must get the variable all by itself on one side of the equal sign. It does not matter which side.

Think About It! Why doesn't it matter which side of the equal sign the variable is on?

So, what is the process used to solve an equation?

We can think of an equation as if it were a balance scale. The two sides must be in balance at all times. For example, 5 = 5 is an equation. If you added two to one side of the equation and not the other, the equation would be "off balance." It would say 7 = 5. In other words, the two sides would no longer be equal to each other. If we add two to *both* sides of the equation though, we would have 7 = 7. This equation is still balanced.

You can keep an equation "balanced" by doing the same thing to *both* sides of the equation. We call this idea a *property of equality*. Figure 3 shows the properties of equality that you will use in this lesson.

The properties of equality really just say that whatever operation you do to one side of an equation you must do the same operation to the other side of the equation to keep it balanced.

So let's return to the problem of buying 3 concert tickets for $81. The equation was 3*p* = 81.

We want to solve the equation by solving for the unknown *p*. We need to get *p* by itself on one side of the equation. Since *p* is multiplied by 3, we need to have an operation that will "undo" multiplication. Division is the *inverse operation*, or opposite operation, of multiplication. To solve this equation, we must divide both sides by 3. When there is only one operation done to a variable that needs to be "undone," the equation is a "one-step" equation.

Property	Definition
addition property of equality	If the same value is added to both sides of an equation, the results are equal.
subtraction property of equality	If the same value is subtracted from both sides of an equation, the results are equal.
multiplication property of equality	If both sides of an equation are multiplied by the same value, the results are equal.
division property of equality	If both sides of an equation are divided by the same value, the results are equal.

Figure 3 | Properties of Equality

$$\frac{3p}{3} = \frac{81}{3} \qquad p = 27$$

We have solved for the value of p. We call 27 a *solution* of the equation, because it makes the two sides of the equation equal. We can check our solution by replacing 27 for the value of p in the original equation. If we are correct, both sides will be equal.

$$3(27) = 81$$
$$81 = 81$$

We just solved a one-step equation, because there was only one inverse operation needed to solve it. However, we solved the equation in a series of four steps. You should always follow these steps to solve a one-step equation. The steps are:

1. Locate the variable.
2. Identify the operation being done to the variable.
3. Do the inverse operation on both sides of the equation.
4. Check the solution.

Reminder!! When solving an equation that has a variable, the goal is to "undo" what was done to the variable.

Here are some examples using the above steps.

Example:

▸ Solve $x - 12 = 61$.

Solution:

$x - 12 = 61$	12 is being subtracted from x. The inverse operation is addition.
$x - 12 + 12 = 61 + 12$	Use the addition property of equality.
$x = 73$	Complete the addition.

Check:

$73 - 12 = 61$	Replace x with 73 in the original equation.
$61 = 61$	Check for balance.

Example:

▸ Solve $5p = 80$.

Solution:

$5p = 80$	p is being multiplied by 5. The inverse operation is division.
$\frac{5p}{5} = \frac{80}{5}$	Use the division property of equality.
$p = 16$	Complete the division.

Check:

$5(16) = 80$	Replace p with 16 to check the solution.
$80 = 80$	Check for balance.

Example:

▸ Find n for $\frac{n}{8} = 16$.

Solution:

$\frac{n}{8} = 16$	n is being divided by 8. The inverse operation is multiplication.
$8 \cdot \frac{n}{8} = 16 \cdot 8$	Use the multiplication property of equality.
$n = 128$	Complete the multiplication.

Check:

$\frac{128}{8} = 16$	Replace n with 128 to check the solution.
$16 = 16$	Check for balance.

Now let's solve some word problems by writing and solving one-step equations. We will need to combine the skill of translating with the skill of solving equations.

1. Use a variable to represent the unknown in the problem. (What are we asked to find?)
2. Write an equation.
3. Locate the variable in the equation.

SELF TEST 1: Equations with Real Numbers

Complete the following activities (6 points, each numbered activity).

1.01 *X* - 6 translates as "a number subtracted from 6."
- ○ True
- ○ False

1.02 One hundred is the solution of the equation *m* - 61 = 39.
- ○ True
- ○ False

1.03 4*b* + 19 can be translated as "nineteen more than the product of 4 and a number."
- ○ True
- ○ False

1.04 "Five times a number" can be written as 5*x*.
- ○ True
- ○ False

1.05 To solve 38 = 6*x* - 16, you would subtract 16 and then divide by 6.
- ○ True
- ○ False

1.06 Which of the following expressions represents "5 less than 3 times the number *m*"?
- ☐ 5 - 3*m*
- ☐ 3*m* - 5
- ☐ 5*m* - 3
- ☐ 3 - 5*m*

1.07 Which of the following steps could be used to solve the equation 93 = *x* + 58?
- ☐ Add 58 to both sides.
- ☐ Multiply both sides by 58.
- ☐ Subtract 58 from both sides.
- ☐ Divide both sides by 58.

1.08 Maria paid $48 for herself and two of her friends to go to a concert. The tickets are all the same price. What equation could be used to find the price of one ticket?
- ☐ p = 3 • $48
- ☐ p = $48 - 3
- ☐ 3 = $48p
- ☐ $48 = 3p

1.09 Which of the following is a solution for the equation 7*x* + 21 = 105?
- ☐ 105
- ☐ 84
- ☐ 15
- ☐ 12

1.010 The senior class had a car wash to raise money for the senior trip. It cost them $35 for supplies. They washed 75 cars and charged $6 for each car. Which of the following expressions represents the money they raised for their trip?
- ☐ $35 + 75 • $6
- ☐ 75 • $6 - $35
- ☐ $35 - 75 • $6
- ☐ 75 - $35 • $6

1.011 All of the following represent the same expression *except* _____.

- ☐ 19 less than 25
- ☐ 19 subtracted from 25
- ☐ 25 less than 19
- ☐ 25 minus 19

1.012 This model shows the solution to an equation.
What property of equality was used to solve the equation?

$$x - 34 = 13$$
$$x - 34 + 34 = 13 + 34$$
$$x = 47$$

- ☐ addition
- ☐ multiplication
- ☐ subtraction
- ☐ division

1.013 The cost to rent a video game for 3 days was $10. It costs the same amount each day. How could one find the cost to rent the video for one day?

- ☐ The cost to rent for one day is the product of $10 and 3.
- ☐ The cost to rent for one day is the quotient of $10 and 3.
- ☐ The cost to rent for one day is the sum of $10 and 3.
- ☐ The cost to rent for one day is the difference of $10 and 3.

1.014 Clarence solved the equation $3x + 15 = 33$ and showed the following work.

$3x + 15 = 33$
$3x + 15 - 15 = 33$
$3x = 33$
$\frac{3x}{3} = \frac{33}{3}$
$x = 11$

Which of the following is true?

- ☐ x is 11.
- ☐ x should be 16.
- ☐ x should be 6.
- ☐ x should be 99.

1.015 All of the following have the same solution *except* _____.

- ☐ $50 = 5m$
- ☐ $b + 2 = 8$
- ☐ $x - 6 = 4$
- ☐ $\frac{w}{5} = 2$

72/90 **SCORE** ________ **TEACHER** ________ ________
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