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MATH

STUDENT BOOK

8th Grade | Unit 5

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Math 805
More with Functions

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More with Functions

Introduction

This unit investigates functions in more detail. Students learn how to solve equations that are more complex. The equations require students to solve for a specified variable, use the distributive property, combine like terms, and/or rewrite the equation using an equivalent expression.

Students then apply these skills to write linear equations in slope-intercept form. Students learn how to identify different types of slopes as well as how to calculate slopes from two points on a line. They also learn more about graphing a linear function, whether from the slope, the intercepts, or an equation.

Finally, students learn how to identify and extend various number patterns

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Solve literal equations.
- Solve multi-step equations that involve one or more of the following: distributive property, combining like terms, and equivalent expressions.
- Identify the type of slope from a graph.
- Find a slope from a graph, mathematically, or from an equation.
- Find intercepts.
- Graph a line, given the slope and/or intercepts.
- Write equations in slope-intercept form.
- Graph quadratic and absolute value graphs.
- Extend number sequences, including arithmetic, geometric, exponential, and recursive.
- Graph exponential functions.

1. Solving Equations

Rewriting Equations

Every time you say, “Hello,” or “Goodbye,” do you always use those two words? Most of the time you probably use other words that have the same meaning. You might say, “Hi,” to your mom, but, “What’s up,” to one of your friends. “Hello,” “hi,” and “what’s up” all have the same meaning but look different. The same can be said about the word “goodbye.” You might use other words, like “see ya later” and “adios,” to express the same meaning. Math, like the English language, also has statements that have the same meaning but look different.

Objectives

- Rewrite formulas to solve for a specific variable.
- Solve for a missing value in a formula.

Vocabulary

equation—two equivalent expressions connected by an equal sign

inverse operations—opposite operations that undo one another

literal equations—equations and formulas with several variables

property of equality—what happens to one side of an equal sign must also happen to the other side of the equal sign

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Rewriting Equations

We can say hello in different ways. “Hi,” “what’s up,” and “yo” all mean the same thing. Well, there’s a way to write $z = 2xy$ in different ways too and still keep its meaning. Can you write two equivalent *equations* for $z = 2xy$?

Let’s take what we know about solving equations and apply it to an equation that is full of variables. An equation or formula that has several variables is called a *literal equation*. We can solve literal equations for any of the variables found in it.

Look back at the problem of $z = 2xy$. It’s currently solved for the variable z, but what if we wanted it solved for x? Just like when we solved equations before, we are looking to isolate the variable.

The x is currently on the same side as 2 and y. Do you remember how to move each of these? You need to use the *inverse operation* to move part of an equation to the other side of the equal sign. Because the 2, x, and y are all touching, we know that they are being multiplied. To move any of those terms, we will need to use division.

$z = 2xy$	Original equation.
$\frac{z}{2y} = \frac{2xy}{2y}$	Divide both sides by 2*y*.
$\frac{z}{2y} = x$	Simplify the right side.

What would happen if wanted to solve that same original equation for *y*?

$z = 2xy$	Original equation.
$\frac{z}{2x} = \frac{2xy}{2x}$	Divide both sides by 2*x*.
$\frac{z}{2x} = y$	Simplify the right side.

We can now answer our original question. Two equivalent equations to *z* = 2*xy* are $x = \frac{z}{2y}$ and $y = \frac{z}{2x}$. You can solve literal equations for any variable within the equation. Let's look at more examples.

Example:

- Solve for *r* in *D* = *rt*.

Solution:

$D = rt$	Original equation.
$\frac{D}{t} = \frac{rt}{t}$	Divide both sides by *t* to get *r* by itself.
$\frac{D}{t} = r$	Simplify the right side.

Example:

- Solve for *x* in *ax* + *b* = *c*.

Reminder! Always start with the terms not directly touching the variable when solving a multi-step equation.

Solution:

$ax + b = c$ $-b \quad -b$	Subtract *b* from both sides.
$ax = c - b$	Original equation.
$\frac{ax}{a} = \frac{c-b}{a}$	Divide both sides by *a*.
$x = \frac{c-b}{a}$	Simplify the left side.

Example:

- Solve for *b* in $A = \frac{bh}{2}$.

Solution:

$A = \frac{bh}{2}$	Original equation.
$(2)A = \frac{bh}{2}(2)$	Multiply both sides by 2.
$2A = bh$	Simplify the right side.
$\frac{2A}{h} = \frac{bh}{h}$	Divide both sides by *h*.
$\frac{2A}{h} = b$	Simplify the right side.

Example:

- Solve $F = \frac{9}{5}C + 32$ for C.

Solution:

$F = \frac{9}{5}C + 32$ $-32 \quad -32$	Subtract 32 from both sides.
$F - 32 = \frac{9}{5}C$	Complete the subtraction.
$\left(\frac{5}{9}\right)(F-32) = \frac{9}{5}C\left(\frac{5}{9}\right)$	Multiply both sides by $\frac{5}{9}$. Dividing by a fraction is the same as multiplying by its reciprocal.
$\left(\frac{5}{9}\right)(F-32) = C$	Use parentheses around (F - 32) to show $\frac{5}{9}$ being multiplied to all of it.

You can see that solving literal equations is similar to solving equations that you learned about before. We use inverse operations to move something to the other side of the equal sign. We also have to be sure to use the property of equality to keep the equation balanced.

Solving Equations with Given Values

Now, let's see what happens when we are given values for some of the variables.

Example:

- Solve for l in $V = lwh$, when $V = 120$, $w = 3$, and $h = 4$.

Solution 1:

$V = lwh$	Original equation.
$120 = l(3)(4)$	Substitute the values into the equation.
$120 = 12l$	Complete the multiplication on the right.
$\frac{120}{12} = \frac{12l}{12}$	Divide both sides by 12.
$10 = l$	Complete the division.

Solution 2:

$V = lwh$	Original equation.
$\frac{V}{wh} = \frac{lwh}{wh}$	Divide both sides by wh.
$\frac{V}{wh} = l$	Simplify the right side.
$\frac{120}{(3)(4)} = l$	Substitute the values into the new equation.
$\frac{120}{12} = l$	Complete the multiplication in the right denominator.
$10 = l$	Complete the division.

Compare! Notice we got the same value for l. In the first solution, we substituted first and then solved for l. In the second solution, we solved for l first and then did the substitution. You can do these problems either way.

Example:

- What is the temperature in Celsius if it is 86°F? Use the equation $F = \frac{9}{5}C + 32$.

Solution 1:

$F = \frac{9}{5}C + 32$	Original equation.
$86 = \frac{9}{5}C + 32$	Substitute 86 in for F.
$86 = \frac{9}{5}C + 32$ $-32 \qquad -32$	Subtract 32 from both sides of the equation.
$54 = \frac{9}{5}C$	Complete the subtraction.
$\left(\frac{5}{9}\right)54 = \frac{9}{5}C\left(\frac{5}{9}\right)$	Multiply both sides by $\frac{5}{9}$.
$30 = C$	Complete the multiplication.

Reminder! Dividing by a fraction is the same as multiplying by its reciprocal.

Solution 2:

$F = \frac{9}{5}C + 32$	Original equation.
$F = \frac{9}{5}C + 32$ $-32 \qquad -32$	Subtract 32 from both sides of the equation.
$F - 32 = \frac{9}{5}C$	Complete the subtraction.
$\left(\frac{5}{9}\right)(F - 32) = \frac{9}{5}C\left(\frac{5}{9}\right)$	Multiply both sides by $\frac{5}{9}$.
$\left(\frac{5}{9}\right)(F - 32) = C$	Complete the multiplication.
$\left(\frac{5}{9}\right)(86 - 32) = C$	Substitute 86 in for F.
$\left(\frac{5}{9}\right)(54) = C$	Complete the subtraction.
$30 = C$	Complete the multiplication.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Literal equations have several variables.
- Literal equations can be solved for any variable.
- Literal equations are solved using inverse operations and the property of equality.

Complete the following activities.

1.1 Solve for *m* in $D = \frac{m}{v}$, if *D*= 5.1 and *v* = 0.3. *m* = ____________

1.2 Solve for *w* in *P* = 2*w* + 2*l*, if *P* = 38 and *l* = 12. *w* = ____________

1.3 Solve for *t* in *d* = *rt*, if *d* = 57 and *r* = 30. *t* = ____________

1.4 Solve for *V* in $V = s^3$, if *s* = 4. *V* = ____________

1.5 Solve for b_2 in $A = \frac{1}{2}h(b_1+b_2)$, if *A* = 16, *h* = 4, and $b_1 = 3$. b_2 = ____________

1.6 The formula $F = \frac{9}{5}C + 32$ is used to convert Celsius to Fahrenheit temperature. If the temperature is 20°C, what is it in Fahrenheit?

☐ 28.8°F ☐ 43.1°F ☐ 68.0°F ☐ 93.6°F

1.7 Which of the following statements explains how to solve for *l* using the formula *A* = *lw*, when $A = 28\frac{1}{4}$ and $w = 2\frac{1}{2}$?

☐ Divide $28\frac{1}{4}$ by $2\frac{1}{2}$.
☐ Multiply $28\frac{1}{4}$ by $2\frac{1}{2}$.
☐ Divide $2\frac{1}{2}$ by $28\frac{1}{4}$.
☐ Subtract $2\frac{1}{2}$ from $28\frac{1}{4}$.

1.8 Convert 77 degrees in Fahrenheit to Celsius temperature using the formula $F = \frac{9}{5}C + 32$.

☐ 72°C ☐ 61°C ☐ 35°C ☐ 25°C

1.9 Using the formula $P = \frac{F}{A}$, find *F* if *P* = 27 and *A* = 4.

☐ 108 ☐ 6.75 ☐ 0.148 ☐ 29

1.10 All of the following are equivalent *except* _____.

☐ $d = rt$ ☐ $dt = r$ ☐ $\frac{d}{t} = r$ ☐ $\frac{d}{r} = t$

1.11 Solve *C* = *AB* + *D* for *B*.

☐ $\frac{C+D}{A} = B$ ☐ $AC\text{-}D = B$ ☐ $\frac{C}{A} - D = B$ ☐ $\frac{C-D}{A} = B$

1.12 In which of the following solutions would you multiply both sides of the equation by *n*?

☐ Solve $\frac{m}{n} = p$ for *m*.
☐ Solve *mn* = *p* for *m*.
☐ Solve *m* - *n* = *p* for *m*.
☐ Solve *m* + *n* = *p* for *m*.

1.13 Which of the following statements explains how to solve for *w* in the equation *A* = *lw*?

☐ Multiply both sides by *l*.
☐ Multiply both sides by *A*.
☐ Divide both sides by *l*.
☐ Divide both sides by *A*.

1.14 Solve for b in the formula 3*a* + 2*b* = *c*.

☐ $b = c - 3a$ ☐ $b = \frac{c-2}{3a}$ ☐ $b = \frac{c-3a}{2}$ ☐ $b = \frac{2c}{3a}$

Combine Like Terms

When you have a lot of something, like clothes, it is often helpful to have your things organized. This lesson will show how organizing math terms can help you simplify expressions.

Objectives

- Identify like terms in an algebraic expression.
- Combine like terms in an algebraic expression.

Vocabulary

coefficient—the number in front of a variable in a term

constant—a number; a term containing no variables

like terms—terms that have the same variable(s), with each variable raised to the same exponent

term—parts of expressions and equations separated by operation symbols and/or equal signs

Like Terms

Consider the following: x, $2x^2$, 4, $5y$, $3x$, $7x^3$, y, 7, *and* 9. Each of these items is called a *term*. A term is part of an expression or equation that is separated by operation symbols and/or equal signs.

Some of our terms have variables, some are just numbers called *constants*, and some have both. If a term has a variable and a number, the number in front of the variable is called a *coefficient*. A coefficient tells us how many of that variable there are. For example, the term $5y$ means that there are five y's.

Let's separate our terms into two groups. Our first group will be terms that have variables. The second group will be the terms that do not have variables.

With Variables		Without Variables	
x	$2x^2$	4	9
$5y$	$3x$	7	
$7x^3$	y		

Our group without variables is good, but look at our group with variables. When we look closer, we notice that we have a couple different kinds of variables in it. Some of the terms have x's, and some have y's. Let's break our variable group down again, separating terms with x's from terms with y's.

x's		y's
x	$2x^2$	$5y$
$7x^3$	$3x$	y

We now have three groups. The first group has the terms with x's, the second group has the terms with y's, and the third group is the terms without variables. Look closely at our group of terms with x's. Notice that they have different exponents. Let's separate that group into terms with the same exponent.

With Variables			Without Variables	
x's		y's		
x	$2x^2$	$5y$	4	9
$7x^3$	$3x$	y	7	

We have now separated our terms into what we call *like terms*. Like terms are terms with the same variable to the same exponent. Look at the chart below to see how our original terms have been separated into like terms.

x	$2x^2$	$7x^3$	$5y$	4
$3x$			y	7
				9

Combining Like Terms

Notice that we started with nine different terms, and we still have nine different terms. The only thing we did was organize them into like terms. We can now simplify the like terms by combining them.

$$7x^3 + 2x^2 + x + 3x + 5y + y + 4 + 7 + 9$$
$$7x^3 + 2x^2 + 4x + 6y + 20$$

To combine like terms, you combine the coefficients and keep the variable and exponent the same. When you combine terms, they will have the same total value after they're combined as they did before they were combined.

If terms are not like, you cannot combine them. The following table has examples of *unlike* terms.

Terms	Type	Reason they cannot combine
$7x$ and $3y$	Unlike	Not the same variable
$2x$ and $5x^2$	Unlike	Not the same exponents
$-4x$ and $5xy$	Unlike	Not all the same variables

Let's practice combining like terms.

Example:

- Simplify $4x^2 + 7 + 6x + 9 + x$.

Solution:

$4x^2 + 7 + 6x + 9 + x$	
$4x^2 + 6x + x + 7 + 9$	Rearrange like terms to be next to one another.
$4x^2 + 7x + 16$	Combine like terms.

Reminder! x is the same as $1x$.

Example:

- Simplify $4x + 6y + 2xy + 7y + 9x$.

Solution:

$4x + 6y + 2xy + 7y + 9x$	
$4x + 9x + 2xy + 6y + 7y$	Rearrange like terms to be next to one another.
$13x + 2xy + 13y$	Combine like terms.

- We did not combine the $2xy$ with any of the other terms because none of the other terms have *both* variables.

So far, we have been combining like terms that are all separated by addition signs. We also can combine like terms that are separated by subtraction signs. Let's see what happens when subtraction signs are included.

Combining Like Terms with Different Signs

To combine is to add. If a term is subtracted, just reverse the sign of the coefficient and change the operation to addition. Let's look at some examples.

Example:

- Simplify $6 - 3x + 4x - 9$.

Solution:

$6 - 3x + 4x - 9$	
$-3x + 4x + (-9) + 6$	Rearrange like terms to be next to one another. Notice that we reversed the signs on the coefficients and the operations of the terms that were subtracted.
$x - 3$	Combine like terms.

Example:

- Simplify $-2x + 7 + 4 - 5x + x^2$.

Solution:

$-2x + 7 + 4 - 5x + x^2$	
$x^2 + (-2x) + (-5x) + 7 + 4$	Rearrange terms. Again, take the operation sign in front of each term with the term.
$x^2 - 7x + 11$	Combine like terms. To simplify the expression, change the negative coefficients back to subtraction operations.

Example:

- Simplify $-7y + 8y^2 - 3y - 6 - 9y^2$.

Solution:

$$-7y + 8y^2 - 3y - 6 - 9y^2$$

$$8y^2 + (-9y^2) + (-7y) + (-3y) + (-6).$$

Rearrange like terms, taking the sign with them

$$-y^2 - 10y - 6$$

Combine like terms. To simplify the expression, change the negative coefficients back to subtraction operations.

Let's look at what you've learned. You can make your work easier by shortening the expressions you're working with. One way you can do this is by combining like terms. You can combine like terms if you simply have an expression alone or if you have more than one expression related to each other, as in an equation. If a term is connected by subtraction, take the opposite (negative) of the coefficient and change the operation to addition (combination).

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Like terms are either constants or they are terms with the same variable(s) with the same exponent(s).
- You can simplify an expression by combining like terms.
- When a term is subtracted, change the coefficient to its opposite value and change the operation to addition.

Complete the following activities.

1.15 Determine whether each statement is true or false based on the expression.
$6x^2 - 2x - 14y + 3x$

1. There are 4 terms.
   - ○ True
   - ○ False

2. $6x^3$ and $3x$ are like terms.
   - ○ True
   - ○ False

3. The coefficient on y is 14.
   - ○ True
   - ○ False

4. Simplified, the expression is $6x^2 + 5x - 14y$.
   - ○ True
   - ○ False

5. The commutative property allows the expression to be written as $6x^2 - 14y + 3x - 2x$.
   - ○ True
   - ○ False

SELF TEST 1: Solving Equations

Complete the following activities (6 points, each numbered activity).

1.01 To solve for *m* in the formula $D = \frac{m}{v}$, use the multiplication property of equality.
- ○ True
- ○ False

1.02 To solve for *y* in the equation 2*x* + *y* = 5, subtract 2 from both sides of the equation.
- ○ True
- ○ False

1.03 The equation $r = \frac{u}{st}$ is equivalent to *u* = *rst*.
- ○ True
- ○ False

1.04 The expression $4y - 6 + 4y^2 - 9$ contains three terms.
- ○ True
- ○ False

1.05 $9x^2$ and $5x$ are like terms.
- ○ True
- ○ False

1.06 The sum of four consecutive integers is 74. What is the first integer?
- ☐ 16 ☐ 17 ☐ 18 ☐ 19

1.07 Simplify 8(*x* - 4).
- ☐ 8*x* - 4 ☐ *x* - 32 ☐ 8*x* - 32 ☐ *x* – 4

1.08 Solve 3*x* - 5 + *x* = 31.
- ☐ -12 ☐ -9 ☐ 9 ☐ 12

1.09 A rectangle has an area of 72 square units. The width of the rectangle is 9 units. The length of the rectangle is 2*x* + 4. What is the rectangle's length?
- ☐ 6 ☐ 7 ☐ 8 ☐ 9

1.010 Using the formula *P* = 2(*a* + *b*), find *a* when *P* = 15 and *b* = 5.
- ☐ 2.5 ☐ 5 ☐ 8 ☐ 10

1.011 Solve $-(2x + 4) = 18$.

☐ -11 ☐ -7 ☐ 7 ☐ 11

1.012 In solving the proportion $\frac{x-3}{4} = \frac{1}{4}$, which of the following would be your first step?

☐ $4(x - 3) = 1$ ☐ $4(x - 3) = 4(1)$

☐ $1(x - 3) = 4(4)$ ☐ $1(x - 3) = 4$

1.013 Kobe's overtime pay is $5 an hour more than his regular pay. He worked 8 hours at his regular wage and 3 hours at his overtime wage. He earned $114. What is Kobe's regular wage per hour?

☐ $5 per hour ☐ $14 per hour

☐ $9 per hour ☐ $23 per hour

1.014 Simplify $3x + 6x^2 - 5x - x^2$.

☐ $6x^2 + 2x$ ☐ $6x^2 - 2x$ ☐ $5x^2 - 2x$ ☐ $5x^2 + 2x$

1.015 Which of the terms *cannot* be combined with the others?

☐ $3xy$ ☐ $2x$ ☐ $-5x$ ☐ x

1.016 A triangle has one side that measures 9 units, another side that measures x, and a third side that measures 2 units more than x. The perimeter is 29 units. Which equation would we use to find the value of x?

☐ $x + x + 2 = 29$ ☐ $x + x + x + 2 = 29$

☐ $x + x + 2 + 9 = 29$ ☐ $x + x + x + 2 + 9 = 29$

1.017 When simplified, which of the following expressions has a coefficient of 5?

☐ $-4x - 9x$ ☐ $-4x + 9x$ ☐ $4x - 9x$ ☐ $4x - (-9x)$

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