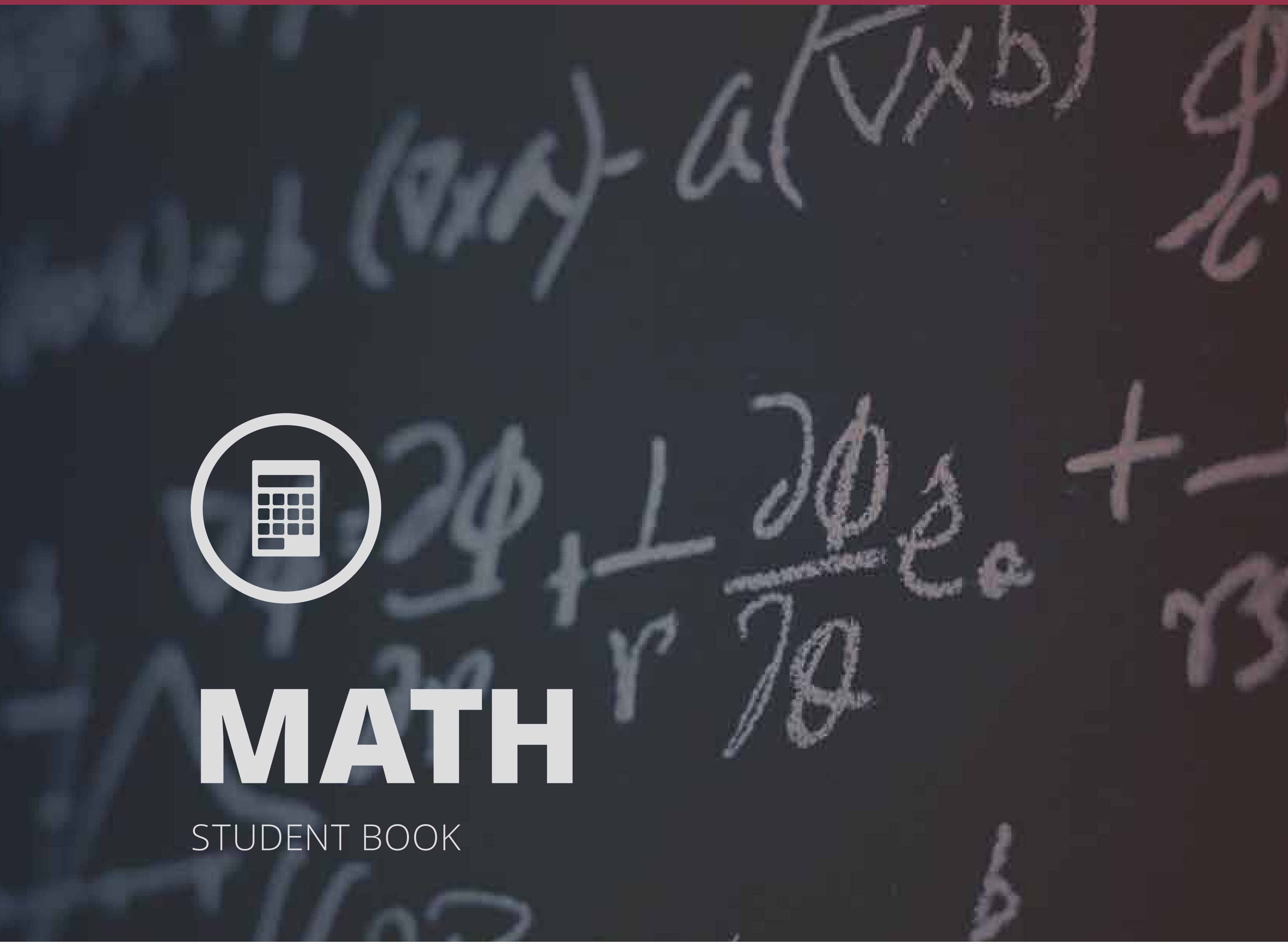
LIFEPAC®



MATH

STUDENT BOOK

8th Grade | Unit 7

Alpha Omega PUBLICATIONS

Math 807

Plane Geometry

Introduction |3

1. Perimeter and Area 5

Perimeter and Circumference |**5**
Area of Parallelograms |**12**
Area of Triangles and Trapezoids |**18**
Area of Circles |**23**
Composite Figures |**27**
Effects of Dimensional Changes |**33**
SELF TEST 1: Perimeter and Area |**42**

2. Symmetry and Reflections 45

Symmetry |**45**
Distance and Midpoint |**55**
Reflections |**63**
SELF TEST 2: Symmetry and Reflections |**71**

3. Other Transformations 77

Translations |**77**
Tessellations |**85**
Rotations |**90**
Dilations |**96**
SELF TEST 3: Other Transformations |**103**

4. Review 109

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Plane Geometry

Introduction

This unit is on geometry in the coordinate plane. Students will determine the area of parallelograms, trapezoids, and circles. Students analyze how change in dimension affects the perimeter and area of a figure. Symmetry is defined in this unit, and students are expected to identify lines of symmetry of various shapes, as well as lines of reflection. The Pythagorean theorem is revisited in the context of the coordinate system. Students are then shown how to find the distance between two points. Lastly, different transformations are examined in the coordinate plane.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Find the perimeter, circumference, or area of a plane figure.
- Use the formulas for perimeter, circumference, or area to find a missing measure of a plane figure.
- Classify parallelograms, triangles, and trapezoids.
- Find the area of parallelograms, triangles, and trapezoids.
- Determine how changes in dimension affect the perimeter or area of a plane figure.
- Determine if shape has line symmetry or rotational symmetry.
- Find the distance and midpoint of two points on a number line or coordinate plane.
- Recognize the four types of transformations and how to find the coordinates of an image or pre-image.

1. Perimeter and Area

Perimeter and Circumference

What do the following situations have in common?

- Running laps around a field.
- Tying a ribbon around a present.
- Making a border for a bulletin board.

Each of these situations involves the outside of a shape. If you wanted to know how far you were running, how much ribbon you needed, or the length of border, you would want to determine the perimeter of the shapes.

Objectives

- Find the circumference or perimeter of a figure.
- Estimate the circumference or perimeter of a figure.
- Find unknown dimensions of a figure by solving algebraic equations.

Vocabulary

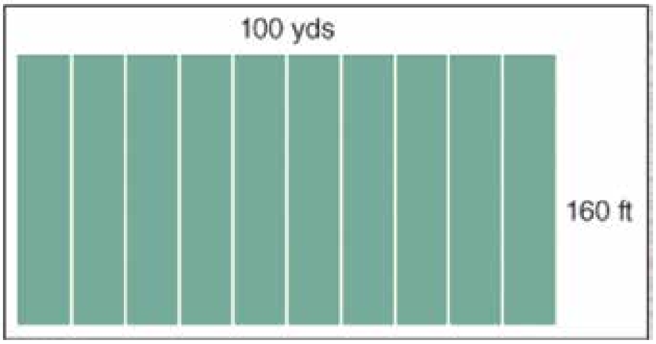
circumference—the distance around the outside of a circle
dimensions—the size of something (e.g., length, width, or height)
perimeter—the distance around the outside of a plane figure
plane figure—a two-dimensional figure

Perimeters of Polygons

Todd's gym class has to run two warm-up laps around the football field at the start of class every day. He started wondering just how far he runs on these daily warm-up laps. Let's help him figure it out.

Since Todd is running around a football field, he is running around the *perimeter* of a rectangle. Perimeter is the distance around the outside of a *plane figure*. To find the perimeter of any polygon, you simply add up the measures of all its sides. On a rectangle, the length and width of the sides are called *dimensions*. A football field has dimensions of 100 yards by 160 feet.

Vocabulary! A plane figure is any two-dimensional figure. Polygons and circles are examples of plane figures. Remember that a polygon is any closed figure made up of line segments.



There are two things we need to be careful of when solving Todd's problem. The first idea may seem simple, but it's easy to forget. When going all the way around a rectangle, there are two lengths and two widths. So, we'll need to add four numbers together. That's true for the perimeter of any polygon: the number of sides of the polygon should be the same as the number of addends.

Keep in mind! Sometimes we represent repeated addition using multiplication. For example, since there are two lengths of 100 yards on our rectangle, we could represent both sides by 2(100 yards) or 200 yards. But, the measure of every side should still be reflected in the sum.

The other thing to notice is that the length is in yards and the width is in feet. We need to convert all measurements to feet *or* yards in order to get an answer that makes sense.

Dimensions must have common units in order to calculate the perimeter.

Let's convert all measurements to yards.

This might help! To convert feet to yards, set up the following proportion, and cross multiply to solve: $\frac{160\text{ ft}}{x} = \frac{3\text{ ft}}{1\text{ yd}}$.

There are 3 feet in a yard, so 160 feet is equal

to $53\frac{1}{3}$ yards. Now, add the four sides together to find the perimeter.

$P = 100\text{ yd} + 53\frac{1}{3}\text{ yd} + 100\text{ yd} + 53\frac{1}{3}\text{ yd}$

$P = 306\frac{2}{3}\text{ yd}$

So, Todd runs $306\frac{2}{3}$ yards each lap around the field. But, wait! The problem said that he runs around the field *twice* each day. So, multiply the perimeter of the field by two.

$2(306\frac{2}{3}\text{ yd}) = 613\frac{1}{3}\text{ yd}$

Todd actually runs $613\frac{1}{3}$ yards each day.

S-T-R-E-T-C-H! There are 1,760 yards in a mile. What part of a mile does Todd run each day?

No matter what shape the polygon is, the perimeter is the sum of the lengths of every side. If you're not given a diagram to use, it's usually helpful to draw one for yourself. Diagrams are a great tool to help you see what you're working with and what you're trying to figure out.

You can also use equations as a tool for helping you solve perimeter problems. Although you might be able to figure out a problem in your head, writing equations for your work will help you with more difficult problems in the future. They also help you organize your thinking, so that you can see exactly what you need to do to solve the problem.

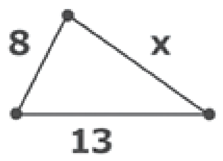
Let's look at a couple more examples. We'll use drawings and equations to help us solve the problems.

Example:

Julia is putting a triangular garden in her backyard. She wants one side of the garden to be 13 feet and another side to be 8 feet. If Julia bought 30 feet of border, what is the longest the third side can be?

Solution:

Let's first draw a picture. We don't know the exact dimensions of the garden, but drawing a picture can give us an idea of what it might look like.



Now, we can write an equation to solve this problem. You know that the perimeter is equal to the sum of the three sides.

Perimeter = 1st side + 2nd side + 3rd side	Equation for the perimeter of a triangle.
30 ft = 13 ft + 8 ft + x	Substitute in the known values.
30 ft = 21 ft + x	Add the constants on the right.
9 ft = x	Subtract 21 ft from both sides.

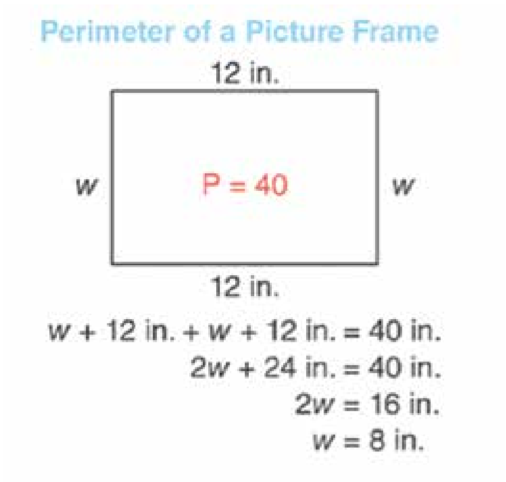
The third side can be up to 9 feet in length.

Example:

- Tony has a narrow piece of wood that is 40 inches long. He wants to use it to make a frame for a picture. Rafael wants the pieces for the top and bottom sides to be 12 inches. How long will the pieces for the width be?

Solution:

- Let's start by drawing a picture and writing what we know on the picture. Then, we can write an equation to solve the problem.



- So, the picture frame will be 12 inches by 8 inches, with a total perimeter of 40 inches.

Complete the following activities.

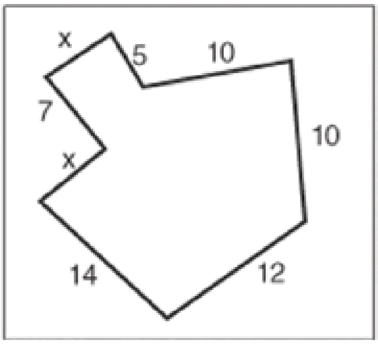
1.1 Find the perimeter of a square that is $3\frac{3}{4}$ inches on a side.

- ☐ $7\frac{1}{2}$ inches
- ☐ $12\frac{3}{4}$ inches
- ☐ 15 inches
- ☐ 30 inches

1.2 The measures of two sides of a triangle are 7.3 centimeters and 2.94 centimeters. The triangle's perimeter is 14.8 centimeters. Which equation could *not* be used to find the length of the third side?

- ☐ $7.3 + 2.94 + x = 14.8$
- ☐ $14.8 = x + 10.24$
- ☐ $14.8 - x = 7.3 + 2.94$
- ☐ $x - 14.8 = 2.94 + 7.3$

1.3 The perimeter of this figure is 70 feet. How long are the unknown sides?



- ☐ 5 ft
- ☐ 6 ft
- ☐ 10 ft
- ☐ 12 ft

1.4 The length of a rectangle is 14 meters. If the perimeter of the rectangle is 44 meters, what is the width?

- ☐ 8 m
- ☐ 16 m
- ☐ 22 m
- ☐ 7 m

Circumference of a Circle

The distance around a polygon is called its perimeter. We can find the perimeter of a circle, too, but it has a different name: *circumference*. Since circles don't have sides, we also have a different way to find the distance.

Connections! Numbers that continue infinitely, or go on forever, are irrational numbers. So, pi is an irrational number. For more information about rational and irrational numbers, see Unit 1.

All circles are similar. That means that they all have the same shape, but not necessarily the same size. Similar figures are proportional. With circles, that means that the ratio of the circumference to the diameter is always the same. In fact, that ratio has a special name: pi. Pi, or π, is approximately 3.14, although it's actually a decimal number that continues infinitely.

So, π is always the ratio of the circumference of a circle to its diameter.

Vocabulary! The *diameter* of a circle is the length of the distance across a circle, going through the center. The *radius* is half that distance, or the length of the segment from any point on the circle to the center.

$$\pi = \frac{C}{d}$$

Since we actually want to find the circumference, we need to solve for *C*. We can multiply both sides of the equation by *d* in order to get *C* by itself.

$$\pi \cdot d = \frac{C}{\cancel{d}} \cdot \cancel{d}$$

So, to find the circumference of a circle, we multiply pi by the diameter of the circle.

$$C = \pi \cdot d \text{ or } C = 3.14 \cdot d$$

Our formula for circumference is written using the diameter of the circle. Sometimes you may be given the *radius* of the circle instead. Diameter is two times the radius, so another version of this formula is $C = 2 \cdot \pi \cdot r$.

Let's try an example.

Example:

- What is the circumference of a circle that has a radius of 4 inches?

Solution:

- Since the radius of the circle is 4 inches, the diameter of the circle is 8 inches. Substitute 8 in for *d* in the formula.

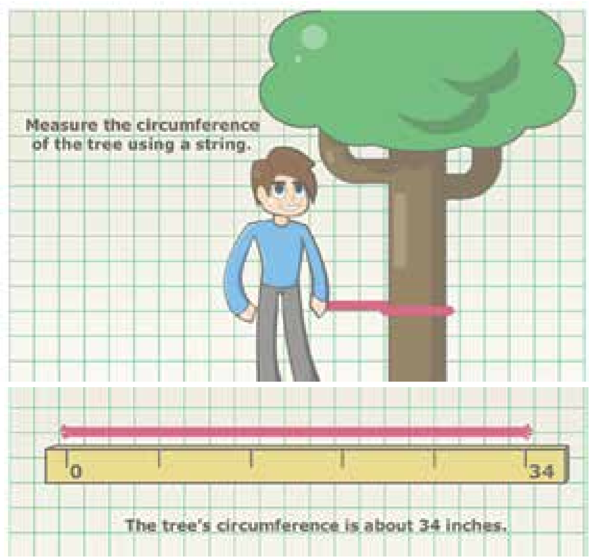
Make note! If you're using the pi button on your calculator, you'll probably get a different answer than in the example. That's because the calculator doesn't round to 3.14. It actually gives a more accurate value for pi. In your problem sets, though, use 3.14 for π.

- $C = \pi \cdot d$
- $C = (3.14)(8)$
- $C = 25.12$
- The circumference of the circle is 25.12 inches.

Let's look at one more example.

Example:

- There's a large tree in Marc's yard. He wants to know what its diameter is. He decides to use what he knows about circumference and pi to find it.



$$C = \pi d$$
$$34 = 3.14d$$
$$\frac{34}{3.14} = \frac{3.14d}{3.14}$$
$$\boxed{10.8 = d}$$

Solution:

- The diameter of Marc's tree is about 11 inches.
- Does Marc's answer make sense? To quickly check if Marc's answer is reasonable, round pi to 3. The

diameter of the tree times 3 should be close to the circumference. Eleven times 3 is 33, which is close to 34. So, Marc's answer makes sense.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- The perimeter of any polygon is the sum of its sides.
- To find the circumference of a circle, multiply the diameter by pi.
- Drawings and equations are helpful tools when finding an unknown measurement.
- Answers can be checked by rounding the dimensions and estimating.

Complete the following activities.

1.5 Jim has 20-inch wheels on his bicycle (their diameter is 20 inches). The tires are worn out, and he would like to cut them into pieces to use as a bumper on something he's making. *Estimate* the length of rubber Jim would get from his two bicycle tires.

☐ ≈ 60 in. ☐ ≈ 40 in. ☐ ≈ 120 in. ☐ ≈ 126 in.

1.6 Sonja bought a 14-inch pizza. How much crust is around its edge? (Use 3.14 for pi.)

☐ 21.98 in. ☐ 43.96 in. ☐ 47.74 in. ☐ 87.92 in.

1.7 The circumference of a circle is 19.468 meters. Find the radius of the circle.

☐ 3.1 m ☐ 6.2 m ☐ 16.328 m ☐ 12.4 m

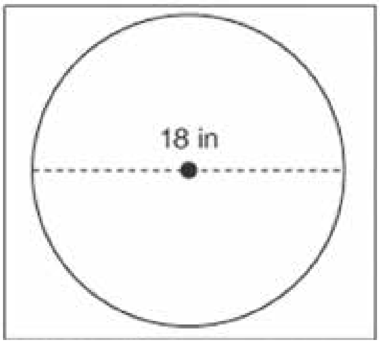
1.8 Jose wants to build a rectangular kennel for his dog. His dad will let him use a spot in the corner of the yard that is 12 feet 3 inches by 8 feet 11 inches. Estimate how much fencing he will need to buy for the kennel.

☐ ≈ 21 ft ☐ ≈ 33 ft ☐ ≈ 42 ft ☐ ≈ 84 ft

1.9 Carl is going to make a frame for a picture. The dimensions of the frame need to be 2 feet by 18 inches. How long must the board be that Carl uses for his frame?

☐ 20 in. ☐ 20 ft ☐ $3\frac{1}{2}$ ft ☐ 7 ft

1.10 What is the circumference of the following figure? Be sure to convert your answer to yards.



- ☐ 56.52 yd
- ☐ 1.57 yd
- ☐ 4.71 yd
- ☐ 28.26 yd

Area of Parallelograms

Do you see any parallelograms in this photo? There are lots of rectangles and a few squares. But, parallelograms? Actually, they all are! That's because rectangles and squares are special types of parallelograms. We're going to look at how these quadrilaterals are related and how to find their area.

Objectives

- Classify parallelograms based on their properties.
- Calculate the area of a parallelogram.
- Find a missing side length or height of a parallelogram.

Vocabulary

area—a measurement of the space inside a plane figure

parallelogram—a quadrilateral with two sets of parallel and congruent sides

rectangle—a parallelogram with four right angles

rhombus—a parallelogram with four congruent sides

square—a parallelogram with four right angles and four congruent sides

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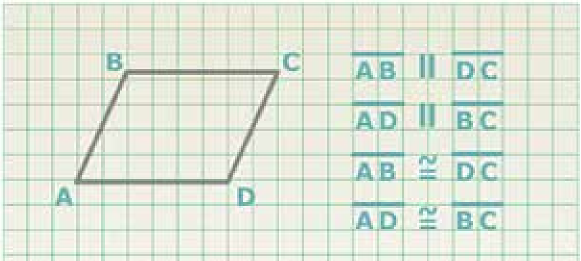
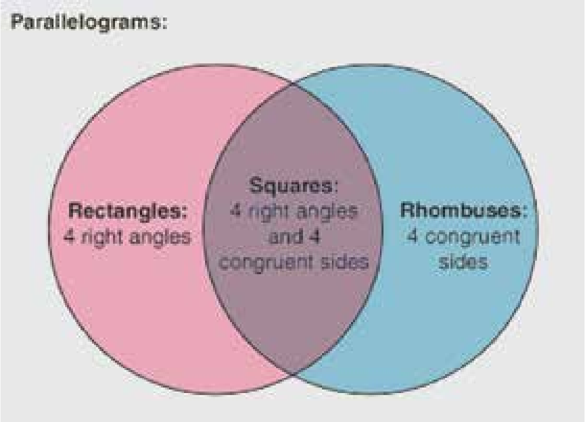
Parallelograms

A *parallelogram* is a special type of quadrilateral. It has two sets of parallel sides. Within each of the two pairs of opposite sides, the parallel sides are also congruent.

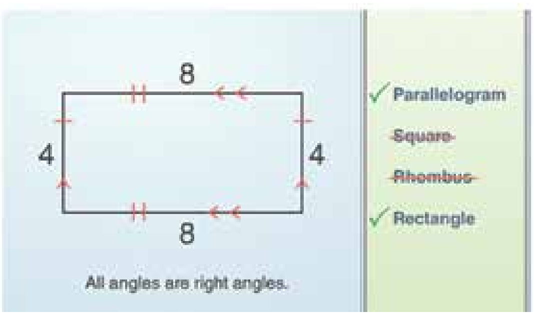


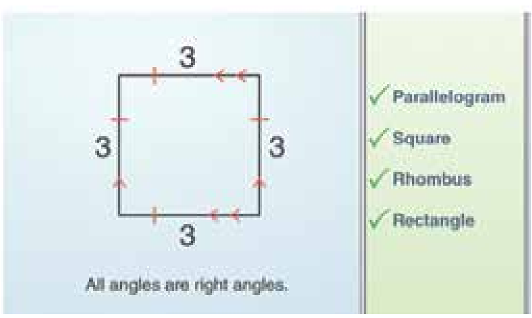
Rectangles, *rhombuses*, and *squares* are all types of parallelograms because they meet the conditions of having two pairs of parallel and congruent sides. Look at the diagram to see how these shapes are related.

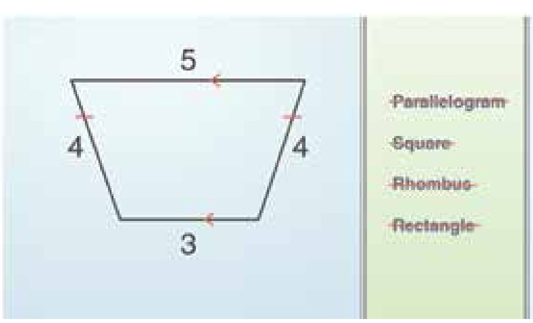


This might help! The Parallelograms Venn diagram shows us how the quadrilaterals are related. Rectangles and rhombuses are types of parallelograms because their circles are inside the set of parallelograms. Squares are a type of rectangle and a type of rhombus because they are represented by the overlapping portion of the circles.

Determining which sets a shape belongs to is called "classifying" the shape. Let's try classifying a few quadrilaterals together.

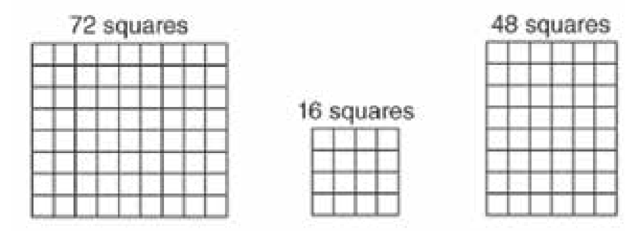






You know how to find the distance around the outside of a parallelogram, but how do you find the measure of the space *inside* the figure? That measurement is called the *area* of the parallelogram. For example, in the following figures, the area is the number of squares inside the figure.

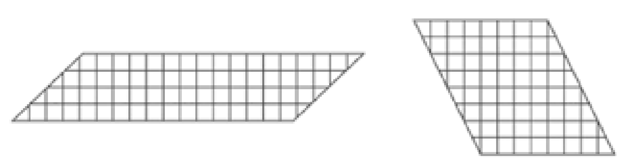
This might help! The distance around the outside of a polygon is its perimeter. It can be found by adding the lengths of the polygon's sides.



The area of a square or rectangle is easy to find. We can determine the number of squares by multiplying the number of squares along the base by the number of squares along the height. The formula is:

Area = base x height $\rightarrow A = b \cdot h$

But, what about a parallelogram that isn't a square or rectangle? It's not nearly as easy to determine the number of squares in the following figures:



There's a special property about parallelograms that we can use to help us. Let's see what it is.

A parallelogram can be cut apart and rearranged to make a rectangle.

SELF TEST 1: Perimeter and Area

Complete the following activities (6 points, every numbered activity).

1.01 Match the terms to their definition.

________	perimeter	the distance from the center to any point on the circle
________	area	a quadrilateral with two pairs of parallel sides
________	quadrilateral	any four-sided polygon
________	trapezoid	the distance from one point to another on a circle, going through the center
________	parallelogram	the distance around the outside of a circle
________	circumference	the distance around a polygon
________	radius	a quadrilateral with one pair of parallel sides
________	diameter	the space inside a figure

1.02 The circumference of a circle can be found using the formula $C = \pi \cdot r^2$.

○ True

○ False

1.03 All rhombuses are squares.

○ True

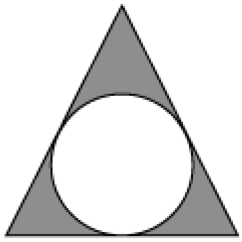
○ False

1.04 If the dimensions of a parallelogram are doubled, the perimeter will also be doubled.

○ True

○ False

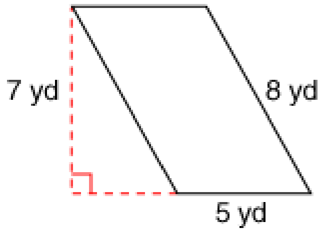
1.05 The shaded area of the following shape can be found by subtracting the area of the circle from the area of the triangle.



○ True

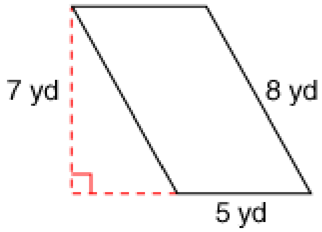
○ False

1.06 Find the perimeter of the parallelogram shown.



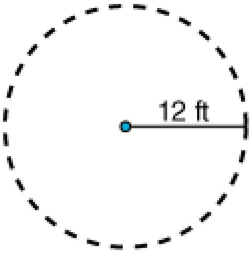
- ☐ 35 yd
- ☐ 24 yd
- ☐ 26 yd
- ☐ 40 yd

1.07 Find the area of the parallelogram shown.



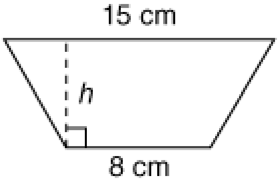
- ☐ 35 yd^2
- ☐ 24 yd^2
- ☐ 26 yd^2
- ☐ 40 yd^2

1.08 Wyatt's dog is tethered to a post in his yard. If the rope tied to the dog is 12 feet long, how many *square yards* does the dog have access to? (Use 3.14 for π.)



- ☐ 452.16 yd^2
- ☐ 50.24 yd^2
- ☐ 75.36 yd^2
- ☐ 25.12 yd^2

1.09 If the area of the following figure is 46 square centimeters, what is the height of the trapezoid?

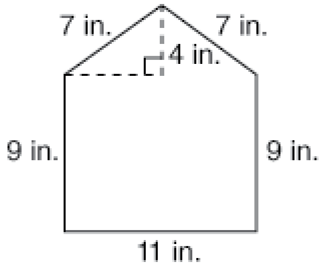


- ☐ 4 cm
- ☐ 2 cm
- ☐ 5.75 cm
- ☐ 3.1 cm

1.010 A rectangle has a base of 5 inches and a height of 7 inches. If the dimensions are cut in half, what will happen to the area of the rectangle?

- ☐ It will be half of the original size.
- ☐ It will be the same as the original size.
- ☐ It will be four times the original size.
- ☐ It will be one-fourth the original size.

1.011 Find the area of the following figure. The base angles are right angles.

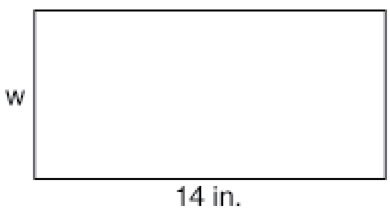


☐ 143 $in.^2$
☐ 137.5 $in.^2$
☐ 121 $in.^2$
☐ 113 $in.^2$

1.012 Which of the following is the best estimate of the circumference of a circle that has a diameter of 7 feet?

☐ 11 ft
☐ 14 ft
☐ 42 ft
☐ 21 ft

1.013 The perimeter of the following rectangle is 40 inches. What is the length of *w*?



☐ 2.9 in.
☐ 6 in.
☐ 12 in.
☐ 26 in.

1.014 An octagon has equal sides that each measure 11 centimeters. Its perimeter is 66 centimeters.

○ True
○ False

1.015 A rectangle has a base of 3 inches and a height of 9 inches. If the dimensions are doubled, what will happen to the area of the rectangle?

☐ It will be half of the original size.
☐ It will be the same as the original size.
☐ It will be four times the original size.
☐ It will be one-fourth the original size.

72/90 **SCORE** ________ **TEACHER** ________ ________
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