



## • 8th Grade | Unit 8



## **Math 808**

## Measures of Solid Figures

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**LIFEPAC Test is located in the center of the booklet**. Please remove before starting the unit. Author: Glynlyon Staff

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# **Measures of Solid Figures**

### Introduction

This unit introduces students to geometric solids. Students begin by learning about the characteristics of solids and how to use the characteristics to classify the solids. Next, students learn how to calculate the surface areas and volumes of common solids, as well as for composite figures. Finally, students learn how changes in the dimensions of figures affect volume.

### Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Identify geometric solids from three-dimensional, pictorial representations.
- Identify geometric solids from net representations.
- Identify the number of faces, bases, lateral faces, edges, and vertices for geometric solids.
- Compute the number of faces, vertices, and edges of a solid with Euler's formula.
- Calculate the surface area of geometric solids and composite figures.
- Calculate the volume of geometric solids and composite figures.

# 1. Surface Area

#### SOLID FIGURES

The world around you is full of geometric solids. Just think about the grocery store. The shelves are full of packages in all different shapes. Canned vegetables and pasta are packaged in two different shapes. Now, think about gym class. Most of the



# games you play in gym class use a ball that is another shape.

This lesson focuses on the different solids that make up our everyday world and how you can classify them by their characteristics.



#### Objectives

- Classify a three-dimensional figure by its characteristics.
- Name a three-dimensional figure by its base(s).
- Identify the number of faces, edges, and vertices for a figure.
- Identify the net of a three-dimensional figure.

#### Vocabulary

base of a solid—a special face of a solid figure

**cone**—three-dimensional figure with a circular base and lateral surface that tapers to a point

cube—three-dimensional figure made of six congruent squares

**cylinder**—three-dimensional figure with two parallel, equal circular bases and a curved surface

face—a plane figure that is one side of a solid figure

lateral face—any face that is not a base

**net**—a two-dimensional representation of a three-dimensional shape when unfolded

plane figure—a two-dimensional shape

**prism**—three-dimensional figure with two parallel, congruent polygon faces and all other faces are parallelograms

**pyramid**—three-dimensional figure with a polygon as a base and all other faces are triangles with a common vertex

**solid**—geometric figure with three dimensions (3D)

**sphere**—three-dimensional figure where all points are the same distance from the center **rectangular prism**—three-dimensional figure with six faces; has two rectangular parallel, congruent bases and four rectangular lateral faces

volume—the amount of space in a three-dimensional figure

#### **Geometric Solids**

As you know, the objects that you encounter in real life are all three dimensional (3D), meaning that they have a length, width, and height. Each of the shapes we see around us has a specific name. Like anything else, we will use the characteristics of the shape to determine its name.

The *face* of a *solid* is a *plane figure* that's one side of the figure. Some solids have no faces, while others may have 12 faces. Take a look at the following pictures. Each face has been shaded.



You can classify faces into two categories bases and lateral faces. Bases of solids are special faces that help name the solid, while a lateral face is any face that is not a base. Again, some solids may not have a base, while others may have one or two bases. There are never more than two bases on a solid.

In the following pictures, the base(s) has(have) been shaded.



These pictures show the lateral faces.



**Make note!** Two of the figures do not have lateral faces. A lateral face has to be a flat surface in one plane.

#### Prisms

Let's see how faces, more specifically bases, play a key role in identifying the different solids. We'll start with one of the most common 3D shapes, the *prism*.

You see prisms all the time and may not even realize it. Every time you see a box, you're actually looking at a prism. A prism is a 3D figure that has two parallel, congruent bases and has parallelograms for its remaining faces. Here are some pictures of prisms. Note that they don't all have the same-shaped bases.



Just like boxes, prisms come in all different shapes and sizes, as you see illustrated. Let's learn a little more about the different types of prisms, and about how we can distinguish their differences.

A rectangular prism is a prism that has rectangles as its bases. Remember that rectangles include squares! Take a look at some examples of rectangular prisms.





There is a special type of rectangular prism called a *cube*. A cube is a rectangular prism whose six faces are all the same square. A common example of a cube is dice.



**Make note!** Dice are not perfect cubes if their sides and corners (vertices) are rounded.

Another type of prism is the triangular prism. This time, the two parallel, congruent bases are triangles. Here are some examples.



There are other types of prisms. Again, we name them by the shape of their bases. Look at some examples of other prisms.







Pentagonal Prism

Hexagonal Prism

Octagonal Prism

#### **Pyramids**

*Pyramids* differ from prisms because they only have one base, and all of their lateral faces are triangles. Look at the following examples of pyramids. Again, notice that they have different bases, but all of the lateral faces are always triangles.



This time, let's begin with the triangular pyramid. A triangular pyramid has all triangular faces, including the base.



Rectangular pyramids have a base in the shape of a rectangle, or square, and lateral faces in the shape of triangles.



Like prisms, there are other pyramids named by the shape of their bases. Take a look at some examples of other pyramids.







Pentagonal Pyramid

Hexagonal Pyramid

Octagonal Pyramid

#### Cylinders

You probably have some food in a *cylinder* in your kitchen right now. Any time food is packaged in a can, whether its vegetables, tuna, or even tomato sauce, the can is made in the shape of a cylinder. A cylinder is a 3D figure with two parallel, congruent circular bases and a curved surface connecting the two bases.

**Make note!** Cylinders do not have lateral faces because their sides are curved, rather than flat.



#### Cones

A *cone* looks like a cross between a cylinder and a pyramid. A cone has one base in the shape of a circle. The base is attached to a curved surface that comes to a point above the base. You often see cones in construction zones or with ice cream in them. Here are some examples of cones that you will see in math.



#### **Spheres**

In many of the sports we play, we use *spheres*. A sphere is a 3D figure where all of the points are the same distance from the center. We know them better as balls. Think about the games you play, like basketball, baseball, softball, soccer, etc. They all use a sphere-shaped object.



#### Nets

Sometimes it's helpful to "unfold" a 3D shape. By unfolding the figure and laying it flat, we can see the entire surface of the shape at once. The unfolded figure is called the *net* of the solid. Let's take a look at the nets of some of the more-common geometric solids.





Nets will become more useful as you continue learning about solids.

#### Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Each solid can be identified and named by its characteristics.
- The shape of the base names the solid.
- A net is a flat representation of a 3-D solid.



- **1.1** Match the descriptions to their correct solid name.
  - \_\_\_\_\_ 3D figure where all points are the same distance from the center
  - \_\_\_\_\_ 3D figure made of all triangles
  - \_\_\_\_\_ 3D figure with six congruent squares
  - \_\_\_\_\_ 3D figure with two triangle bases and rectangles for the rest of its faces
  - \_\_\_\_\_ 3D figure with two circular bases
  - 3D figure with a rectangle as a base and triangles for the rest of its faces
  - \_\_\_\_\_ 3D figure with one circular base and one vertex
  - \_\_\_\_\_ 3D figure with rectangles for all of its faces

cone sphere triangular prism triangular pyramid cube rectangular prism rectangular pyramid cylinder

**1.2** A cereal box is an example of a:



- **1.3** A volleyball is an example of
  - a:\_\_\_\_\_



**1.5** A tent is an example of



**1.4** The Great Pyramid is an example of



**1.6** What shape can be created by the given net?





**1.7** What shape can be created by the given net?



□ square pyramid

□ cube

- □ triangular prism
- □ triangular pyramid

**1.8** What shape can be created by the given net?



- □ triangular pyramid
- □ square pyramid
- □ triangular prism
- rectangular prism

**1.9** What shape can be created by the given net?



- pyramidcylindercone
  - 🗌 cube

**1.10** What shape can be created by the given net?



- hexagonal prism
- pentagonal prism
- rectangular prism
- □ triangular prism

**1.11** What shape can be created by the given net?



- □ triangular prism
- Cube
- rectangular prism
- □ square pyramid

**1.12** What shape can be created by the given net?



- triangular prism
- rectangular prism
- □ square pyramid
- 🗌 cube

1.13



Describe how the figures are alike.

Describe how the figures are different.

## **SELF TEST 1: Surface Area**

**Complete the following activities** (6 points, every numbered activity).

Students, please check with your teacher to determine if a formula sheet can be used with this self test.

- **1.01** Surface area is always measured in square units.
  - O True
  - O False
- **1.02** Which of the following is *not* the net of a cube?



**1.03** A cone is a solid that has one circular base and only one face.

- O True
- O False

**1.04** What is the surface area of the cone? (Use 3.14 for  $\pi$ .)



- **1.05** The surface area of a geometric solid is the amount of space inside the solid.
  - O True
  - O False





920 ft<sup>2</sup>
 1,040 ft<sup>2</sup>
 760 ft<sup>2</sup>

**860 ft**<sup>2</sup>

**1.07** What is the surface area of the rectangular prism?





**1.08** The square pyramid pictured has a surface area of \_\_\_\_\_.



 $\square$  144 m<sup>2</sup>  $\square$  120 m<sup>2</sup>  $\square$  189 m<sup>2</sup>  $\square$  126 m<sup>2</sup>

**1.09** The net of a solid is the view from above the solid.

- O True
- O False
- **1.010** What is the surface area of a sphere with a radius of 7 feet? (Use 3.14 for  $\pi$ .)

    $\Box$  175.84 ft<sup>2</sup>
    $\Box$  615.44 ft<sup>2</sup>
    $\Box$  351.68 ft<sup>2</sup>
    $\Box$  703.36 ft<sup>2</sup>
- **1.011** Identify the two solids that create the composite figure shown.



- cone
- square pyramid
- 🗆 cube
- rectangular prism

1.012	<b>1.012</b> What is the radius of a sphere if its surface area is 2,122.64 square inches? (Use 3.14 for $\pi$ .)			
	□ 11 in.	🗖 13 in.	🔲 121 in.	🔲 169 in.
1.013	What is the slant height of a square pyramid that has a surface area of 189 square feet and a side length of 7 feet?			
	□ 14 ft	□ 12 ft	□ 10 ft	🔲 8 ft
1.014	What is the height of cylinder with a surface area of 226.08 square meters and a radius of 3 meters? (Use 3.14 for $\pi$ .)			
	27 meters	□ 18 meters	□ 3 meters	9 meters
<b>1.015</b> What is the surface area of the composite figure?				
	8 cm		<b>608</b> cm <sup>2</sup>	
			□ 672 cm <sup>2</sup>	
	11 cm		<b>570</b> cm <sup>2</sup>	
	8 cm		<b>544</b> cm <sup>2</sup>	

**1.016** Which container has the greatest surface area? (Use 3.14 for  $\pi$ .)







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