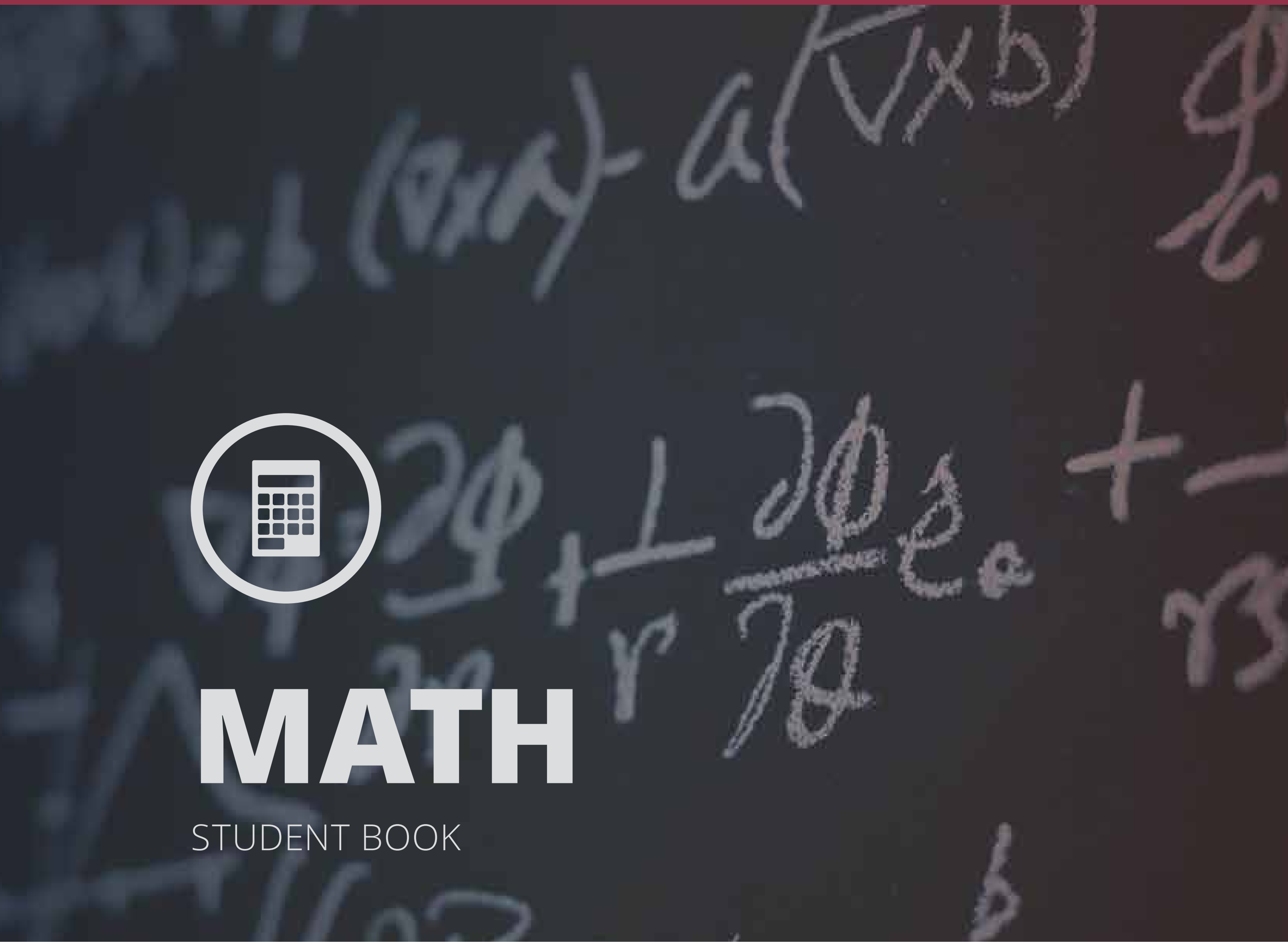
LIFEPAC®



MATH

STUDENT BOOK

8th Grade | Unit 10

Alpha Omega
PUBLICATIONS

Math 810
Probability

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Probability

Introduction

This unit addresses outcomes of events and the probabilities of different outcomes. The first section introduces efficient methods for determining outcomes, such as tree diagrams and formulas. It explains the distinction between permutations and combinations. The lessons let students work with the formulas that allow us to compute the number of possible permutations or combinations. Section 2 covers theoretical and experimental probabilities. Lessons in this section show how to find probabilities for disjointed and overlapping events, as well as independent and dependent events.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Determine the number of possible outcomes using tree diagrams and the fundamental counting principle.
- Identify and evaluate permutation and combination problems.
- Find theoretical and experimental probabilities.
- Identify and compute probabilities of independent and dependent events.
- Identify and compute probabilities of overlapping and disjointed events.

1. Outcomes

Tree Diagrams and the Counting Principle

Many science classes ask you to perform experiments as part of your assignments. You might even be asked to complete a lab report about your experiment, specifically about your outcome. Outcomes can also happen in experiments in math.

This lesson will help you understand the role that outcomes play in a probability experiment. You will also learn that knowing all possible outcomes can help you make predictions.

Objectives

- Identify all the possible outcomes for a given situation.
- Use tree diagrams to identify probabilities.
- Use the counting principle to identify probabilities.

Vocabulary

counting principle—uses multiplication to find the possible number of outcomes

outcome—a possible event

tree diagram—an organizational tool that uses branches to list choices

Outcomes

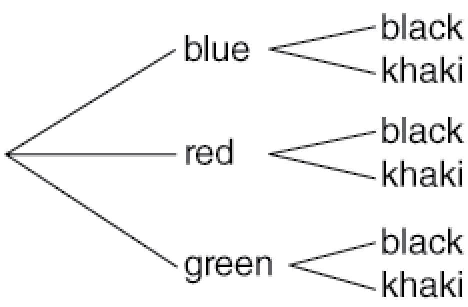
The *outcome* of a science experiment is the final result you get after completing the experiment. Outcomes, in probability experiments, are similar to those of a science experiment. The outcomes of a probability experiment are the possible events that can occur.

There are a number of ways to determine and list the outcomes for a probability experiment. This lesson is going to focus on using *tree diagrams*. Tree diagrams both help you determine the number of possible outcomes that can occur and allow you to see all the possible outcomes for a specific experiment. This lesson will also introduce you to the *counting principle*, which helps you determine the total number of possible outcomes for an experiment.

Tree Diagrams

Sometimes, it is hard to determine all the possible outcomes for an experiment without using some type of organizational tool. This is where tree diagrams are useful. A tree diagram is an organizational tool that uses branches to list choices/possible combinations. The tree diagram allows you to list all possible outcomes while seeing each possible combination of outcomes.

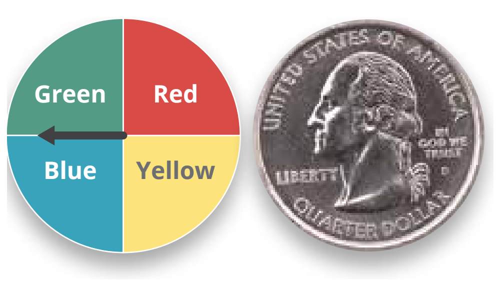
What are all the possible outcomes for combining a shirt with a pair of pants if you have three shirts (one blue, one red, and one green) and two pairs of pants (one black and one khaki)?



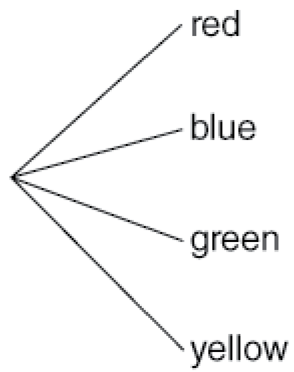
The tree diagram helps us see that there are six possible outcomes. The possible outcomes are:

- blue shirt with black pants
- blue shirt with khaki pants
- red shirt with black pants
- red shirt with khaki pants
- green shirt with black pants
- green shirt with khaki pants

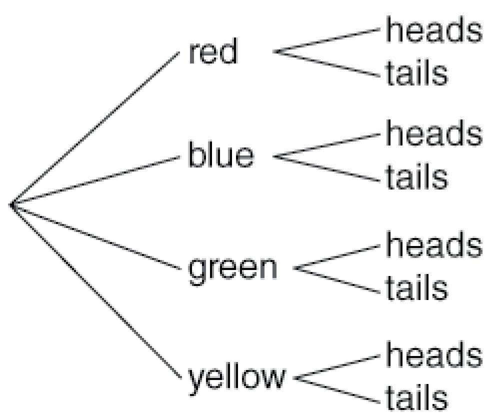
Let's take a look at another experiment. What are all the possible outcomes if you spin the following spinner and then flip the coin?



Let's make a tree diagram of the possible outcomes. The first step is to list all the possible outcomes for the spinner.



The next step is to draw two branches from each color. One branch should be labeled heads, and the other should be labeled tails. This adds eight new branches to our tree, resulting in the following diagram.

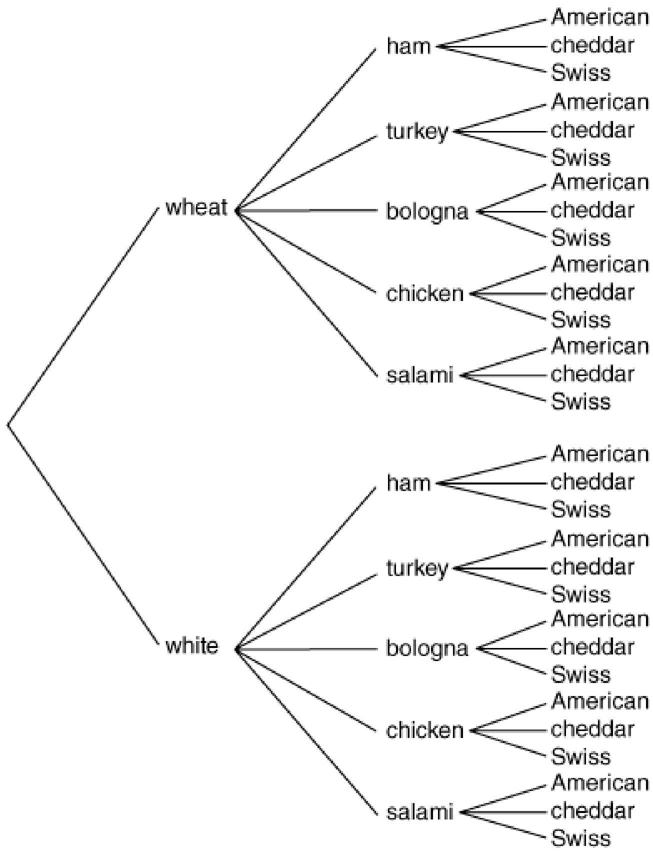


You can now see that there are eight possible outcomes. The possible outcomes are:

- red and heads
- red and tails
- blue and heads
- blue and tails
- green and heads
- green and tails
- yellow and heads
- yellow and tails

Let's take a look at a tree diagram for an experiment that has three different sets of options.

What are all the possible outcomes if you can choose from two different breads (wheat and white), five meats (ham, turkey, bologna, chicken, and salami), and three cheeses (American, cheddar, and Swiss)?



Take a second and count up the number of possible outcomes that exist when you choose one type of bread, one meat choice, and one cheese choice. There are 30 possible outcomes for these choices. A few possible outcomes are:

- wheat bread with turkey and cheddar cheese
- wheat bread with salami and Swiss cheese
- white bread with turkey and American cheese
- white bread with bologna and cheddar cheese
- white bread with chicken and Swiss cheese

Remember, this list isn't all the possible outcomes, just a few of them. If you want to know all the possible outcomes, read across each branch of the tree diagram.

The Counting Principle

As you saw in the last example, sometimes the tree diagrams can become quite large and complicated. Sometimes, you will be asked only to determine the total number of possible outcomes for a given experiment. In this case, you don't need to draw a tree diagram if you can find another way to determine the number of possible outcomes. This is where the counting principle is a useful tool. The counting principle uses multiplication to find the number of possible outcomes.

Let's look back at our last example using a tree diagram. The experiment started with two bread choices, five meat choices, and three cheese choices. Once we drew our tree diagram, we could see that there were 30 possible outcomes. But, what if we didn't care about what the outcomes were, just how many there were? We could use the counting principle to help us determine this.

Number of Breads		Number of Meats		Number of Cheeses		Total number of outcomes
2	•	5	•	3	=	30

The counting principle is easy to use—the only thing that you have to do is multiply the number of items in each category to determine the total possible number of outcomes. Let's take a look at some other examples.

Example:

- How many combinations are possible if you have four pairs of pants, nine shirts, and five sweaters?

Solution:

- 4 · 9 · 5 = 180 possible outfits
- Who would have thought that you would be able to create 180 different outfits from just four pairs of pants, nine shirts, and five sweaters! Let's go through a few more examples.

Example:

- A restaurant offers eight appetizers, 12 main courses, and five desserts. How many possible combinations exist on their menu?

Solution:

- 8 · 12 · 5 = 480 different combinations

Imagine drawing a tree diagram for the last example. There would be eight branches to represent the number of appetizers. Then, from each of those eight branches would be 12 branches to represent the main dishes. Finally, five branches would be drawn from each of the previous branches to represent the desserts. In all, you would have 480 branch stems to follow to see all the different combinations!

Example:

- Your new lock allows you to pick the combination. You want to make sure that there are a number of combinations to pick from. The lock has three dials. The first dial has 10 possible numbers. The second dial has 10 possible numbers, and the third dial has five possible numbers. Determine how many different outcomes you can pick from for your combination.

Solution:

- 10 · 10 · 5 = 500 different combination possibilities

Example:

- In choosing a license plate, you were offered the chance to design your own combination of letters and digits. How many different license plate combinations can be created if the license plate will have two letters and three digits, in that order, with repetition?

Solution:

- 26 · 26 · 10 · 10 · 10 = 676,000 different combination possibilities

Example:

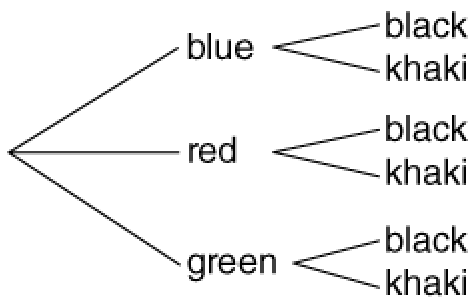
- After signing up for a new email account, you are asked to choose a secure password with three letters and two digits. How many different password combinations are you able to create with three letters and two digits, in any order, without repetition?

Solution:

- 26 · 25 · 24 · 10 · 9 = 1,404,000 different combination possibilities

Making Predictions

Knowing the number of possible outcomes can help you when you are asked to make predictions about an experiment. Let's look at a tree diagram to see how knowing all possible outcomes can affect your predictions.



The tree diagram shows all the possible combinations of three shirts (blue, red, and green) and two pairs of pants (khaki and black). We see that there are six possible combinations.

If someone asked you to predict the color of pants that would randomly be chosen, you can see from the tree diagram that khaki pants are found in three of the combinations and black pants are found in the other three combinations. This means that there is an equal chance of khaki or black pants being chosen.

What if you were asked to predict what color of shirt would be chosen? Does one shirt color appear in the combinations more than the other two? Looking at the tree diagram, we can see that each shirt color also has an equal chance of being picked.

Not all experiments will have outcomes that all have equal chances of happening. As you learn more about probability, you will see examples where some combinations have a higher—or lesser—chance of happening.

Let's Review

Before moving on to the practice problems, make sure you understand the main points of this lesson.

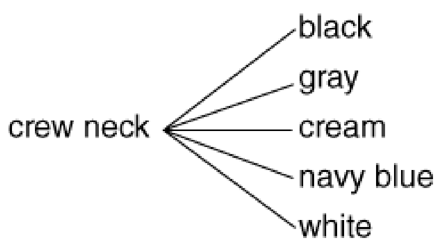
- Outcomes play a key role in any probability experiment.
- A tree diagram helps you see all the possible combinations and if any combination has a better chance of happening than the other combinations.
- The counting principle provides a shortcut for determining the number of combinations that can occur.
- To find the number of possible combinations, multiply the number of options in each category to get the total number of possible outcomes.

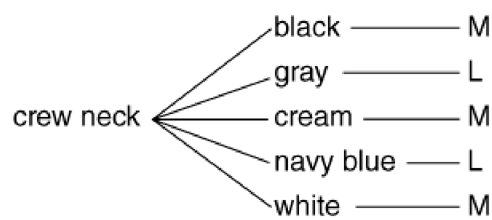
Complete the following activities.

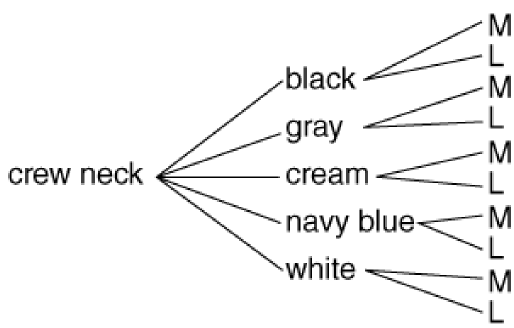
1.1 The counting principle lets you add the number of options per category to find the total number of possible outcomes.

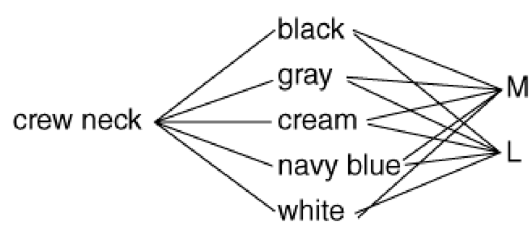
- ○ True
- ○ False

1.2 Santiago wants to buy a new sweater. He can choose the style (crew neck, v-neck, cardigan, or zip up), the color (black, gray, cream, navy blue, or white), and the size (medium or large). Which tree diagram shows all of the possible outcomes for a crew neck sweater.

☐ 

☐ 

☐ 

☐ 

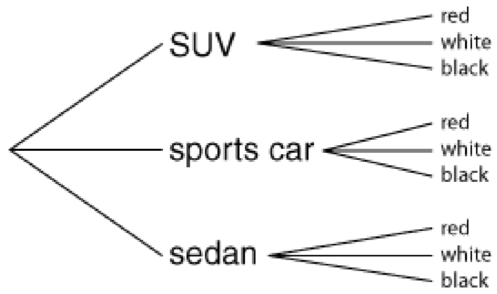
1.3 How many five-digit numbers can be created using the digits 0-9? A number can be repeated for different digits.

1.4 Many states license plates have three letters and three digits (0-9). How many different license plate combinations are available if repetition is allowed?

- ☐ 175,760
- ☐ 17,576,000
- ☐ 1,757,600
- ☐ 175,760,000

1.5 You and your friends are trying to decide on a movie. You can choose from eight movies, five show times, and three locations. How many combinations are possible?

1.6 Which problem matches the tree diagram given?



- ☐ A customer can choose from four car models offered in three colors.
- ☐ A customer can choose from three car models offered in four colors.
- ☐ A customer can choose from four car models offered in four colors.
- ☐ A customer can choose from three car models offered in three colors.

1.7 Alejandria has four shirts and three pairs of pants. She says that she has seven possible combinations.

- ○ True
- ○ False

1.8 A code consists of two letters (they can be repeated) followed by one digit (0-9). The number of possibilities is _________ .

Permutations

Any time you want to open a combination lock, you have to know the correct order of the numbers. It's not good enough to just know the three numbers. If you use them in the wrong order, you will not be able to open the lock. Because you must know the numbers in the correct order, this is a permutation.

This lesson will help you understand permutations and their use in practical applications.

Objectives

- Use permutations to count all possible outcomes.

Vocabulary

counting principle—uses multiplication to find the possible number of outcomes

factorial—the product of a natural number and all of the natural numbers less than itself

permutations—arrangement of objects in which order is important

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Permutations

Permutations are arrangements of objects in which order is important, just as in the case of the combination lock. It was not enough to know the three numbers of the combination. You also had to know the correct order of the three numbers. This is a good example of a permutation.

A permutation acknowledges that 1-2-3 is different from 3-2-1, 2-1-3, 2-3-1, 3-1-2, or 1-3-2. Although the same three numbers make up all the different sets of numbers, each is different from the others because of their order, thus making them permutations.

An easy way to find the number of permutations is to use the *counting principle*. The counting principle uses multiplication to find the total number of possible outcomes. You multiply the number of objects per group to find the total number of possible outcomes. Let's look at an example of using the counting principle to help find the number of permutations.

Example:

- ▸ Kelly, Cassandra, LaTasha, and Marisol are running in a race. In how many different ways can the four girls finish the race?

Solution:

- 4 girls have the possibility of coming in first. 3 girls would be left to come in second, 2 girls could come in third and 1 girl would be fourth.
  - 4 · 3 · 2 · 1 =
  - 24
- Using the counting principle, there are 24 different orders in which the four girls could finish the race.

Factorials

In the previous example, we used 4 · 3 · 2 · 1 to help us determine the number of different ways four girls could finish a race. There is another way for us to write that expression involving the use of *factorials*. We can rewrite 4 · 3 · 2 · 1 as 4! Anytime you see a number followed by an exclamation point, it means to multiply all of the natural numbers up to, and including, the number. Remember, the natural numbers begin with one.

Connections! Think of 4! as a short cut for saying 4 · 3 · 2 · 1, similar to how you use g2g, bff, and lol as shortcuts when texting your friends on your cell phone.

Let's evaluate some factorial expressions.

Example:

- Evaluate 6!

Solution:

- 6!
- 6 · 5 · 4 · 3 · 2 · 1
- 720

Example:

- Evaluate 9!

Solution:

- 9!
- 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1
- 362,880
- Let's see if the factorial method works for our original ordering of numbers for a combination lock. A combination lock uses three numbers in a particular order. Three factorial (3!) equals 3 · 2 · 1, which is six. That is the same number of permutations we found previously!

Permutations using Subsets of a Larger Set

Sometimes, you may not need to use all of the numbers available in the set. If this happens, you will have to apply your knowledge of factorials to a permutation formula.

- $_nP_r = \frac{n!}{(n-r)!}$

In this formula, the *n* stands for the total number of objects. The *r* stands for the number of objects taken at a time. Let's apply this notation to an example.

Example:

- Imagine you have eight books you want to put on a shelf, but your shelf can only hold three books. How many different ways can you organize the eight books into sets of three?

SELF TEST 1: Outcomes

Complete the following activities (6 points, each numbered activity).

1.01 Evaluate ${}_9P_4$.

☐ 15,120 ☐ 36 ☐ 3,024 ☐ 504

1.02 Evaluate ${}_6C_2$.

☐ 30 ☐ 720 ☐ 120 ☐ 15

1.03 Louisa has five shirts (white, gray, black, green, and blue), four pairs of pants (black, khaki, navy, and jeans), and two pairs of shoes (flip flops and sneakers). How many different outfits can she create?

☐ 10 ☐ 20 ☐ 40 ☐ 60

1.04 Which is greater, ${}_7P_5$ or ${}_7C_5$?

☐ ${}_7P_5$ ☐ ${}_7C_5$

1.05 How many ways can six kids line up in a single row to have their picture taken?

1.06 How many sandwiches can be created from three types of bread, six meats, and eight toppings, if you can only use one item from each category?

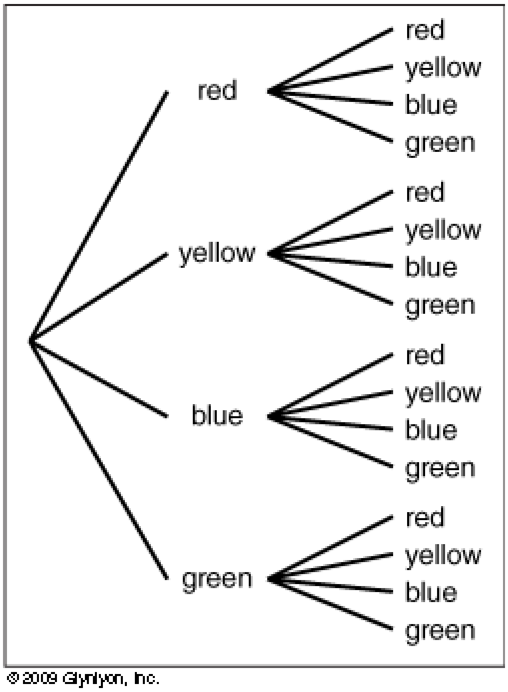
☐ 17 ☐ 144 ☐ 120 ☐ 72

1.07 How many combinations are there of the five letters *j*, *k*, *l*, *m*, and *n* using three letters at a time?

☐ 10 ☐ 120 ☐ 12 ☐ 60

1.08 Evaluate ${}_9P_2$.

1.09 How many different outcomes can happen from spinning a four-section spinner twice, according to the diagram given?



☐ 4
☐ 8
☐ 12
☐ 16

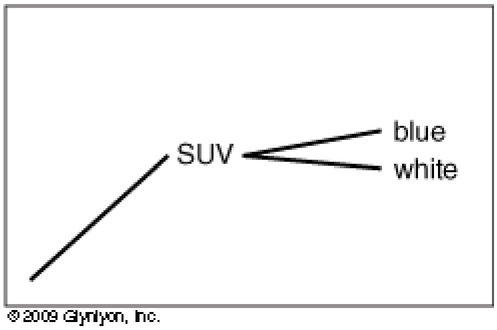
1.010 Your English teacher has given you a list of 10 possible topics for essays. You need to pick four topics from the list to write about over the school year. How many different groups of four essay topics can you choose?

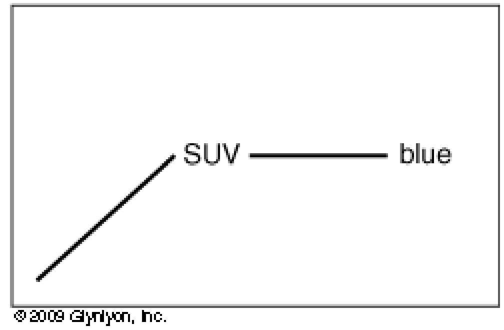
1.011 Which scenario would result in 12 possible outcomes?

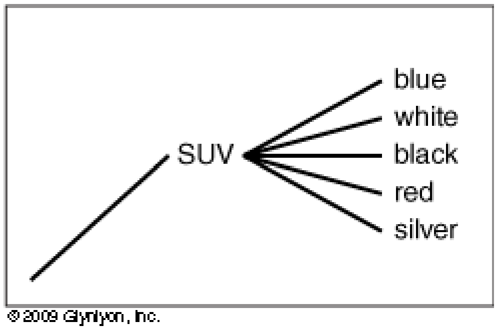
☐ You roll a number cube twice.
☐ You flip a coin twice.
☐ You roll a number cube, and then you flip a coin.
☐ You roll a number cube once.

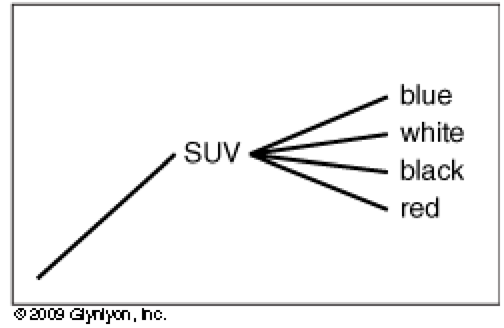
1.012 Evaluate ${}_5C_4$.

1.013 A car company offers four different styles (SUV, sedan, convertible, and hatchback). Each style is offered in five colors (blue, white, black, red, and silver). Choose the diagram that shows how many different versions of an SUV can be created.

☐ 

☐ 

☐ 

☐ 

1.014 From a stack of seven movies, you can choose any two that you want to watch. How many different combinations can you choose from?

☐ 2 ☐ 21 ☐ 42 ☐ 5,040

1.015 Use the counting principle to determine the number of possible outfits that can be created using one item from each category.

7 shirts
6 pairs of pants
6 pairs of socks
4 hats

☐ 1,512
☐ 23
☐ 5,040
☐ 1,008

72/90 **SCORE** ________ **TEACHER** ________ initials ________ date

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