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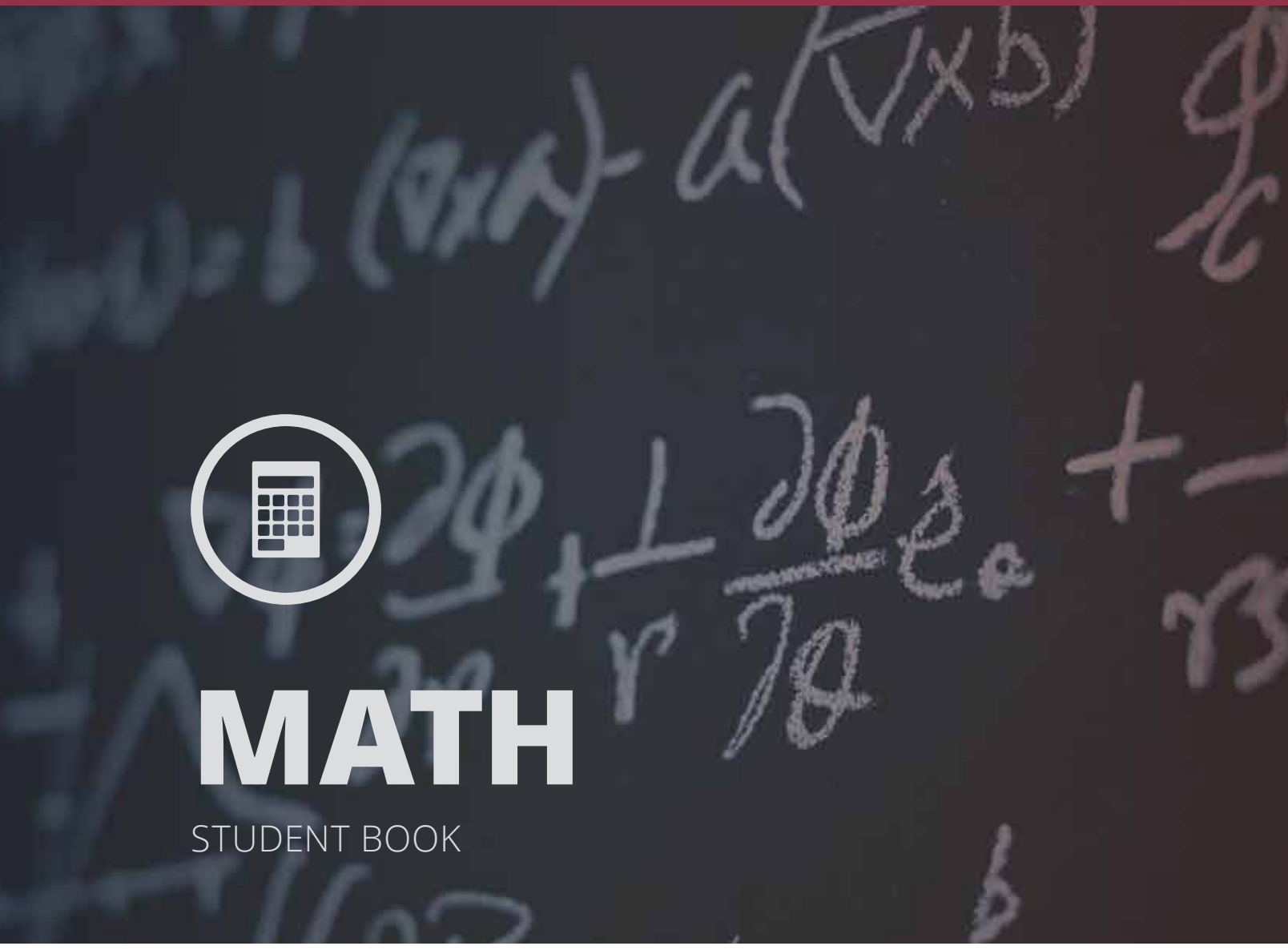

MATH

STUDENT BOOK

9th Grade | Unit 2

Alpha Omega
PUBLICATIONS

Math 902

SOLVING EQUATIONS AND INEQUALITIES

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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SOLVING EQUATIONS AND INEQUALITIES

INTRODUCTION

Algebra is one of several *systems* of mathematics; other systems are arithmetic, geometry, trigonometry, and calculus. Algebra expresses ideas in symbolic language—equations, inequalities, and formulas—with the use of positive and negative numbers and letters.

One of the most important and useful functions of algebra is to simplify the process of solving problems. Many mathematical problems are too complex to solve by arithmetic methods, and require translating the problem into *symbolic* language. When the translation into algebraic sentences has been done, we must solve the sentence to obtain the solution to the problem. In this LIFEPAC® we will learn how to solve many kinds of algebraic sentences.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Evaluate expressions.
2. Interpret and evaluate formulas.
3. Translate a verbal statement into symbolic language.
4. Solve an equation.
5. Solve an inequality.
6. Graph the solutions of sentences involving absolute value.

1. SENTENCES AND FORMULAS

Equations and inequalities are mathematical sentences. A formula is a sentence which expresses a principle.

Verbal sentences that express quantitative relationships may be translated into equations or inequalities.

OBJECTIVES

When you have completed this section, you should be able to:

1. Evaluate expressions.
2. Interpret and evaluate formulas.
3. Translate a verbal statement into symbolic language.

NUMERICAL SENTENCES

Expressions such as "two times six equals twelve" and "six is greater than four" are verbal sentences. If we translate these sentences into numerical symbols, we have $2 \cdot 6 = 12$ and $6 > 4$, respectively. A numerical sentence must contain one of three symbols (>, <, or =) to make it complete. A numerical sentence may be either true or false.

$6 > 10$ is a numerical sentence; however, it is a false statement. Likewise, $8 = 5$ is also false. In our study of algebra and algebraic sentences, we will be concerned primarily with true sentences and merely be aware of sentences that are false.

Write true or false.

1.1 _____ $6 + 2 = 10 - 2$

1.2 _____ $3(5 + 3) = 5(3 + 3)$

1.3 _____ $7 + 5 > 7 + 4$

1.4 _____ $8 - 6 > 8 - 5$

1.5 _____ $6 \cdot 5 - 3 > 7 \cdot 5 + 2$

1.6 _____ $-6 - 2 < 6 + 2$

1.7 _____ $10 - 15 < 15 - 10$

1.8 _____ $3(5 + 2) = 3 \cdot 7$

1.9 _____ $\frac{16}{4} = \frac{100}{25}$

1.10 _____ $3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$

1.11 _____ $5(7 - 3) = 5 \cdot 7 - 5 \cdot 3$

1.12 _____ $6 \cdot 5 = 5 \cdot 6$

1.13 _____ $5^2 + 7 > 7^2 - 5$

1.14 _____ $\frac{15}{5} - 2 = 0$

1.15 _____ $\frac{8}{8} + 1 > 2$

PROPERTIES OF ZERO

Mathematical sentences including zero are common. Several properties of zero are important in working with these sentences.

PROPERTIES OF ZERO:

A. Any number times zero equals zero.

B. Zero divided by any nonzero number equals zero.

C. Zero canot be used as a divisor.

D. Zero added to a number equals that number.

E. A number added to its opposite equals zero.

A. **Models:** $7 \cdot 0 = 0$ and $0 \cdot x = 0$

B. **Models:** $\frac{0}{6} = 0$ and $\frac{0}{y} = 0$ if $y \neq 0$.

C. **Models:** $\frac{3}{0}$ is impossible and $\frac{a}{0}$ is impossible

These quotients are undefined.

D. **Models:** $8 + 0 = 8$ and $0 + b = b$

E. **Models:** $3 + (-3) = 0$ and $-c + c = 0$

PROPERTIES OF ONE

Mathematical sentences including one are also common. These properties of one can be helpful in working with these sentences.

PROPERTIES OF ONE:

A. One times any number equals the number.

B. Any nonzero number divided by itself equals one.

C. Any number divided by one equals the number.

A. **Models:** $12 \cdot 1 = 12$ and $1 \cdot R = R$

B. **Models:** $\frac{4}{4} = 1$ and $\frac{T}{T} = 1$ if $T \neq 0$

C. **Models:** $\frac{9}{1} = 9$ and $\frac{N}{1} = N$

Evaluate each of the following expressions. Work in space below problem, and circle your answer.

Models: $4(1 - 0) = 4(1) = 4$

$\frac{A}{1} - A \cdot 0 = A - 0 = A$

1.16 $6 \cdot 0$

1.17 $0 \cdot 5$

1.18 $6(1 - 1)$

1.19 $\frac{10}{10} + 2$

1.20 $0 \cdot 15 - 15$

1.21 $7 \cdot \frac{1}{7}$

1.22 $A - A + 1$

1.23 $3 \cdot 1 - 1 \cdot 3$

1.24 $\frac{8}{1} + \frac{8}{8}$

1.25 $\frac{15}{0} + 6$

1.26 $\frac{0}{15} + 6$

1.27 $x \cdot 1 + \frac{x}{1}$

1.28 $7\ (B - B)$

1.29 $13\ (8 - 8)$

1.30 $(7 + 3)\ 0$

ABSOLUTE VALUE

Sentences including absolute values are less common than those with 1 or 0. Absolute value is, however, a very important concept in mathematics.

The absolute value of a number is the number without regard to sign.

The expression $|5| = 5$ is an *absolute value sentence*. The vertical bar on each side of a number is the symbol for absolute value.

Models: $|7| = 7$ $\quad |-8| = 8$

$|12 - 9| = |3| = 3$

$|4 - 7| = |-3| = 3$

The more formal definition of absolute value is

$|x| = x$ if x is positive and

$|-x| = - -x = x$ if x is negative

$|x| = 0$ if x is zero.

Evaluate each problem. Show work in space below each problem, and circle your answer.

Models: $|6 - 9| = |-3| = 3$

$|6| - |9| = 6 - 9 = -3$

1.31 $|6|$

1.32 $|6 - 1|$

1.33 $|8 - 9|$

1.34 $|5 - 15|$

1.35 $|3 + 2|$

1.36 $|2 - 10|$

1.37 $3 \cdot |-3|$

1.38 $-4\,|-4|$

1.39 $-8\,|2|$

1.40 $7\,|8 - 8|$

1.41 $|5| - |-5|$

1.42 $|x| - |-x|$

1.43 $0 \cdot |-6|$

1.44 $2\,|P| - |P|$

1.45 $|3 - 5 + 7|$

SENTENCES WITH VARIABLES

The sentence $n + 8 = 10$ is an algebraic sentence. This sentence may be true or false, depending on the value used for the variable n. If n is replaced by 2, the sentence is true; however, if n is replaced by any other number, the sentence is false.

We call his type of sentence a *conditional* sentence or an *open* sentence.

Hereafter, we will refer to an open sentence having an equal sign (=) as an equation. We desire to find the value of the variable that will satisfy the equation; that is, a value that will make the sentence true.

Model: Solve $N - 10 = 30$

Solution: Find the number N such that when 10 is subtracted from it, the difference is 30.

The solution is $N = 40$.

The check is $40 - 10 = 30$

$30 = 30$ True

Model: Solve $5N = 30$

Solution: 5 times what number is 30?

The answer is $N = 6$

The check is $5 \cdot 6 = 30$

$30 = 30$ True

Model: Solve $\frac{x}{12} = 8$

Solution: What number divided by 12 is 8?

The answer is $x = 96$

The check is $\frac{96}{12} = 8$

$8 = 8$ True

Circle the number that makes the sentence true.

1.61	$N + 5 = 10$	5, -5, 0	**1.66**	$3A = -15$	-5, 5, -12
1.62	$x - 6 = 10$	6, 10, 16	**1.67**	$\frac{P}{5} = 20$	60, 100, 15
1.63	$A + 15 = 10$	5, -5, 10	**1.68**	$\frac{R}{2} + 5 = 10$	20, 10, -8
1.64	$x - 4 = -8$	4, -2, -4	**1.69**	$5B - 2 = 18$	8, 10, 4
1.65	$2x = 14$	8, 6, 7	**1.70**	$2x + 1 = 21$	10, 12, 14

At this time you are solving equations by inspection; that is, the value of the letter—called the *unknown*—is fairly evident because the equation is simple. Also, in activities 1.61 through 1.68, you merely had to try each of the choices until one of them fit.

But how would you approach the equation $\frac{4}{5}x - 13 - 2x = \frac{1}{5}$?

A systematic way must exist to solve such complicated equations. In fact there is, and it will be introduced in Section II.

See if you can figure out a system for solving equations as you work the next set of exercises.

Write the solution to each sentence. Work in the space below the problem, and circle your answer. (Hint: Using the number line may be helpful in solving some of the following equations.)

Model: $-30 = N - 5$

Solution: Rewrite the subtraction $N - 5$ as the corresponding addition $N + (-5)$. Use the number line to determine what number must be added to -5 to obtain -30. Thus, $N = -25$.

1.71	$3x = 12$	**1.73**	$7x = 21$
1.72	$5p = 10$	**1.74**	$x - 5 = 10$

1.75 $A - 6 = -10$

1.76 $B + 5 = -8$

1.77 $7x = 49$

1.78 $-8x = 24$

1.79 $-5x = -25$

1.80 $\frac{B}{5} = 10$

1.81 $\frac{x}{3} = -10$

1.82 $\frac{P}{12} = 2$

1.83 $2x + 1 = 11$

1.84 $3x - 2 = 10$

1.85 $5x - 5 = 5$

1.86 $\frac{x}{2} = 9$

1.87 $\frac{a}{6} = 9$

1.88 $\frac{n}{5} = 3$

1.89 $\frac{x}{6} = 8$

1.90 $9a = 81$

1.91 $8x = 72$

1.92 $9x = 108$

SELF TEST 1

Evaluate. Show work in space below each problem, and circle your answer (each answer, 4 points).

1.01 $|3|$

1.02 $|-3|$

1.03 $|8 + 7|$

1.04 $|7 - 8|$

1.05 $3|-4| + |2|$

1.06 $5\ |7 - 9| - 2$

1.07 $-3|-4| - |-5|$

1.08 $8(5 - 5)$

1.09 $7 \cdot 0 \cdot 8 \cdot 5$

1.010 $9 \cdot 0 + 9 \cdot 1$

1.011 $6 \cdot 1 - \frac{6}{1}$

1.012 $7(x - x)$

1.013 $(9 + 2)\ 0$

1.014 $\frac{8}{8} - 1$

1.015 $\frac{10}{10} - \frac{10}{1}$

1.016 $|6| - |-6|$

Solve. Show work in space below each problem, and circle your answer (each answer, 4 points).

1.017 $x + 5 = 15$

1.018 $x - 6 = 12$

1.019 $x - 8 = -20$

1.020 $2x + 1 = 19$

1.021 $3x - 1 = 20$

1.022 $5x + 5 = 30$

1.023 $\frac{x}{2} = 10$

1.024 $\frac{x}{5} = 20$

1.025 $\frac{x}{12} = 3$

1.026 $\frac{2x}{5} = \frac{2}{5}$

1.027 $\frac{3x}{8} = \frac{3}{4}$

1.028 $\frac{1}{4}x + \frac{3}{4} = 1$

1.029 $5x = 30$

1.030 $8x = 72$

1.031 $10x = 120$

1.036 $V = lwh$

a. $l = 10, w = 8, h = 6$

b. $l = 5.1, w = 7.3, h = 12.5$

c. $l = 18\frac{1}{2}, w = 10, h = 5\frac{1}{2}$

Write an open sentence for each statement (each answer, 5 points).

1.037 20 times a number is 73. _____

1.038 The sum of 8 and a number is 100. _____

1.039 The difference between a number and 50 is less than 200. _____

_____

1.040 8 less than a number is 75. _____

1.041 6 times the sum of a number and 12 is 40. _____

153/191 **SCORE** _____ **TEACHER** _____ _____
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