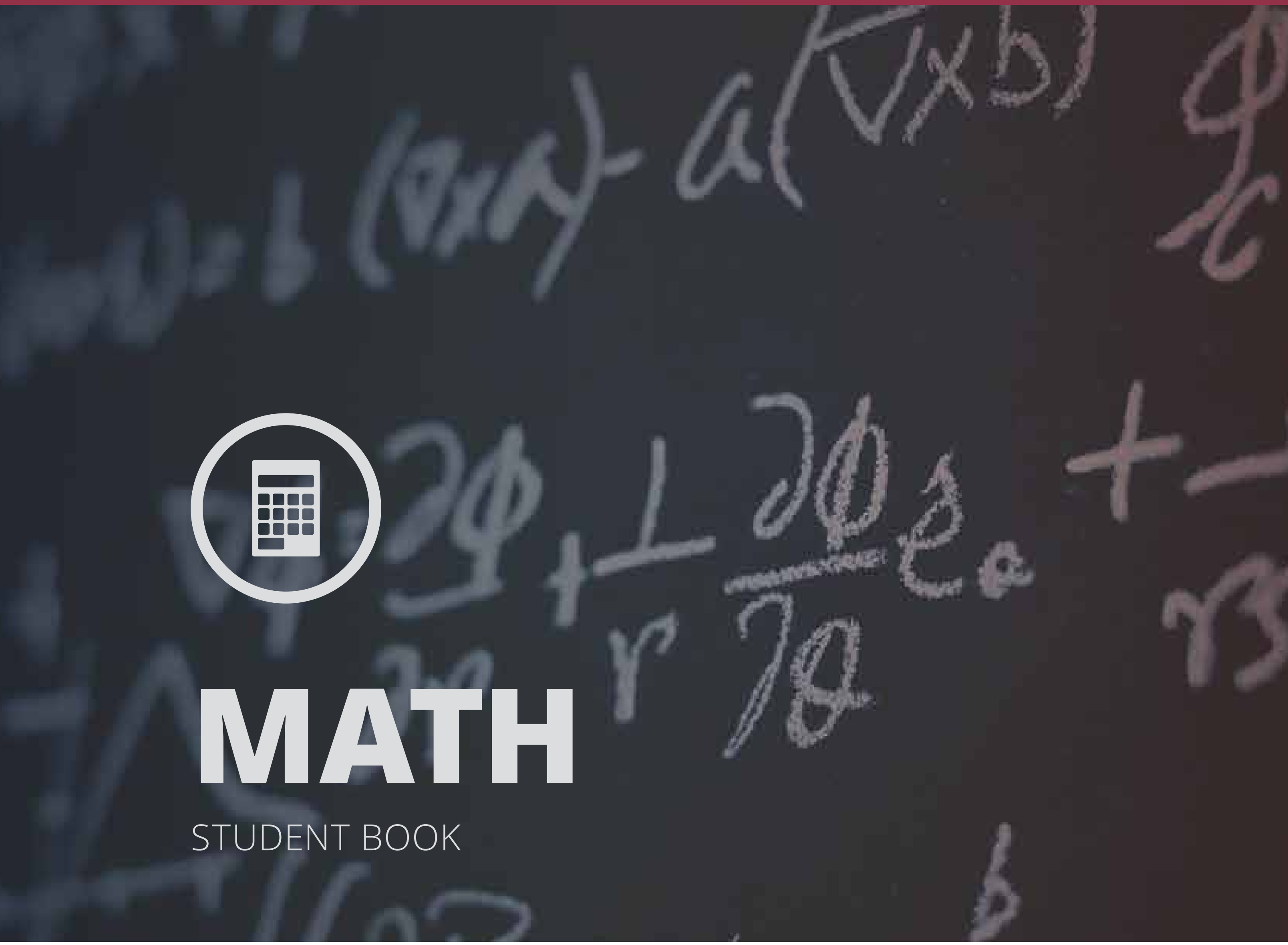
LIFEPAC®



MATH

STUDENT BOOK

9th Grade | Unit 5

Alpha Omega PUBLICATIONS

Math 905
Factors

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Factors

INTRODUCTION

In this LIFEPAC® you will continue your study in the mathematical system known as algebra by learning an important procedure—*factoring*. You have already learned that the answer obtained from multiplying numbers or polynomials is called *product*; these numbers or polynomials then are called *factors* of that product. You will need a solid foundation in factoring methods for success in your subsequent study in algebra. In this LIFEPAC you will learn to find greatest common factors, binomial factors, and complete factorizations of polynomials.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Factor numerical terms and literal terms.
2. Find the greatest common factor of monomials.
3. Factor polynomials by finding the greatest common factor.
4. Multiply binomials mentally.
5. Factor trinomials of the form $ax^2 + bx + c$.
6. Factor differences of two squares.
7. Factor binomials and trinomials completely.
8. Factor four-term polynomials.
9. Solve verbal problems involving factoring.

1. FINDING THE GREATEST COMMON FACTOR

Any expression has 1 and itself as factors, of course; if these numbers are its only factors, then the expression is called *prime*. However, if a number or a polynomial is not prime, you need to be able to find its other factors. In this section you will work first with numerical terms, then with literal terms, and finally with polynomials. The key definition for this section is the definition of the greatest common factor.

OBJECTIVES

When you have completed this section, you should be able to:

1. Factor numerical terms and literal terms.
2. Find the greatest common factor of monomials.
3. Factor polynomials by finding the greatest common factor.

VOCABULARY

Greatest common factor (GCF)—The greatest common factor of two or more expressions is the largest value that will divide each expression exactly.

Models: The GCF of 8 and 12 is 4 (since 4 is the largest value that will divide both 8 and 12 exactly).

The GCF of 12 and 30 is 6.

The GCF of 8, 12, and 30 is 2.

The GCF of ab and bc is b.

The GCF of $9a$, $12b$, and $16c$ is 1.

NUMERICAL TERMS

The GCF for numerical terms may be found in two ways. All the factors may be listed; or prime factorizations may be used.

LISTS OF FACTORS

One method for finding the greatest common factor of numerical terms is to list all the factors (positive integral divisors) of each number and then to choose the largest factor that is common to every list.

Model: Find the GCF of 28 and 42 by listing all the factors.

Solution: The factors of 28 are 1, 2, 4, 7, 14, and 28.

The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

The common factors are 1, 2, 7, and 14.

∴ The GCF of 28 and 42 is 14.

List all the factors of each number.

1.1 20 ______________________

1.2 100 ______________________

1.3 36 ______________________

1.4 23 ______________________

1.5 30 ______________________

Using the lists you made in Problems 1.1 through 1.5, find the GCF of the following sets of numbers.

1.6 20 and 100 ______________________

1.7 100 and 36 ______________________

1.8 36 and 23 ______________________

1.9 36 and 30 ______________________

1.10 20, 100, 36, and 30 ______________________

PRIME FACTORIZATIONS

Another method for finding the greatest common factor of numerical terms is to obtain the prime factorization of each number and then to use the product of any common prime factors. If no common prime factors exist, then the GCF is 1.

Model 1: Find the GCF of 28 and 42 by prime factorizations.

Solution:

$28 = 2 \cdot 14$ $\quad$ $42 = 2 \cdot 21$

$= 2 \cdot 2 \cdot 7$ $\quad$ $= 2 \cdot 3 \cdot 7$

The common prime factors are 2 and 7.

The GCF of 28 and 42 is $2 \cdot 7 = 14$.

Model 2: Find the GCF of 72 and 90.

Solution:

$72 = 2 \cdot 36$ $\quad$ $90 = 2 \cdot 45$

$= 2 \cdot 2 \cdot 18$ $\quad$ $= 2 \cdot 3 \cdot 15$

$= 2 \cdot 2 \cdot 2 \cdot 9$ $\quad$ $= 2 \cdot 3 \cdot 3 \cdot 5$

$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

The product of the common prime

factors is $2 \cdot 3 \cdot 3$.

$\therefore$ The GCF of 72 and 90 is 18.

Factor each number into the product of primes.

1.11 210 ____________________

1.12 27 ____________________

1.13 198 ____________________

1.14 170 ____________________

1.15 325 ____________________

Using the prime factorizations from Problems 1.11 - 1.15, find the GCF of the following sets of numbers.

1.16 210 and 27 ____________________

1.17 27 and 198 ____________________

1.18 210 and 170 ____________________

1.19 198 and 325 ____________________

1.20 210, 170, and 325 ____________________

If prime factorizations are written in *exponential form*, then the product of the highest common powers of prime factors will be the GCF of the numerical terms. In the preceding Model 2, $72 = 2^3 \cdot 3^2$ and $90 = 2^1 \cdot 3^2 \cdot 5^1$. The highest common power of 2 is 2^1, and the highest common power of 3 is 3^2. (Five is not a common prime factor.) Thus, the GCF of 72 and 90 is $2^1 \cdot 3^2 = 2 \cdot 9 = 18$.

Model: Find the GCF of 1,560 and 2,548.

Solution:

$1{,}560 = 2 \cdot 780$
$= 2 \cdot 2 \cdot 390$
$= 2 \cdot 2 \cdot 2 \cdot 195$
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 65$
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 13$
$= 2^3 \cdot 3^1 \cdot 5^1 \cdot 13^1$

$2{,}548 = 2 \cdot 1{,}274$
$= 2 \cdot 2 \cdot 637$
$= 2 \cdot 2 \cdot 7 \cdot 91$
$= 2 \cdot 2 \cdot 7 \cdot 7 \cdot 13$
$= 2^2 \cdot 7^2 \cdot 13^1$

The highest common power of 2 is 2^2.

The highest common power of 13 is 13^1.

$\therefore$ The GCF of 1,560 and 2,548 is $2^2 \cdot 13^1 = 52$.

Find the GCF in exponential form of the following sets of numbers.

1.21 $2^5 \cdot 5 \cdot 11$ and $2^3 \cdot 5^2 \cdot 7$ ______________

1.22 $13 \cdot 17^2 \cdot 19^3 \cdot 23^4$ and $13^4 \cdot 19^3 \cdot 23^2$ ______________

1.23 48 and 60

____

1.25 726 and 1210

____

1.24 81 and 45

____

LITERAL TERMS

To find the greatest common factor of literal terms, apply either of the two methods used for numerical terms.

Model 1: Find the GCF of x^2y^3 and x^4y.

Solution 1: List all the factors. The factors of x^2y^3 are 1, x, x^2, y, y^2, y^3, xy, xy^2, xy^3, x^2y, x^2y^2, and x^2y^3. The factors of x^4y are 1, x, x^2, x^3, x^4, y, xy , x^2y, x^3y, and x^4y.

The common factors are 1, x, x^2, y, xy, and x^2y.

$\therefore$ The GCF of x^2y^3 and x^4y is x^2y.

Solution 2: Use the exponential forms.

The literal terms x^2y^3 and x^4y are in exponential form.

The highest common power of x is x^2.

The highest common power of y is y^1.

$\therefore$ The GCF of x^2y^3 and x^4y is $x^2 \cdot y^1 = x^2y$.

The second solution is preferred for literal terms since they are usually written (or can easily be written) in exponential form.

Model 2: Find the GCF of $aabccc$ and ab^4c^5.

Solution: First rewrite $aabccc$ as a^2bc^3.

The highest common power of a is a^1.

The highest common power of b is b^1.

The highest common power of c is c^3.

$\therefore$ The GCF of c^2bc^3 and ab^4c^5 is $a^1 \cdot b^1 \cdot c^3 = abc^3$.

List all the factors of each literal term.

1.26 ab ______________________

1.27 x^3 ______________________

1.28 mnp ______________________

1.29 q^2r ______________________

Write each literal term in exponential form.

1.30 $kkkkknn$ ______________________

1.31 $abcbabc$ ______________________

1.32 $xyyzzz$ ______________________

Find the GCF of the following sets of literal terms.

1.33 x^5y and x^4y^2 ______________________

1.34 $abcde$ and $cdefg$ ______________________

1.35 $m^7n^4p^3$ and $mn^{12}p^5$ ______________________

1.36 $xxyyyzz$ and $xxxxzzz$ ______________________

In general, the greatest common factor of two or more monomials is the product of the GCF of the numerical factors and the GCF of the literal factors.

Model 1: Find the GCF of $8m^3n^2$ and $6m^2n^3$.

Solution: The GCF of 8 and 6 is 2.

The GCF of m^3n^2 and m^2n^3 is m^2n^2.

$\therefore$ The GCF of $8m^3n^2$ and $6m^2n^3$ is $2 \cdot m^2n^2 = 2m^2n^2$.

Model 2: Find the GCF of $2abc$, $3bcd$, and $4cde$; then write each monomial as the product of the GCF and the remaining factors of that monomial.

Solution: The GCF of 2, 3, and 4 is 1.

The GCF of *abc, bcd,* and *cde* is *c*.

$\therefore$ The GCF of $2abc$, $3bcd$, and $4cde$ is $1 \cdot c = c$.

The monomials are written $2abc = c(2ab)$,

$3bcd = c(3bd)$, and $4cde = c(4de)$.

Model 3: Find the GCF of $-9x^2y^7$, $12x^5y^5$, and $-30x^3y^{10}$; then write each monomial as the product of the GCF and the remaining factors of that monomial.

Solution: The GCF of -9, 12, and -30 is 3.

The GCF of x^2y^7, x^5y^5, and x^3y^{10} is x^2y^5.

$\therefore$ The GCF of $-9x^2y^7$, $12x^5y^5$, and $-30x^3y^{10}$ is $3x^2y^5$.

The monomials are written $-9x^2y^7 = 3x^2y^5\,(-3y^2)$,

$12x^5y^5 = 3x^2y^5\,(4x^3)$, and $-30x^3y^{10} = 3x^2y^5\,(-10xy^5)$.

Find the GCF of the following sets of monomials.

1.37 $5a$, $5b$, and $5c$ ________________

1.38 $4pq$, $3pq$, and $2pq$ ________________

1.39 $9x^2y^2$ and $6xy^3$ ________________

1.40 $-50m^4n^7$ and $40m^2n^{10}$ ________________

Find the GCF of each pair of monomials; then write each monomial as the product of the GCF and the remaining factors of that monomial.

		GCF	PRODUCTS		
	Model: $8abc$ and $-12ac^2$	$4ac$	$4ac(2b)$		$4ac(-3c)$
1.41	$4wxy$ and $6xyz$	_______	____________________	;	______________
1.42	$-10c^2d$ and $15cd^2$	_______	____________________	;	______________
1.43	$38m$ and $57n$	_______	____________________	;	______________
1.44	$12x^5y^9$ and $-35x^7y^3$	_______	____________________	;	______________
1.45	$-80a^3bc^5$ and $-200a^2bc^7$	_______	____________________	;	______________

POLYNOMIALS

The greatest common factor of a polynomial is the GCF of all the terms that make up that polynomial.

Model 1: Find the GCF of $8x^3y^4 - 4x^3y^2 - 6x^2y^2 + 2xy^3$.

Solution: The four terms are $8x^3y^4$, $-4x^3y^2$, $-6x^2y^2$, and $2xy^3$.

The GCF of 8, -4, -6, and 2 is 2.

The GCF of x^3y^4, x^3y^2, x^2y^2, and xy^3 is xy^2.

∴ The GCF of the polynomial is $2xy^2$.

The distributive property, $AB + AC = A(B + C)$, is used to factor out or separate the greatest common factor from a polynomial.

Model 2: Factor $10x + 5y$ by separating the GCF.

Solution: The GCF of $10x + 5y$ is 5, and $10x + 5y = 5 \cdot 2x + 5 \cdot y$

∴ The factorization is $5(2x + y)$.

Model 3: Factor out the GCF of $9x^3 + 10x^2 - 11x$.

Solution: The GCF is x, and $9x^3 + 10x^2 - 11x = x(9x^2) + x(10x) - x11$

∴ The factorization is $x(9x^2 + 10x - 11)$.

To check the factorization of a polynomial that results from separating the GCF, two steps should be followed. First, be certain that the GCF of the polynomial in the parentheses is 1. Second, be certain that the original polynomial is obtained when the separated GCF is multiplied by the polynomial in the parentheses.

Model 4: $2(3m - 6mn)$ is *not* the correct factorization of $6m - 12mn$ since the GCF of $3m - 6mn$ is $3m$, not 1. The correct factorization is $6m - 12mn = 6m \cdot 1 - 6m \cdot 2n = 6m(1 - 2n)$.

Model 5: $4(-x + 8)$ is *not* the correct factorization of $-4x + 8$ since $4(-x + 8) = -4x + 32$. The correct factorization is $-4x + 8 = 4(-x) + 4(2) = 4(-x + 2)$

You should note that the binomial $-4x + 8$ will often be factored as $-4(x - 2)$ since $-4x \cdot x$ is $-4x$ and $-4(-2)$ is 8.

Model 6: The polynomial $-10a^3 + 15a^2b - 30ab + 5a$ may be factored as $5a(-2a^2 + 3ab - 6b + 1)$ or preferably as $-5a(2a^2 - 3ab + 6b - 1)$

Model 7: Factor $8x^3y^4 - 4x^3y^2 - 6x^2y^2 + 2xy^3$, and check.

Solution: The GCF of this polynomial is $2xy^2$, and $8x^3y^4 - 4x^3y^2 - 6x^2y^2 + 2xy^3 = 2xy^2 \cdot 4x^2y^2 - 2xy^2 \cdot 2x^2 - 2xy^2 \cdot 3x + 2xy^2 \cdot y = 2xy^2(4x^2y^2 - 2x^2 - 3x + y)$.

Check: The GCF of the polynomial in the parentheses is 1, and the product of $2xy^2$ with each term gives the original polynomial.

$\therefore$ The factorization is $2xy^2(4x^2y^2 - 2x^2 - 3x + y)$.

Factor each polynomial by separating the GCF; check your factors.

1.46 $14a + 7b$ ______________________

1.47 $3y^2 - 4y$ ______________________

1.48 $10x^3 + 8x^2 - 6x$ ______________________

1.49 $4n^4 + n^3$ ______________________

1.50 $-3x - 3y - 3z$ ______________________

1.51 $a^2bc + ab^2c + abc^2$ ______________________

1.52 $5k^2 - 35k^3$ ______________________

1.53 $x^4y^2 + x^3y^3$ ______________________

1.54 $16p^5 - 24p^4$ ______________________

1.55 $-4d^3 + 28d^2 - 4d$ ______________________

1.56 $30y^2z + 12yz^2 - 18yz$ ______________________

1.57 $x^8 + 3x^5$ ______________________

1.58 $2a^5b + 2a^4b^2 + 2a^3b^3$ ______________________

1.59 $48m - 80n$ ______________________

1.60 $-24a^3b^3c^3 - 84a^4b^2c$ ______________________

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1

Complete these activities (each numbered item, 10 points).

1.01 List all the factors (positive integral divisors) of 48.

____

1.02 Write the prime factorization of 7700 in exponential form.

____

1.03 Find the greatest common factor of 270 and 360. (Give your answer in exponential form and in simplified form).

______________ = ______________

1.04 List all the factors of mn^2.

____

1.05 Write *pqqqqrr* in exponential form.

____

1.06 Find the greatest common factor of $8a^3b^2$ and $12ab^4$.

____

1.07 Write each monomial in Problem 1.06 as the product of the GCF and the remaining factors of the monomial.

________________ ; ________________

1.08 Factor $9x - 27y$ by separating the GCF.

____

1.09 Factor $3a^3 + 7a^5$.

____

1.010 Factor $-20m^3n^2 + 28m^2n^2 - 44m^2n^3$.

____

80/100 **SCORE** ____________ **TEACHER** ____________ ____________
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