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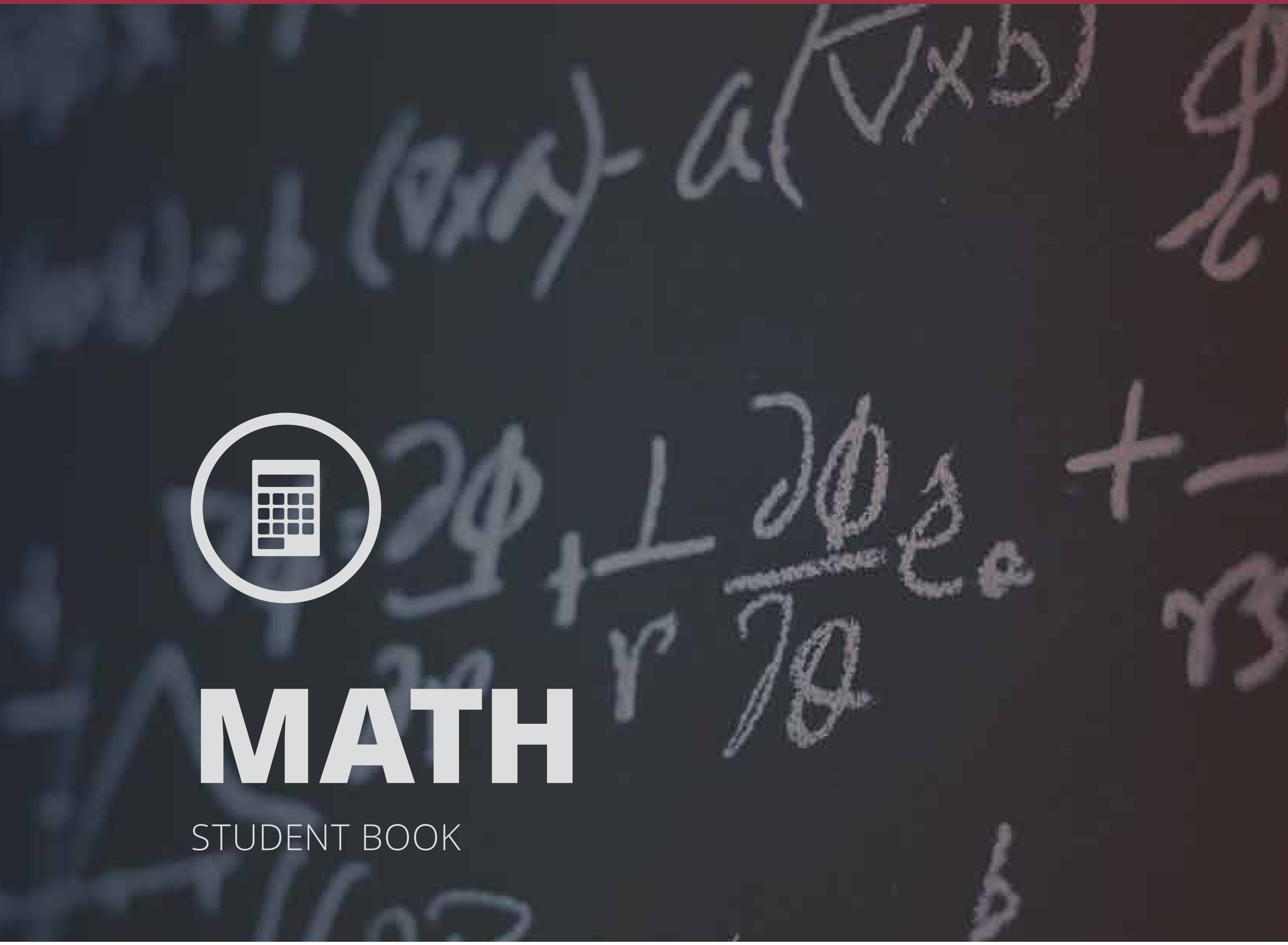

# MATH

STUDENT BOOK

## 9th Grade | Unit 6

Alpha Omega PUBLICATIONS

# Math 906

## Algebraic Fractions



**LIFEPAC Test is located in the center of the booklet**. Please remove before starting the unit.

**Author:**
Arthur C. Landrey, M.A.Ed.

**Editor-In-Chief:**
Richard W. Wheeler, M.A.Ed.
**Editor:**
Robin Hintze Kreutzberg, M.B.A.
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Andi Graham
Jerry Wingo

Alpha Omega PUBLICATIONS

**804 N. 2nd Ave. E.**
**Rock Rapids, IA 51246-1759**

# Algebraic Fractions

## INTRODUCTION

In this LIFEPAC® you will continue your study in algebra. You will apply what you have learned so far to fractions having polynomial numerators or denominators or both. The factoring techniques that you learned in Mathematics LIFEPAC 905 will be used when performing the basic operations with these fractions. Then you will solve open sentences containing fractions by methods that are quite similar to those you have already used. Finally you will have another opportunity to solve verbal problems, this time in applications that involve fractions.

## Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Determine the excluded value(s) for a fraction.
2. Reduce a fraction to lowest terms.
3. Find sums and differences of fractions.
4. Find products and quotients of fractions.
5. Simplify complex fractions.
6. Solve equations containing fractions.
7. Solve inequalities containing fractions.
8. Change the subject of a formula containing fractions.
9. Solve problems requiring the use of fractions.

# 1. OPERATIONS

As you work through this first section, keep in mind that the basic concepts of reducing, adding, subtracting, multiplying, dividing, and simplifying the fractions of algebra are the same as those used for the fractions of arithmetic.

We will begin by defining algebraic fractions, since we must know what they are in order to be able to work with them.

### OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Determine the excluded value(s) for a fraction.
2. Reduce a fraction to lowest terms.
3. Find sums and differences of fractions.
4. Find products and quotients of fractions.
5. Simplify complex fractions.

### VOCABULARY

**Algebraic fraction**—an indicated quotient of two polynomials written in the form $\frac{A}{B}$. *A* is the numerator of the algebraic fraction and *B* is the denominator.

**Terms**—the numerator and denominator of a fraction.

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**Models:** $\frac{2}{x + 3}$ $\frac{-y^2 - 3y + 1}{5 - 2y}$ $\frac{a + b + c}{m - n}$ $\frac{k - 3}{7}$

Algebraic fractions can be reduced, using similar methods as for reducing arithmetic fractions. Addition, subtraction, multiplication, division, and simplification are also possible with algebraic fractions.

# REDUCING FRACTIONS

Algebraic fractions can be reduced by finding the lowest terms. First, however, we need to discuss the circumstances under which algebraic fractions may not even exist!

## EXCLUDED VALUES

Since a fraction indicates division ($\frac{A}{B} = A \div B$) and since division by zero is undefined, the denominator of a fraction must be nonzero ($B \neq 0$). If a denominator contains any variables, then a value that would result in zero for that denominator must be *excluded* for the fraction to exist.

In the preceding models, the denominators are $x + 3$, $5 - 2y$, $m - n$, and 7, respectively. The excluded values are $x = -3$ for the first model ($-3 + 3 = 0$), $y = 2.5$ for the second ($5 - 2 \cdot 2.5 = 0$), and $m = n$ for the third ($m - m$ or $n - n = 0$); since the denominator of the fourth fraction is the constant 7 and $7 \neq 0$, that fraction has no excluded values.

In determining the excluded values for the fraction $\frac{x-3}{x^2-4}$, you may be able to see immediately that $2^2 - 4 = 0$; thus, $x = 2$ is an excluded value. However, $(-2)^2 - 4 = 0$ is also true; thus, $x = -2$ is an excluded value as well.

In Mathematics LIFEPAC 905 you learned to factor, and now factoring can be used to find both these excluded values. Since the denominator $x^2 - 4$ is a difference of two squares, it has factors $(x + 2)(x - 2)$. The first factor, $x + 2$, would become zero if $x = -2$; likewise, the second factor, $x - 2$, would become zero if $x = 2$. The excluded values are then $x = 2$ and $x = -2$. In this method we have made use of an important property in mathematics.

**Property**

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (or both); if a product of factors is zero, then at least one of the factors must be zero.

**Model 1:** Find the excluded value(s) for the fraction $\frac{a + 5}{a(b + 3)(c - 2)}$.

**Solution:** The denominator is already factored, so each of the three factors is set equal to zero.

$$a = 0 \qquad \begin{aligned} b + 3 &= 0 \\ b &= -3 \end{aligned} \qquad \begin{aligned} c - 2 &= 0 \\ c &= 2 \end{aligned}$$

$\therefore$ The excluded values are $a = 0$, $b = -3$, and $c = 2$.

**Model 2:** Find the excluded value(s) for the fraction $\frac{7}{d^2 - 5d - 24}$.

**Solution:** The factors of $d^2 - 5d - 24$ are $(d - 8)(d + 3)$.

$$\begin{aligned} d - 8 &= 0 \\ d &= 8 \end{aligned} \qquad \begin{aligned} d + 3 &= 0 \\ d &= -3 \end{aligned}$$

$\therefore$ The excluded values are $d = 8$ and $d = -3$.

**Check:**

If $d = 8$, $d^2 - 5d - 24$
$= (8)^2 - 5(8) - 24$
$= 64 - 40 - 24 = 0.$

If $d = -3$, $d^2 - 5d - 24$
$= (-3)^2 - 5(-3) - 24$
$= 9 + 15 - 24 = 0.$

**Write the excluded value(s) for each fraction, or none if that is the case.**

**1.1** $\frac{a}{b - 2}$ ________

**1.2** $\frac{4x + 3}{x}$ ________

**1.3** $\frac{y^2 - y + 5}{y + 4}$ ________

**1.4** $\frac{3}{5n}$ ________

**1.5** $\frac{-2x}{17}$ ________

**1.6** $\frac{a}{3 - 2a}$ ________

**1.7** $\frac{x + 3}{y(z + 5)}$ ________

**1.8** $\frac{k^2 + 5k + 1}{k^2 - 9}$ ________

**1.9** $\frac{7b^3}{b^2 - 7b + 10}$ ________

**1.10** $\frac{x + 11}{3x^2 + 5x - 2}$ ________

As you work through this LIFEPAC, you are to assume that all fractions do exist; that is, any value(s) that would make a denominator zero are understood to be excluded. However, from time to time (as in the preceding activities), you will be asked to identify these excluded values.

## LOWEST TERMS

Now you are ready to begin working with these algebraic fractions. A basic property of fractions will be used in much of this work.

### Property

$\frac{A}{B} = \frac{AC}{BC}$ (or $\frac{AC}{BC} = \frac{A}{B}$) for $C \neq 0$; if the numerator and the denominator of a fraction are both multiplied (or divided) by the same nonzero value, then an **equivalent** fraction is obtained.

In arithmetic you learned that the fraction $\frac{1}{2}$ has the same value as the fraction $\frac{5}{10}$, since both the numerator and the denominator of $\frac{1}{2}$ are multiplied by 5. Similarly, the fraction $\frac{12}{18}$ is equivalent to the fraction $\frac{2}{3}$ since both the numerator and the denominator of $\frac{12}{18}$ are divided by 6; this latter procedure is known as *reducing*. An algebraic fraction is reduced to lowest terms when the greatest common factor of its numerator and denominator is 1.

**Model 1:** Reduce $\frac{24m^2n}{21mp^2}$ to lowest terms.

**Solution:** The GCF of $24m^2n$ and $21mp^2$ is $3m$. Divide both the numerator and the denominator by $3m$.

$$\frac{24m^2n \div 3m}{21mp^2 \div 3m} = \frac{8mn}{7p^2},$$

the equivalent reduced fraction since the GCF of $8mn$ and $7p^2$ is 1.

**Model 2:** Reduce $\frac{4y - 20}{12y}$ to lowest terms.

**Solution:** $4y - 20$ factors into $4(y - 5)$, and $12y$ is $4 \cdot 3y$. Divide both the numerator and the denominator by the common factor 4.

$$\frac{4y - 20}{12y} = \frac{4(y - 5)}{12y}$$
$$= \frac{4(y - 5) \div 4}{12y \div 4}$$
$$= \frac{y - 5}{3y},$$

the equivalent reduced fraction since the GCF of $y$ - 5 and $3y$ is 1.

*NOTE:* The $y$'s cannot be reduced since $y$ is a term (not a factor) of the numerator $y - 5$. Only common factors can be reduced!

**Model 3:** Reduce $\frac{r^2 - 3r + 2}{r^2 - 1}$ to lowest terms.

**Solution:** Since $r^2$ is a term (not a factor) of both the numerator and denominator, to try to reduce this fraction by dividing by $r^2$ would be wrong, even though very tempting. You must avoid this type of mistake that so many beginning students make.

Factor the trinomial numerator and the binomial denominator; then divide by the common factor. (This reducing is often shown by drawing lines through these factors.)

$$\frac{r^2 - 3r + 2}{r^2 - 1} = \frac{(r - 2)(r - 1)}{(r + 1)(r - 1)}$$
$$= \frac{(r - 2)\cancel{(r - 1)}}{(r + 1)\cancel{(r - 1)}}$$
$$= \frac{r - 2}{r + 1}$$

**Model 4:** Reduce $\frac{6m + 6n}{9n + 9m}$ to lowest terms.

**Solution:**
$$\frac{6m + 6n}{9n + 9m} = \frac{6(m + n)}{9(n + m)}$$
$$= \frac{2 \cdot \cancel{3}\cancel{(m + n)}}{3 \cdot \cancel{3}\cancel{(n + m)}}$$
$$= \frac{2}{3}$$

In Model 4, the binomials $m + n$ and $n + m$ are equal and reduce as part of the GCF $3(m + n)$. If, however, the binomials had been $m - n$ and $n - m$, they would not have reduced in quite the same way since they are opposites. $\frac{A}{-A} = -1$; if two expressions are opposites, they divide (or reduce) to negative one.

**Model 5:** Reduce $\frac{6m - 6n}{9n - 9m}$ to lowest terms.

**Solution:**

$$\frac{6m - 6n}{9n - 9m} = \frac{6(m - n)}{9(n - m)}$$

$$= \frac{\overset{2}{\cancel{6}}\overset{(-1)}{\cancel{(m - n)}}}{\underset{3}{\cancel{9}}\cancel{(n - m)}}$$

$$= -\frac{2}{3}$$

*Note:* The (-1) is included in the answer as a minus sign before the fraction.

**Model 6:** Reduce $\frac{16 - a^2}{a^2 + 20 - 9a}$ to lowest terms.

**Solution:**

$$\frac{16 - a^2}{a^2 + 20 - 9a} = \frac{16 - a^2}{a^2 - 9a + 20}$$

$$= \frac{(4 + a)\overset{(-1)}{\cancel{(4 - a)}}}{(a - 5)\cancel{(a - 4)}}$$

$$= -\frac{4 + a}{a - 5}$$

**Model 7:** Reduce $\frac{a + 3b + c}{a^2 - 9b^2}$ to lowest terms.

**Solution:**

$$\frac{a + 3b + c}{a^2 - 9b^2} = \frac{a + 3b + c}{(a + 3b)(a - 3b)},$$

but nothing can be reduced since $a + 3b$ is not a factor of the numerator.

$\therefore \frac{a + 3b + c}{a^2 - 9b^2}$ is in lowest terms.

**Reduce each fraction to lowest terms.**

**1.11** $\frac{75a^2b}{25ab^2}$ ____________

**1.12** $\frac{12m^4}{28m^3}$ ____________

**1.13** $\frac{-5jk}{35j^2k^2}$ ____________

**1.14** $\frac{84y^3}{36y^4}$ ____________

**1.15** $\frac{38x^2yz^2}{-19xy^2z^3}$ ____________

**1.16** $\frac{6x + 2}{8}$ ____________

**1.17** $\frac{x^3 - x^2}{x^4}$ ____________

**1.18** $\frac{27a}{ab + ac}$ ____________

**1.19** $\frac{y + 5}{2y + 10}$ ____________

**1.20** $\frac{n + 2}{n^2 - 4}$ ____________

**1.21** $\frac{5r - 5s}{5r + 5s}$ ____________

**1.22** $\frac{8a + 8b}{12c + 12d}$ ____________

**1.23** $\frac{x^2 - y^2}{8x - 8y}$ ____________

**1.24** $\frac{a + 5}{a^2 - 25}$ ____________

**1.25** $\frac{7 - y}{y - 7}$ ____________

**1.26** $\frac{x^2 - 4x - 12}{36 - x^2}$ ____________

**1.27** $\frac{m^2}{m^2 - n^2}$ ____________

**1.28** $\frac{-5k + 15}{k^2 - 9}$ ____________

# SELF TEST 1

**Give the excluded value(s) for each fraction** (each answer, 3 points).

**1.01** $\frac{2}{x\,(x - 3)}$ ____________________

**1.02** $\frac{y + 5}{y^2 + 4y - 32}$ ____________________

**1.03** $\frac{-7z}{4z + 1}$ ____________________

**Reduce each fraction to lowest terms** (each answer, 3 points).

**1.04** $\frac{6a^2b^3}{8ab^4}$ ____________________

**1.05** $\frac{3 - k}{k - 3}$ ____________________

**1.06** $\frac{n^2 - 7n - 44}{n^2 - 121}$ ____________________

**Perform the indicated operations** (each answer, 4 points).

**1.07** $\frac{4x}{2x + y} + \frac{2y}{2x + y}$

**1.08** $\frac{d + 3}{8d} - \frac{2d + 1}{10d^2}$

**1.09** $\frac{3}{n^2 - 9} + \frac{7}{3 - n}$

**1.010** $\frac{m}{n} \cdot \frac{n}{p} \div \frac{p}{q}$

**1.011** $\frac{4x^2yz^3}{9} \cdot \frac{45y}{8x^5z^3}$

**1.012** $\frac{k + 5}{k^2 + 3k - 10} \div \frac{7k + 14}{4 - k^2}$

**Simplify each complex fraction** (each answer, 3 points).

**1.013** $\dfrac{\frac{1}{x}}{\frac{1}{y}}$

**1.015** $\dfrac{\frac{5}{a} + 1}{\frac{a}{5} - \frac{5}{a}}$

**1.014** $\dfrac{\frac{m}{5} - \frac{1}{6}}{\frac{1}{3}}$

41/51 **SCORE** ________ **TEACHER** ________ ________

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Alpha Omega
PUBLICATIONS
804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759
800-622-3070
www.aop.com