LIFEPAC®

MATH

STUDENT BOOK

9th Grade | Unit 7

Alpha Omega PUBLICATIONS

Math 907
Radical Expressions

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Author:
Arthur C. Landrey, M.A.Ed.

Editor-In-Chief:
Richard W. Wheeler, M.A.Ed.
Editor:
Robin Hintze Kreutzberg, M.B.A.
Consulting Editor:
Robert L. Zenor, M.A., M.S.
Revision Editor:
Alan Christopherson, M.S.

Westover Studios Design Team:
Phillip Pettet, Creative Lead
Teresa Davis, DTP Lead
Nick Castro
Andi Graham
Jerry Wingo

Alpha Omega Publications

804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

Radical Expressions

INTRODUCTION

In this LIFEPAC® you will continue your study in the mathematical system of algebra by learning first about real numbers and then about radical expressions. After becoming familiar with radical expressions, you will learn to simplify them and to perform the four basic operations (addition, subtraction, multiplication, and division) with them. Finally, you will learn to solve equations containing these expressions.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify and work with rational numbers.
2. Identify and work with irrational numbers.
3. Draw number-line graphs of open sentences involving real numbers.
4. Simplify radical expressions.
5. Combine (add and subtract) radical expressions.
6. Multiply radical expressions.
7. Divide radical expressions.
8. Solve equations having irrational roots.
9. Solve equations containing radical expressions.

1. REAL NUMBERS

In this section, you will study the fundamental set of numbers for beginning algebra and geometry—the *real numbers*. You will learn about two other sets, the *rational numbers* and the *irrational numbers*, that make up the real numbers. You will also learn a new property that applies to no other numbers you have studied so far—*completeness*; this property will be used in graphing the real numbers.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify and work with rational numbers.
2. Identify and work with irrational numbers.
3. Draw number-line graphs of open sentences involving real numbers.

RATIONAL NUMBERS

You will begin by classifying some numbers that you are quite familiar with already. You will need to discover what numbers are actually included in this classification according to the definition. Conversion between the different forms that a rational number may take should help to understand the classification better. Then you will be ready to graph rational numbers and study their properties.

DEFINITIONS AND CONVERSIONS

The following definition outlines the classification of numbers known as rational numbers.

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rational number—a number that can be written as a ratio of two integers in the form $\frac{A}{B}$ with $B \neq 0$.

Model 1: $\frac{2}{9}$ is a rational number since it is the ratio of the integers 2 and 9.

Model 2: $4\frac{1}{5}$ is a rational number since it can be written as $\frac{21}{5}$, the ratio of the integers 21 and 5.

Model 3: $-\frac{3}{8}$ is a rational number since it can be written as $\frac{-3}{8}$, the ratio of the integers -3 and 8.

Model 4: 0.283 is a rational number since it can be written as $\frac{283}{1{,}000}$, the ratio of the integers 283 and 1,000.

Model 5: -81.7 is a rational number since it can be written as $-81\frac{7}{10} = -\frac{817}{10} = \frac{817}{-10}$, the ratio of the integers 817 and -10.

Model 6: 17 is a rational number since it can be written as $\frac{17}{1}$, the ratio of the integers 17 and 1.

Model 7: 0 is a rational number since it can be written as $\frac{0}{1}$, the ratio of the integers 0 and 1.

Model 8: -6 is a rational number since it can be written as $-\frac{6}{1} = \frac{-6}{1}$, the ratio of the integers -6 and 1.

From the models, you can see that the common fractions, mixed numbers, and decimals of arithmetic (as well as their negatives) are included in the rational numbers. Also, you can see that the integers themselves are included in the rational numbers.

You may be wondering which numbers are *not* included in this classification. Such numbers will be considered in detail later in this section, but at the present time you should know that not all decimals are rational and not all fractions are rational. For example, a number that you have probably worked with, π, cannot be written as the ratio of two integers and is not rational; therefore, neither is a fraction such as $\frac{\pi}{6}$ rational. You may have used an *approximation* for π , such as 3.14 or $\frac{22}{7}$, in evaluating formulas. These approximations are themselves rational, but π is not!

A fraction that is rational can be converted to an equivalent decimal form, and a decimal that is rational can be converted to an equivalent fraction form. The two equivalent forms, of course, must have the same sign.

Model 1: Convert $\frac{5}{8}$ and $-\frac{5}{8}$ to decimals.

Solution: $\frac{5}{8} = 5 \div 8 = 0.625$,

a *terminating* decimal.

$\therefore \; \frac{5}{8} = 0.625$ and $-\frac{5}{8} = -0.625$

Model 2: Convert -0.24 to a fraction.

Solution: $-0.24 = -\frac{24}{100} = -\frac{\cancel{4} \cdot 6}{\cancel{4} \cdot 25}$

$\therefore \; -0.24 = -\frac{6}{25}$

Model 3: Convert $\frac{1}{3}$ to a decimal.

Solution: $\frac{1}{3} = 1 \div 3 = 0.3333\ldots$,

a *repeating* decimal.

$\therefore \; \frac{1}{3} = .0\overline{3}$

The decimal $0.\overline{3}$ is said to have a *period* of 1 since one number place continues without end; the decimal -0.363636… = $-0.\overline{36}$ has a period of 2 since two number places continue without end. The line drawn above a repeating decimal is called the *repetend* bar, and it should be over the exact number of places in the period of the decimal.

Models: 0.12341234… = $0.\overline{1234}$ and has a period of 4.

0.12343434… = $0.12\overline{34}$ and has a period of 2.

0.12344444… = $0.123\overline{4}$ and has a period of 1.

We saw that the rational number $\frac{1}{3}$ converts to $0.\overline{3}$, but how does $0.\overline{3}$ convert back to $\frac{1}{3}$? The decimal 0.3 converts to $\frac{3}{10}$, but $\frac{1}{3} \neq \frac{3}{10}$; thus, $0.\overline{3} \neq \frac{3}{10}$ either. The following solutions show a procedure for converting repeating decimals to fractions.

Model 1: Convert $0.\overline{3}$ to a fraction.

Solution: Let $n = 0.\overline{3} = 0.333\ldots$

Then $10n = 10(0.333\ldots) = 3.33\ldots$, since multiplying a decimal by ten moves the decimal point one place to the right.

Now subtract:

$$\begin{array}{rcl} 10n & = & 3.33\ldots \\ 1n & = & 0.33\ldots \\ \hline 9n & = & 3.00\ldots \end{array}$$

or $9n = 3$

and $n = \frac{3}{9}$ or $\frac{\cancel{3} \cdot 1}{\cancel{3} \cdot 3}$

$\therefore\ 0.\overline{3} = \frac{1}{3}$

NOTE: The period of $0.\overline{3}$ is 1, and n is multiplied by $10^1 = 10$.

Model 2: Convert $-0.\overline{36}$ to a fraction.

Solution: Since the given decimal is negative, its equivalent fraction will be negative also.

Let $n = 0.\overline{36} = 0.363636\ldots$

Then $100n = 100(0.363636\ldots) = 36.3636\ldots$ since multiplying a decimal by one hundred moves the decimal point two places to the right.

Now subtract:

$$\begin{array}{rcl} 100n & = & 36.3636\ldots \\ 1n & = & 0.3636\ldots \\ \hline 99n & = & 36.0000\ldots \end{array}$$

$$\text{or } 99n = 36$$

$$\text{and } n = \frac{36}{99} \text{ or } \frac{\cancel{9} \cdot 4}{\cancel{9} \cdot 11}$$

$\therefore$ Since $0.\overline{36} = \frac{4}{11}$, then $-0.36 = -\frac{4}{11}$.

NOTE: The period of $0.\overline{36}$ is 2, and n is multiplied by $10^2 = 100$.

Some other results of this procedure are shown in the following models. Try to find a relationship between the repeating decimal, its period, and its equivalent unreduced fraction.

Models:

$0.\overline{1} = \frac{1}{9}$ $\quad 0.\overline{01} = \frac{1}{99}$ $\quad 0.\overline{001} = \frac{1}{999}$

$0.\overline{5} = \frac{5}{9}$ $\quad 0.\overline{53} = \frac{53}{99}$ $\quad 0.\overline{531} = \frac{531}{999} = \frac{59}{111}$

You have seen that a repeating decimal as well as a terminating decimal can be written as the ratio of two integers in the form $\frac{A}{B}$; both types then are rational numbers. Actually, a terminating decimal is just a special type of repeating decimal—one that repeats zero; for example, 0.815 = 0.815000... and -5.3 = -5.3000.... Keeping this fact in mind, consider the following alternate definition and see how it applies to the models from the beginning of this section.

VOCABULARY

rational number—a number that can be written as a repeating decimal.

Models:

$\frac{2}{9} = 0.\overline{2}$

$4\frac{1}{5} = 4\ .2\overline{0}$

$-\frac{3}{8} = -0.375\overline{0}$

$0.283 = 0.283\overline{0}$

$-81.7 = -81.7\overline{0}$

$17 = 17.\overline{0}$

$0 = 0.\overline{0}$

$-6 = -6.\overline{0}$

Write each rational number as the ratio of two integers in the form $\frac{A}{B}$ and then as a repeating decimal.

Model: $4.3 = \frac{43}{10} = 4.3\overline{0}$

1.1 $\frac{3}{4}$ = ________ = ________

1.2 $-7\frac{1}{3}$ = ________ = ________

1.3 $8\frac{1}{2}$ = ________ = ________

1.4 -6.59 = ________ = ________

1.5 10 = ________ = ________

Convert each fraction to its equivalent decimal form.

1.6 $\frac{3}{5}$ = ________

1.7 $\frac{33}{50}$ = ________

1.8 $\frac{2}{3}$ = ______

1.9 $\frac{1}{15}$ = ______

1.10 $\frac{20}{33}$ = ______

1.11 $-\frac{83}{200}$ = ______

1.12 $-\frac{52}{125}$ = ______

1.13 $-\frac{5}{12}$ = ______

1.14 $-\frac{12}{5}$ = ______

1.15 $-\frac{21}{37}$ = ______

Convert each decimal to its equivalent reduced fraction form.

1.16 0.7 = ______

1.17 $0.\overline{7}$ = ______

1.18 -0.8 = ______

1.19 $-0.\overline{8}$ = ______

1.20 0.25 = ______

1.21 $0.\overline{25}$ = ______

1.22 -0.63 = ______

1.23 $-0.\overline{63}$ = ______

1.24 0.135 = ______

1.25 $0.\overline{135}$ = ______

1.26 0.9 = ______

1.27 $0.\overline{9}$ = ______

Model: $.2\overline{5}$

Solution: Let $n = 0.2\overline{5} = 0.25555...$

Then $10n = 2.55...$ and $100n = 25.55...$

Now subtract:

$$\begin{aligned} 100n &= 25.55... \\ 10n &= 2.55... \\ \hline 90n &= 23.00... \end{aligned}$$

or $90n = 23$

and $n = \frac{23}{90}$

$\therefore 0.2\overline{5} = \frac{23}{90}$

Convert each decimal to its equivalent reduced fraction form. Show your work as in the preceding model.

1.28 $0.8\overline{7}$

1.29 $-0.3\overline{8}$

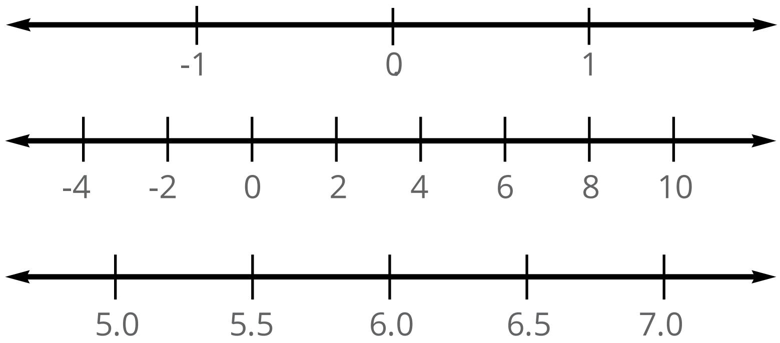
1.30 $0.0\overline{9}$

GRAPHS AND ORDER

Now that rational numbers have been explained, you should be able to graph them on the number line. First, however, a review of some of the basic ideas of graphing may be helpful.

The small vertical line segments drawn on the number line are only reference marks (not graphed points); and the spacing between them, as well as the numbers written below them, may be changed for convenience in graphing.

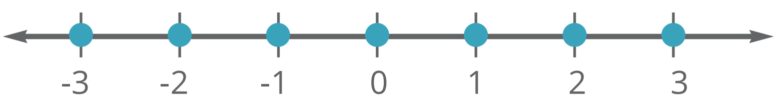
Models:



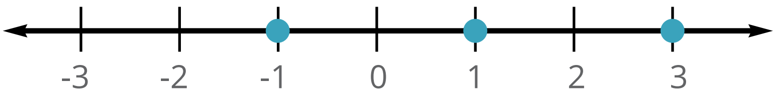
All three lines shown can be thought of as the same number line, but with different reference marks; no points are graphed on these lines.

A point is graphed on the number line by placing a heavy dot on (or between) the appropriate reference mark(s). A darkened arrowhead is used at the end(s) of the line represented on the paper to show a continuation of points.

Model 1: The graph of the integers is shown.



Model 2: The graph of the odd integers larger than -2 is shown.



You have already learned that all the integers are rational numbers since each can be written as the ratio of itself to 1 and as a decimal that repeats zero. From the graph, the order of the integers can be seen to be ...-3 < -2 < -1 < 0 < 1 < 2 < 3...; that is, an integer is less than another integer if it is to the left of the other integer on the number line.

Although infinitely many integers are in the rational numbers, infinitely many nonintegers are also rational, such as $1\frac{1}{4}$ and $-0.\overline{56}$. A nonintegral rational number that would be between two reference marks on the number line is graphed by placing a heavy dot on the approximate corresponding point and writing the number above the dot. The *order* of both integral and nonintegral rational numbers can be determined from the relative positions of their points on the line.

SELF TEST 1

Convert each fraction to its equivalent decimal form, (each answer, 3 points).

1.01 $\frac{2}{5}$ = __________

1.02 $-\frac{16}{33}$ = __________

1.03 $\frac{19}{16}$ = __________

Convert each decimal to its equivalent reduced fraction form (each answer, 3 points).

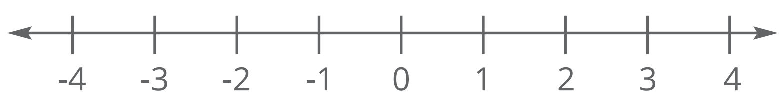
1.04 -0.72 = __________

1.05 $0.\overline{72}$ = __________

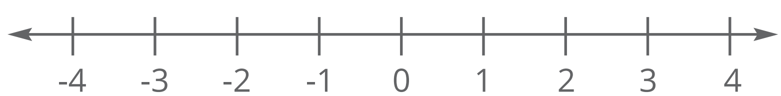
1.06 $0.7\overline{2}$ = __________

Graph the given rational numbers (each location, 1 point).

1.07 $-3\frac{1}{3}$, 0, 1.6



1.08 2.4, $-\frac{5}{4}$, $3.\overline{8}$



Write the order of the given rational numbers (each numbered item, 3 points).

1.09 -3, $\frac{1}{3}$, 0.3 __________ < __________ < __________

1.010 $5.\overline{40}$, $\left(\frac{7}{3}\right)^2$, $5\frac{2}{5}$ __________ < __________ < __________

Find the rational number (each answer, 3 points).

1.011 One-sixth of the way between 7 and 16.

1.012 0.8 of the way between -0.5 and $4\frac{1}{2}$.

Tell whether each infinite decimal is rational or irrational (each answer, 2 points).

1.013 -7.234567... ______________________

1.014 -7.232323... ______________________

Round each number to the nearest hundredth (each answer, 2 points).

1.015 $0.\overline{61}$ ____________ **1.016** 0.292292229... ____________

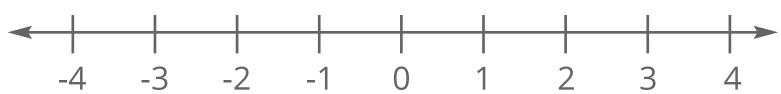
a. Find each square root; b. then tell whether each is rational or irrational (each answer, 3 points).

1.017 $\sqrt{\frac{25}{49}}$ = a.________ b.______________________

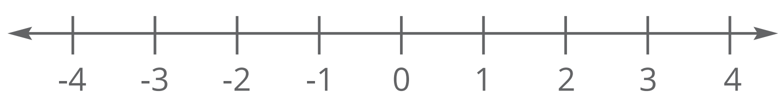
1.018 $\sqrt{32.49}$ = a.________ b.______________________

Draw the graph of each condition (each graph, 3 points).

1.019 $y < 1.\overline{8}$



1.020 $|h| \geq \sqrt{13}$



50/62 **SCORE** ____________ **TEACHER** ____________ ____________
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800-622-3070
www.aop.com