



MATH

STUDENT BOOK

▶ **10th Grade | Unit 3**

MATH 1003

ANGLES AND PARALLELS

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**804 N. 2nd Ave. E.
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Angles and Parallels

Introduction

In this LIFE PAC®, we shall study another basic geometric idea. Lines, segments, and rays can be placed in such a way as to form angles. Angles are present everywhere and are very important in the study of geometry. Many angle relationships will be presented, along with methods for measuring angles. We shall also learn about parallels and how special angles are formed using parallels. Many theorems will be presented in connection with angles and parallels.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFE PAC. When you have finished this LIFE PAC, you should be able to:

1. Identify angles as acute, right, or obtuse.
2. Find the measure of angles with a protractor.
3. Add and subtract measures of angles.
4. Find the measure of angles by their relationship with other angles.
5. Prove theorems about angles.
6. Define terms related to parallels.
7. Prove theorems about parallel and related angles.
8. Classify triangles by their sides and by their angles.
9. Prove theorems about triangles and their related angles.

1. ANGLE DEFINITIONS AND MEASUREMENT

To continue our study of geometry, we must learn basic angle definitions and measurement methods. Definitions will help in identifying and classifying angles; measurement will allow us to add and subtract angles.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Identify angles as acute, right, or obtuse.
2. Find the measure of angles with a protractor.
3. Add and subtract measures of angles.

ANGLE DEFINITIONS

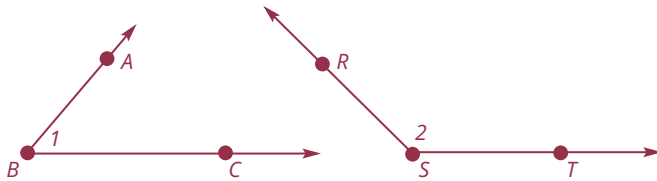
These definitions relate to angles. Make sure you know them, because you will be using them many times in this LIFEPAAC.

DEFINITION

Angle (\angle): the union of two noncollinear rays that have a common end point.

The two rays that form the angle are called its *sides*, and the common end point is called the *vertex* of the angle. The symbol for angle is \angle .

Models:

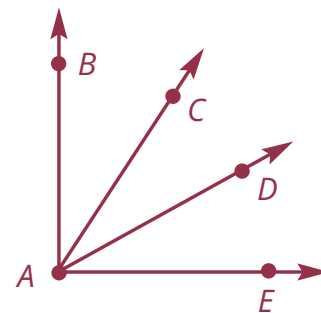


The angle to the left is formed by the union of \vec{BA} and \vec{BC} . Its sides are \vec{BA} and \vec{BC} . Its vertex is B . We can name the angle $\angle ABC$, $\angle CBA$, $\angle B$, or $\angle 1$. The angle to the right is the union of \vec{SR} and \vec{ST} . The vertex is point S . This angle can be called $\angle RST$, $\angle TSR$, $\angle S$, or $\angle 2$.

Notice that when three letters are used to name an angle, the vertex letter is always placed between the other two.

If no confusion will result, the vertex letter can be used alone; or a numeral can be used. You should never use a single letter when several angles have the same vertex.

Model:



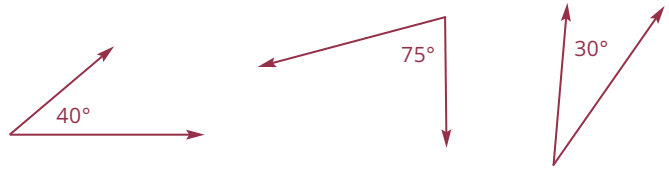
$\angle A$ could mean:

$\angle BAC$, the angle formed by \vec{AB} and \vec{AC} ; or
 $\angle CAD$, the angle formed by \vec{AC} and \vec{AD} ; or
 $\angle DAE$, the angle formed by \vec{AD} and \vec{AE} ; or
 $\angle BAD$, the angle formed by \vec{AB} and \vec{AD} ; or
 $\angle CAE$, the angle formed by \vec{AC} and \vec{AE} ; or
 $\angle BAE$, the angle formed by \vec{AB} and \vec{AE} .

DEFINITION

Acute angle: an angle whose measure is less than 90° .

Models:

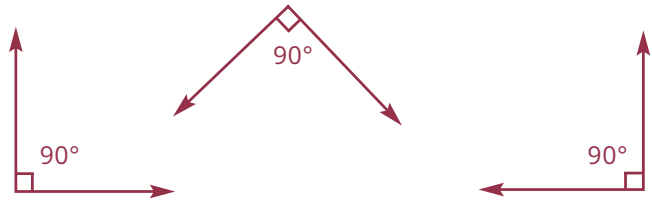


These angles are all acute angles.

DEFINITION

Right angle (rt. \angle): an angle whose measure equals 90° .

Models:

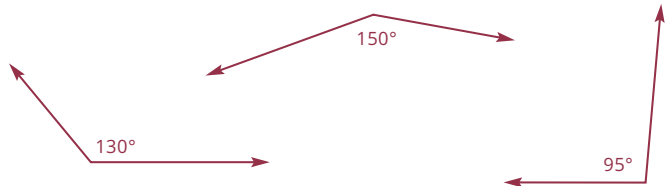


These angles are all right angles. A little box at the vertex indicates a right angle.

DEFINITION

Obtuse angle: an angle with a measure greater than 90° but less than 180° .

Models:

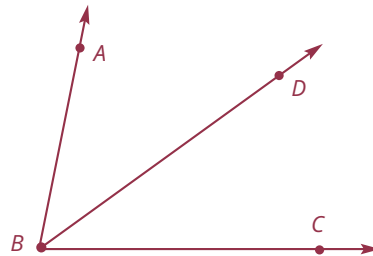


These angles are all obtuse angles.

DEFINITION

Bisector of an angle: a ray that is in the interior of the angle and divides the angle into two angles of equal measure.

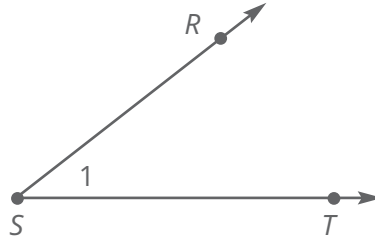
Model:



If the measure of $\angle ABD$ ($m \angle ABD$) equals the measure of $\angle DBC$ ($m \angle DBC$), then \overrightarrow{BD} bisects $\angle ABC$.



Given the following angle, complete the following items.

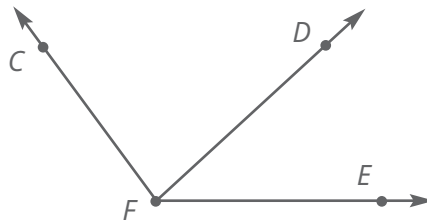


- 1.1 State four ways of naming this angle. _____
- 1.2 What point is the vertex of this angle? _____
- 1.3 Name the sides of this angle. _____

Write your answers on the lines.

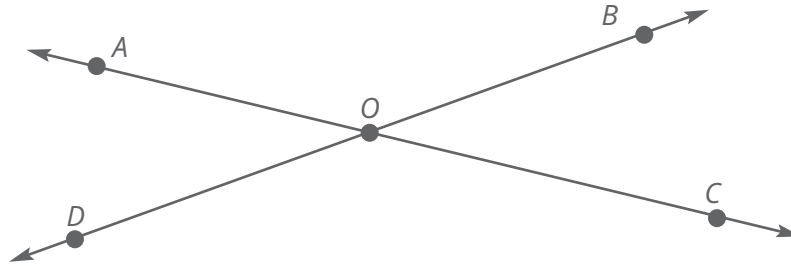
- 1.4 Name the sides of $\angle ABC$. _____
- 1.5 Name the vertex of $\angle ABC$. _____

Given the following angles, complete the following items.



- 1.6 Name three angles with vertex F . _____
- 1.7 \vec{FC} and \vec{FE} are the sides of what angle? _____
- 1.8 What ray is the common side of $\angle CFD$ and $\angle DFE$? _____
- 1.9 If $m\angle DFE = m\angle CFD$, name the bisector of $\angle CFE$. _____

Given the following angles, complete the following items.



1.10 Name the acute angles. _____

1.11 Name the obtuse angles. _____

Complete the following activity.

1.12 In the space provided:

a. Draw a right angle.

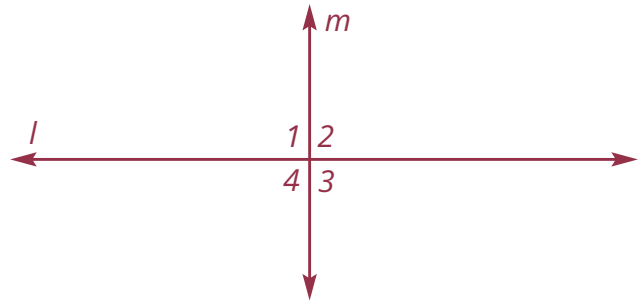
b. Draw a bisector of the right angle. Label all parts.

c. What kind of angles are formed by the bisector?

DEFINITION

Perpendicular lines: two lines that intersect such that the four angles formed are equal to each other.

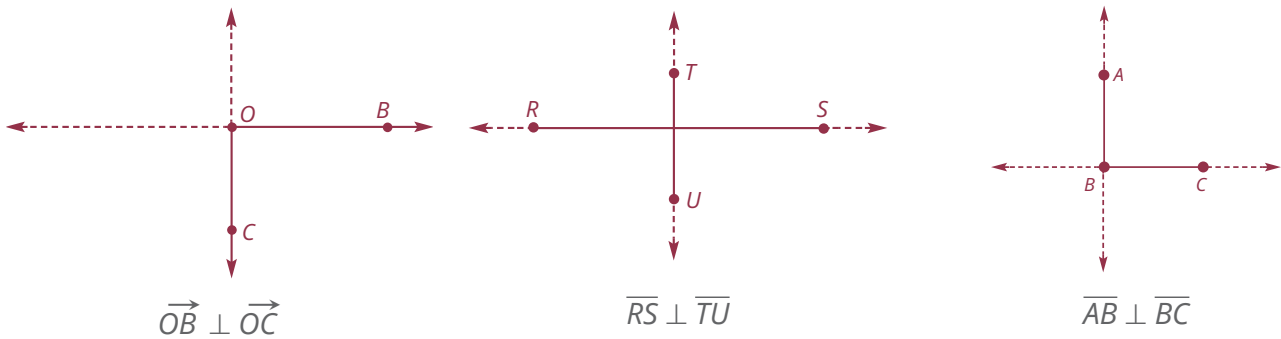
Model:



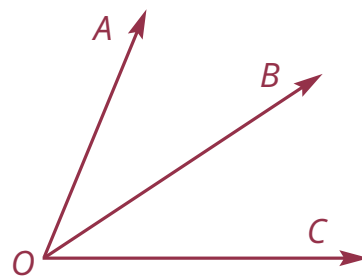
When $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4$, we say that line l is perpendicular (\perp) to line m .

Segments and rays that are parts of lines can also be called perpendicular if the lines of which they are part are perpendicular.

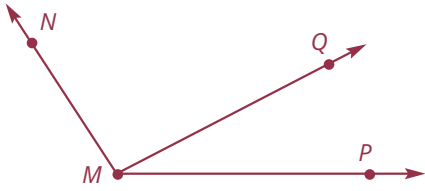
Models:



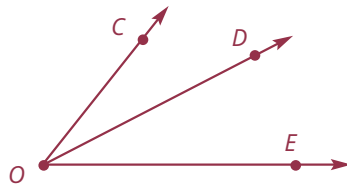
Betweenness of rays means that for any $\angle AOC$, \vec{OB} is between \vec{OA} and \vec{OC} when all three rays have the same end point *and* when OB lies in the interior of $\angle AOC$. Both of these conditions must be true for \vec{OB} to be between \vec{OA} and \vec{OC} .



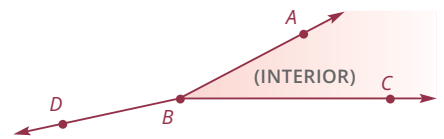
Models



\vec{MQ} is between \vec{MN} and \vec{MP} .



\vec{OD} is between \vec{OC} and \vec{OE} .



\vec{BD} is not between \vec{BA} and \vec{BC} .
It is not in the interior of $\angle ABC$.



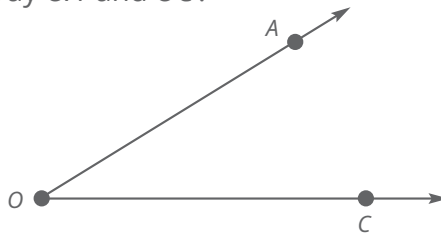
Complete the following activities.

1.13 a. Draw lines \perp to l through points $A, B, C,$ and D .



b. What does inductive reasoning tell you about these lines?

1.14 a. Draw ray \vec{OB} between ray \vec{OA} and \vec{OC} .



b. What does inductive reasoning tell you about the sum of the measures of $\angle AOB$ and $\angle BOC$?

1.15 a. Draw $\vec{RS} \perp \vec{TU}$ at O . Draw \vec{OA} between \vec{OT} and \vec{OS} . Draw \vec{OB} opposite \vec{OA} .



b. What does inductive reasoning tell you about $m \angle AOS$ and $m \angle BOR$?

c. About $m \angle AOT$ and $m \angle BOU$?

1.16 a. Draw several \perp lines.

b. What kinds of angles are formed? (acute, obtuse, or right) _____

1.17 a. Using \vec{OA} , \vec{OB} , \vec{OC} , and \vec{OD} , make a diagram so that $\angle AOB$ is an acute \angle , $\angle BOC$ is an obtuse \angle , and $\angle COD$ is a right \angle .

b. What kind of angle is $\angle AOD$? _____

1.18 What is the measure of a rt. \angle ? _____

1.19 What is the measure of an obtuse \angle ? _____

1.20 What is the measure of an acute \angle ? _____

ANGLE MEASUREMENT

We now need to work with angle measurement. The following postulates will help us to establish a method for measuring angles.

P6 tells us that every angle has a measure that is a real number between 0° and 180° . No angle has a measure of 0° ; no angle has a measure of 180° .

The next postulate tells us how to find the measurement number.

POSTULATE 6

P6: Every angle corresponds with a unique real number greater than 0° and less than 180° .

(angle measurement postulate)

POSTULATE 7

P7: The set of rays on the same side of a line with a common end point in the line can be put in one-to-one correspondence with the real numbers from 0° to 180° inclusive in such a way:

1. that one of the two opposite rays lying in the line is paired with zero and the other is paired with 180° .
2. that the measure of any angle whose sides are rays of that given set is equal to the absolute value of the difference between the numbers corresponding to its sides.

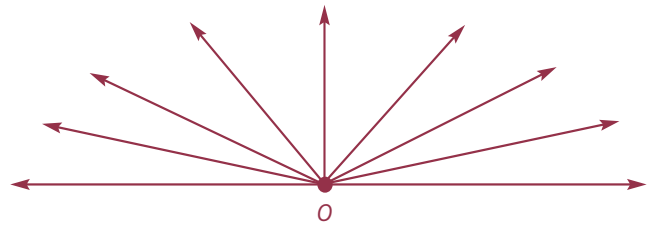
(protractor postulate)

P7 tells us how to build and use a protractor to measure an angle.

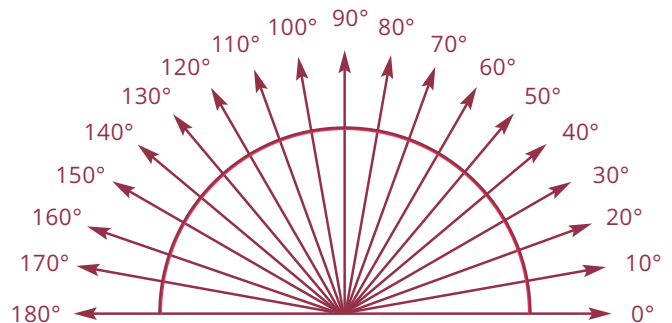
Take a line and a point O on that line.



Place a set of rays all on the same side of the line with common end point O .



Then match the numbers from 0° to 180° inclusive with the rays in such a way that one of the two opposite rays is paired with zero and the other is paired with 180° .

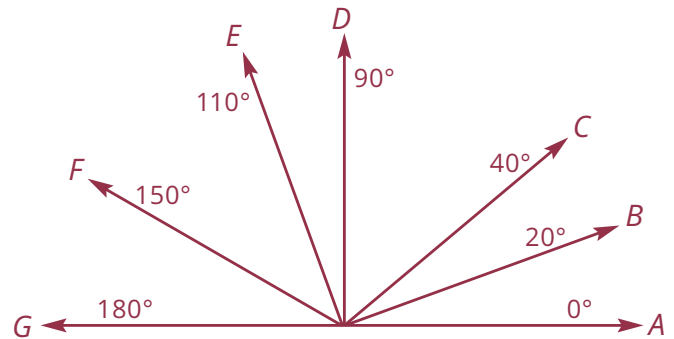


You have built a protractor. Now, to find the measure of an angle, place the sides of the angle along the rays of the protractor, lining up the vertex of the angle with point O . Noting the numbers associated with the rays, subtract one ray number from another and take the absolute value of the difference to make the answer a positive number. This number is the measure of the angle. You can line up one ray of the angle with the zero ray, but this step is not necessary. You only need to have the vertex matching the O mark on the protractor.

$$m\angle AOB = |20^\circ - 0^\circ| = |20^\circ| = 20^\circ$$

$$m\angle BOC = |40^\circ - 20^\circ| = |20^\circ| = 20^\circ$$

$$m\angle COD = |90^\circ - 40^\circ| = |50^\circ| = 50^\circ$$



The order in which you subtract does not affect the measure.

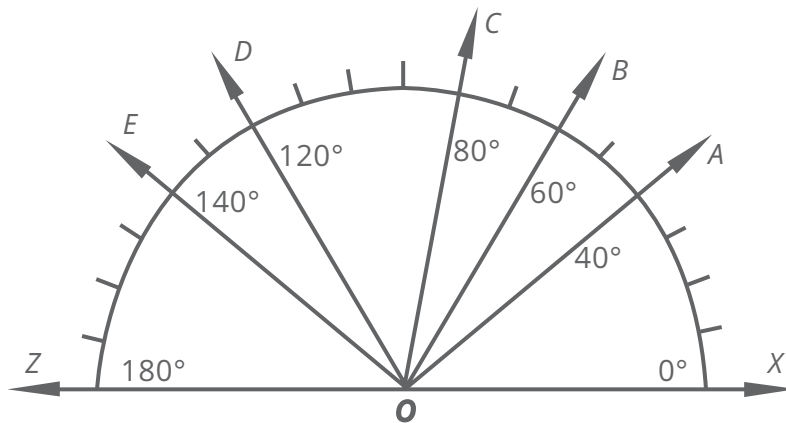
$$m\angle COP = |40^\circ - 90^\circ| = |-50^\circ| = 50^\circ$$

$$m\angle FOC = |40^\circ - 150^\circ| = |-110^\circ| = 110^\circ$$

$$m\angle DOG = |90^\circ - 180^\circ| = |-90^\circ| = 90^\circ$$



Refer to this diagram to complete the following items.



1.21 Write the measure of the following \angle 's.

a. $m\angle AOX =$ _____

b. $m\angle BOX =$ _____

c. $m\angle BOA =$ _____

d. $m\angle COB =$ _____

e. $m\angle DOX =$ _____

f. $m\angle BOE =$ _____

g. $m\angle EOD =$ _____

h. $m\angle AOD =$ _____

i. $m\angle AOC =$ _____

SELF TEST 1

Complete the following items (each answer, 3 points).

1.01 Draw an acute angle and label it so its name is $\angle WON$.

1.02 Name the vertex of $\angle WON$. _____

1.03 Name the sides of $\angle WON$. _____

1.04 Draw \vec{OS} the bisector of $\angle BOT$.



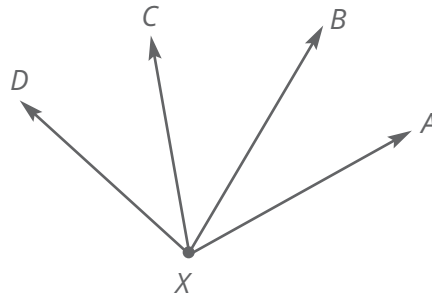
1.05 Draw $\vec{RA} \perp \vec{TU}$ at point Q .



1.06 What is the measure of an obtuse angle? _____

1.07 What is the measure of an acute angle? _____

Given the following diagram, write the required information (each answer, 3 points).



1.08 $m \angle BXA = 30^\circ 20'$ $m \angle CXB = 40^\circ 35'$ $m \angle CXA =$ _____

1.09 $m \angle DXB = 70^\circ 15' 12''$ $m \angle DXC = 30^\circ 30' 20''$ $m \angle CXB =$ _____

1.010 Name the largest angle in the figure. _____

1.011 $m \angle DXB + m \angle BXA =$ _____

Write the required information (each answer, 3 points).

1.012 Can we have an angle with a measure of 0 degrees? _____

1.013 Can we have an angle with a measure of 180 degrees? _____

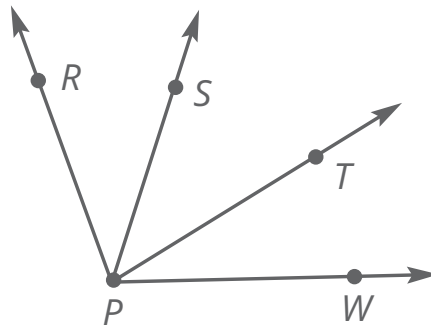
1.014 Find the sum.

$$\begin{array}{r}
 20^\circ \quad 15' \quad 18'' \\
 30^\circ \quad 41' \quad 32'' \\
 + \quad 2^\circ \quad 30' \quad 15'' \\
 \hline
 \end{array}$$

1.015 Find the difference.

$$\begin{array}{r}
 60^\circ \quad 50' \quad 40'' \\
 - \quad 30^\circ \quad 40' \quad 50'' \\
 \hline
 \end{array}$$

Supply the reasons in the following proof (each answer, 4 points).



Given: $m \angle RPS = m \angle TPW$
To Prove: $m \angle RPT = m \angle SPW$

STATEMENT	REASON
1. $m \angle RPS = m \angle TPW$	1.016 _____
2. $m \angle RPS + m \angle SPT =$ $m \angle TPW + m \angle SPT$	1.017 _____ _____
3. $m \angle RPS + m \angle SPT =$ $m \angle RPT$	1.018 _____ _____
4. $m \angle TPW + m \angle SPT =$ $m \angle SPW$	1.019 _____ _____
5. $m \angle RPT = m \angle SPW$	1.020 _____

<div style="border: 1px solid black; padding: 5px; display: inline-block;"> 52 <hr style="width: 50%; margin: 0;"/> 65 </div>	SCORE _____	TEACHER _____ <small>initials date</small>
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800-622-3070
www.aop.com

MAT1003 – Apr '15 Printing

ISBN 978-0-86717-633-9



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