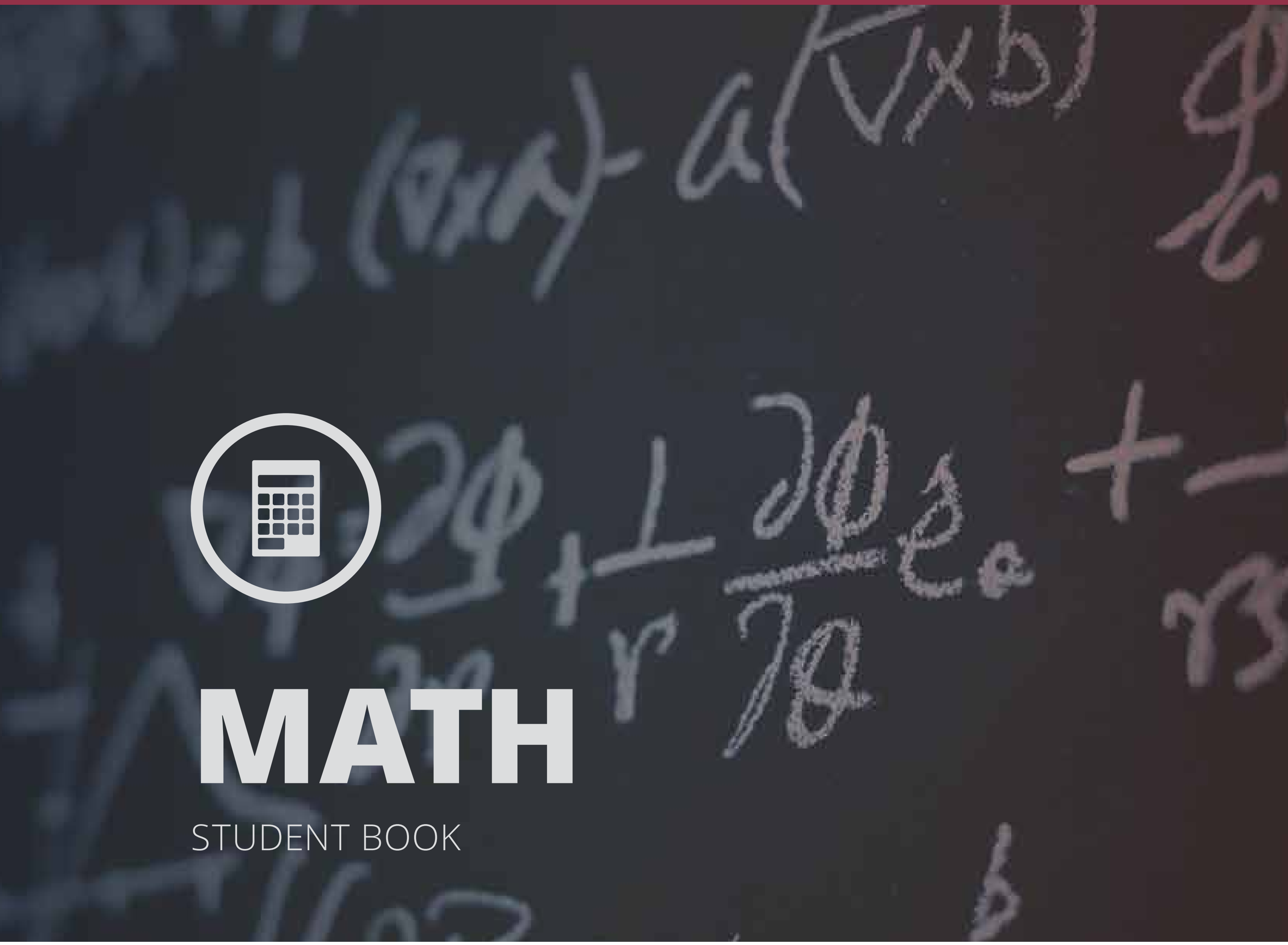
LIFEPAC®



MATH

STUDENT BOOK

10th Grade | Unit 4

Alpha Omega
PUBLICATIONS

MATH 1004
Congruency

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Alpha Omega PUBLICATIONS

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Congruency

Introduction

You have learned in previous LIFEPACs about geometry as a system. You have studied points and lines, induction and deduction, and angle relationships. The next step in your study of geometry is to learn about congruent triangles and some quadrilaterals related to congruent triangles. Theorems and properties relating to these figures will be presented in this LIFEPAC®. Completion of this LIFEPAC should prepare you for studying more complex and interesting geometric concepts.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. State the definition of congruent triangles.
2. Prove triangles congruent by using SSS, SAS, ASA, and AAS statements.
3. Prove right triangles congruent by using HL, LL, HA, and LA statements.
4. Prove corresponding parts equal when triangles are in normal position.
5. Prove corresponding parts equal when triangles are overlapping.
6. Prove properties of isosceles triangles.
7. Prove inequalities in one triangle.
8. Prove inequalities in two triangles.
9. Identify the properties of parallelograms, rectangles, squares, rhombuses, and trapezoids.

1. TRIANGLES

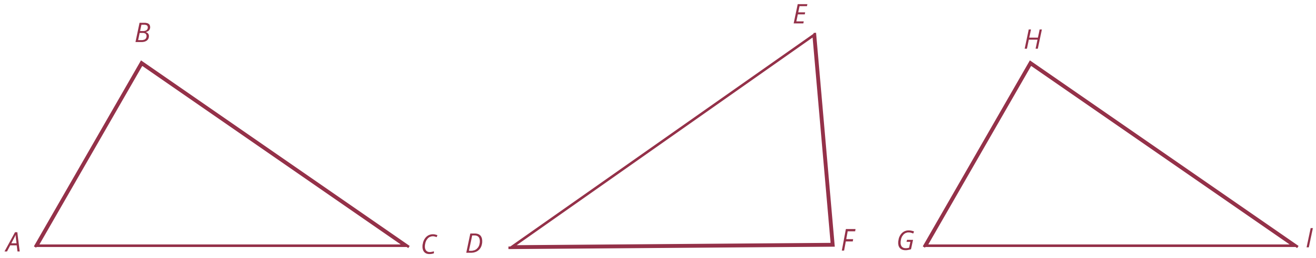
Most of the material goods we use today are mass produced. Every product is produced by the thousands, and all are exactly alike. They are the same size and the same shape. When your car needs a new part, the mechanic can replace the old part with a new one that is exactly the same as the old one. Figures, whether plane or solid, that have the same size and the same shape are called congruent figures.

Section Objectives

Review these objectives. When you have finished this section, you should be able to:

1. State the definition of congruent triangles.
2. Prove triangles congruent by using SSS, SAS, ASA, and AAS statements.
3. Prove right triangles congruent by using HL, LL, HA, and LA statements.

DEFINING CONGRUENT TRIANGLES



All three triangles shown are congruent. One way of describing the situation is to say any one of these triangles can be moved onto any other one in such a way that it fits exactly. To show this fit we can match the vertices of the triangles. This matching can take place in several ways, but only one way will make one triangle fit exactly over the other.

Model 1: $A \leftrightarrow E$

$B \leftrightarrow F$

$C \leftrightarrow D$

When the vertices are matched as in Model 1, then $\triangle$ ABC will fit over $\triangle$ EFD.

Model 2: $A \leftrightarrow G$

$B \leftrightarrow H$

$C \leftrightarrow I$

When the vertices are matched as in Model 2, then $\triangle ABC$ will fit over $\triangle GHI$.

A matching of vertices in this way is called a *one-to-one correspondence* between the vertices of the two triangles. The angles at the vertices that are matched are called *corresponding angles*. Three *corresponding sides* also match.

Model 3: $AB \leftrightarrow GH \leftrightarrow EF$

$BC \leftrightarrow HI \leftrightarrow FD$

$CA \leftrightarrow IG \leftrightarrow DE$

> **DEFINITION**
>
> **One-to-one correspondence:** the situation when each member of a set, such as angles of a triangle, can be paired with one and only one member of another set.
>
> **Corresponding angles:** angles paired with one another in a one-to-one correspondence.
>
> **Corresponding sides:** sides paired with one another in a one-to-one correspondence.

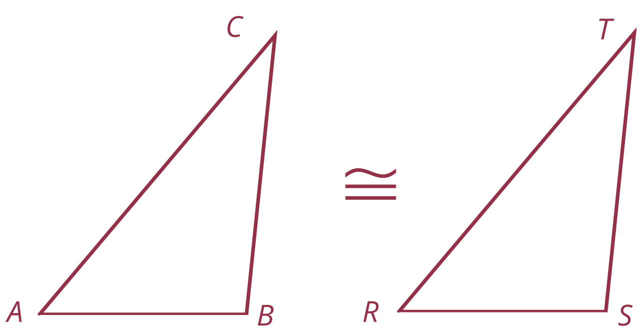
For each part (angle or side) of one triangle, a corresponding part of the other triangle exists. Therefore, we have a one-to-one correspondence between all six parts of one triangle with all six parts of another triangle.

If the one-to-one correspondence of all six parts leads to one triangle fitting over the other exactly, then the triangles are *congruent*. The symbol for congruent is ≅.

> **DEFINITION**
>
> **Congruent Triangles:** If a one-to-one correspondence between the parts of two triangles is such that the corresponding parts are equal, then the triangles are congruent.

To show which parts correspond to each other, we name the triangles in a special way.

Model 4:



First write the name of one triangle, then write the vertices of the other triangle so that the corresponding vertices are in matching position in the name.

$\triangle ABC \cong \triangle RST$

or

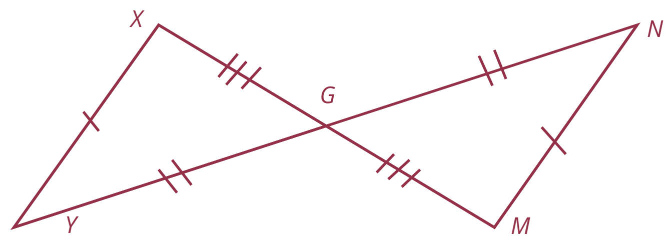
$\triangle BCA \cong \triangle STR$

or

$\triangle CAB \cong \triangle TRS$

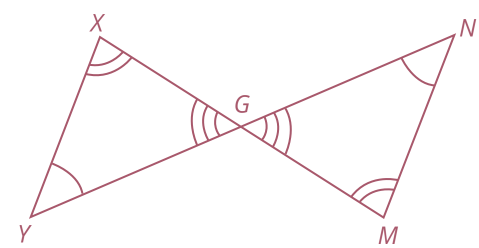
When we draw models of congruent triangles, we often mark pairs of corresponding parts in the same way, to show which parts are equal.

Model 5:



The marks /, //, and /// show that
$YX = MN$, $YG = GN$, and $XG = GM$.

Model 6:



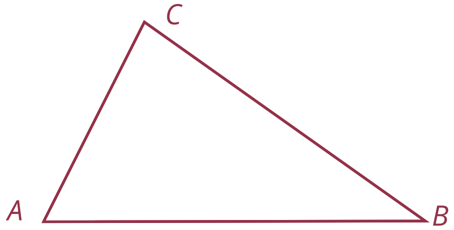
The marks , , and show that
$\angle Y = \angle N$, $\angle X = \angle M$, and $\angle XGY = \angle MGN$.

Two more definitions that we will be using are the definitions for *included angle* and *included side*.

DEFINITION

Included Angle: the angle formed by two sides of a triangle.

Model:

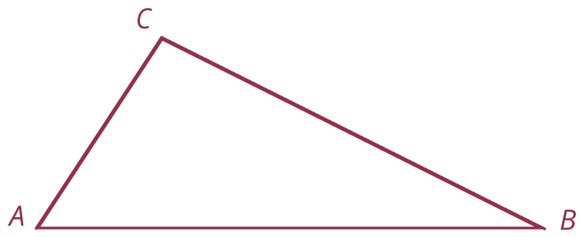


$\angle C$ is the included angle between sides $\overline{AC}$ and $\overline{BC}$. $\angle B$ is the included angle between sides $\overline{AB}$ and $\overline{BC}$. $\angle A$ is the included angle between sides $\overline{AB}$ and $\overline{AC}$.

DEFINITION

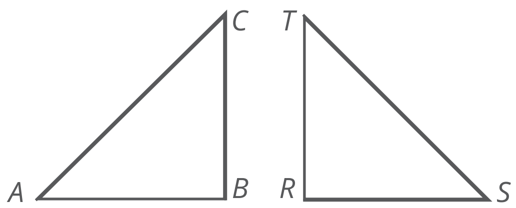
Included Side: the side of a triangle that is formed by the common side of two angles.

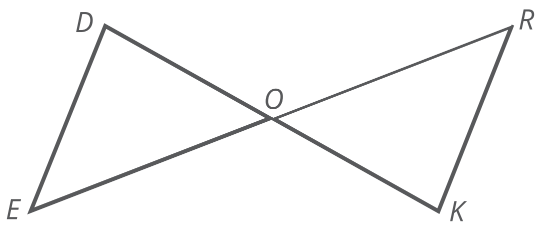
Model:



$\overline{AB}$ is the included side between $\angle A$ and $\angle B$. $\overline{BC}$ is the included side between $\angle B$ and $\angle C$. $\overline{AC}$ is included between $\angle A$ and $\angle C$.

Complete the correspondence so a congruence can be established.





1.1 $A \leftrightarrow$ ____________

1.2 $B \leftrightarrow$ ____________

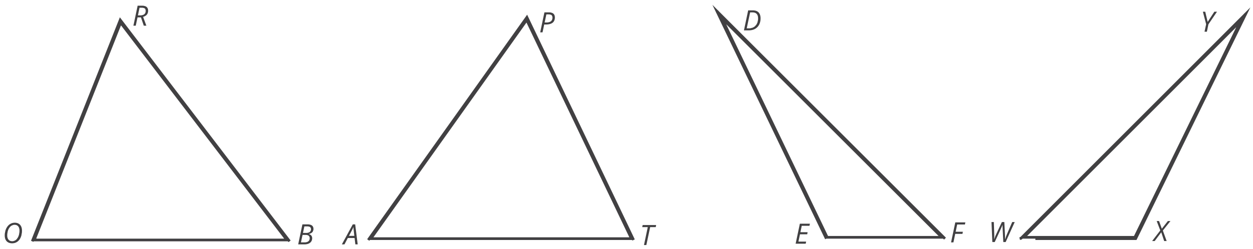
1.3 $C \leftrightarrow$ ____________

1.4 $E \leftrightarrow$ ____________

1.5 $D \leftrightarrow$ ____________

1.6 $O \leftrightarrow$ ____________

Basing your answer on the appearance of the figures, write true or false.



1.7 __________ $\triangle ROB \cong \triangle PTA$

1.8 __________ $\triangle ROB \cong \triangle PAT$

1.9 __________ $\triangle RBO \cong \triangle PTA$

1.10 __________ $\triangle OBR \cong \triangle APT$

1.11 __________ $\triangle DEF \cong \triangle WXY$

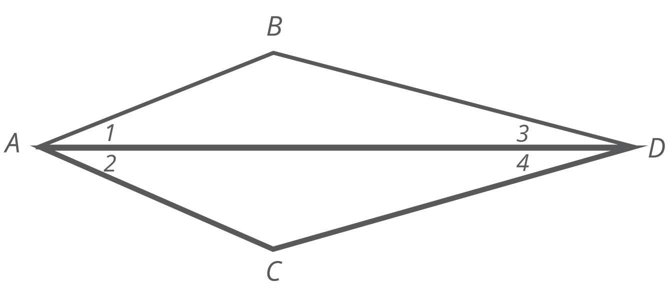
1.12 __________ $\triangle DFE \cong \triangle YWX$

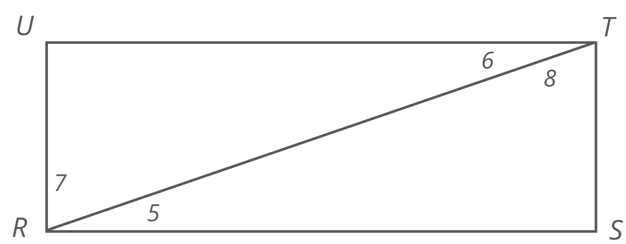
1.13 __________ $\triangle FED \cong \triangle WXY$

1.14 __________ $\triangle PAT \cong \triangle WXY$

1.15 __________ $\triangle ROB \cong \triangle DEF$

In the following pairs of congruent triangles, complete the pairs of corresponding parts.





1.16 $\overline{AB} \leftrightarrow$ __________

1.17 $\overline{CD} \leftrightarrow$ __________

1.18 $\overline{AD} \leftrightarrow$ __________

1.19 $\angle 1 \leftrightarrow$ __________

1.20 $\angle 3 \leftrightarrow$ __________

1.21 $\angle B \leftrightarrow$ __________

1.22 $\overline{RS} \leftrightarrow$ __________

1.23 $\overline{TS} \leftrightarrow$ __________

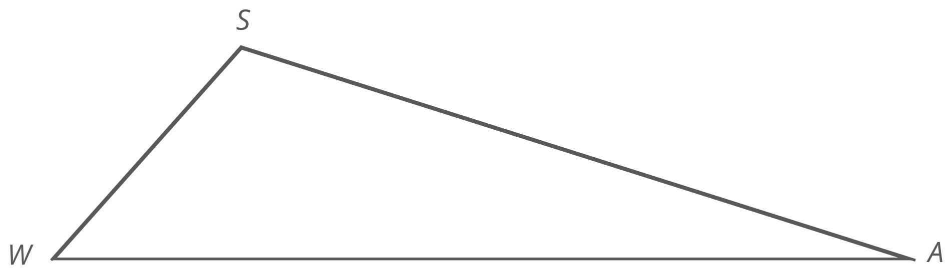
1.24 $\overline{RT} \leftrightarrow$ __________

1.25 $\angle 5 \leftrightarrow$ __________

1.26 $\angle 7 \leftrightarrow$ __________

1.27 $\angle U \leftrightarrow$ __________

Name the included angle or the included side asked for.



1.28 Included angle between $\overline{WA}$ and $\overline{AS}$: ________________

1.29 Included angle between $\overline{SW}$ and $\overline{WA}$: ________________

1.30 Included side between $\angle S$ and $\angle A$: ________________

1.31 Included side between $\angle S$ and $\angle W$: ________________

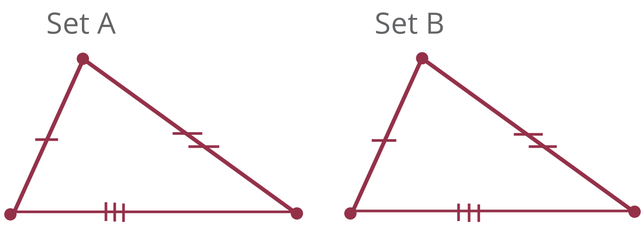
1.32 Included angle between $\overline{SW}$ and $\overline{AS}$: ________________

1.33 Included side between $\angle A$ and $\angle W$: ________________

PROVING TRIANGLES CONGRUENT

Suppose you take two identical sets of three sticks with the sticks in one set the same length as the sticks in the other set.

Connect the three sticks in each set at their end points.



However you put the sticks together, the two Δ's formed will be the same size and shape. The two triangles will be congruent. This result suggests the following postulate.

> **POSTULATE 11**
>
> **P11:** If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.
>
> **(SSS Postulate)**

Postulate 11 states that we only need to show that three sides of one triangle are equal to three sides of the other triangle for the triangles to be congruent. We do not need to know anything about the angles to use this postulate. The following two postulates can be used to prove triangles congruent in other ways.

POSTULATE 12

P12: If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the triangles are congruent.

(SAS Postulate)

POSTULATE 13

P13: If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

(ASA Postulate)

With the ASA postulate we can prove the next congruent triangle statement.

THEOREM 4-1

If two angles and a not-included side of one triangle are equal to the corresponding parts of another triangle, then the triangles are congruent.

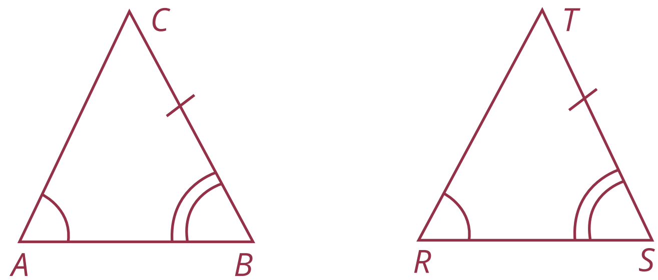
(AAS Theorem)

Note: *We will use $\angle A$ for $m\angle A$.*

Given: $\angle A = \angle R$
$\angle B = \angle S$
$BC = ST$

To Prove: $\triangle ABC \cong \triangle RST$

Plan: Show that $\angle C = \angle T$ and use ASA.

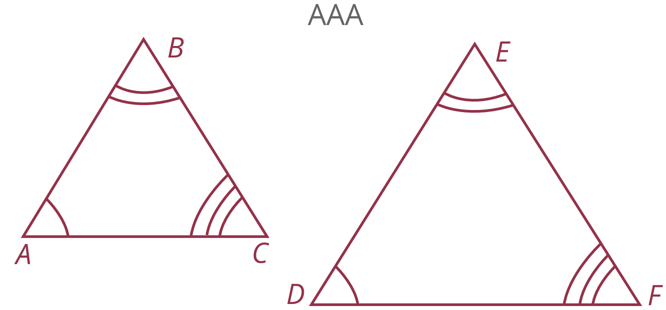


STATEMENT		REASON	
1.	$\angle A = \angle R$ $\angle B = \angle S$ $BC = ST$	1.	Given
2.	$\angle C = \angle T$	2.	If 2 $\angle$'s of one $\triangle$ are = to 2 $\angle$'s of another, then the third $\angle$'s are also =.
3.	$\triangle ABC \cong \triangle RST$	3.	ASA Postulate

We now have four ways that can be used to prove any two triangles congruent: SSS, SAS, ASA, and AAS. When you use these abbreviations, make sure you understand the complete statement.

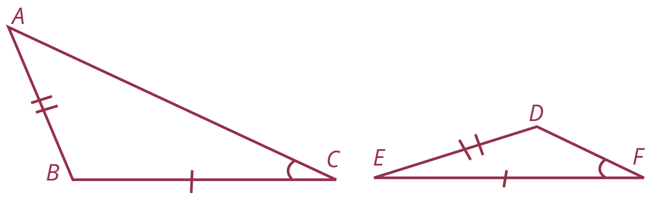
Two other statements about sides and angles of triangles are correspondence statements only, not congruence statements. They are AAA and SSA.

AAA



$\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$ but $\triangle ABC$ does not fit exactly over $\triangle DEF$. The triangles are the same shape, but different sizes. They are not congruent $\triangle$'s.

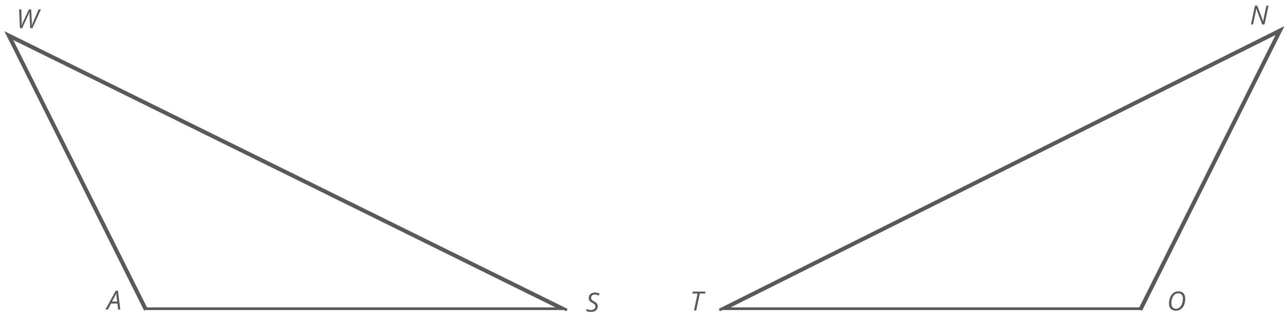
The other statement, SSA, is also not a congruence statement.



$AB = DE$, $BC = EF$, and $\angle C = \angle F$; but $\triangle ABC$ will not fit $\triangle DEF$; therefore, the two triangles are not congruent. These two triangles are not even the same shape.

Remember that when you are asked to write a postulate or theorem, you should write the statement or its abbreviation (such as SSS) rather than writing the number of the theorem or postulate. The statements are easier and more important to learn than are their numbers.

Write the abbreviation of the postulate or theorem that supports the conclusion that $\triangle WAS \cong \triangle NOT$.



Given:

1.34 $\angle A = \angle O$, $WA = NO$, $AS = OT$. __________

1.35 $WA = NO$, $AS = OT$, $SW = TN$. __________

1.36 $\angle A = \angle O$, $\angle W = \angle N$, $SW = TN$. __________

1.37 $WS = NT$, $AS = OT$, $\angle S = \angle T$. __________

1.38 $WA = NO$, $WS = NT$, $\angle W = \angle N$. __________

1.39 $\angle W = \angle N$, $\angle S = \angle T$, $WS = NT$. __________

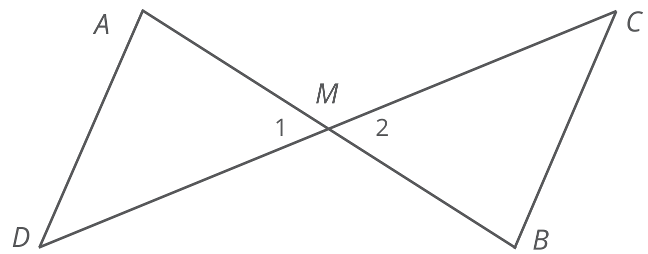
1.40 $\angle W = \angle N$, $\angle S = \angle T$, $WA = NO$. __________

Complete the two-column proofs.

1.41 Given: $AM = BM$

$DM = CM$

Prove: $\triangle AMD \cong \triangle BMC$

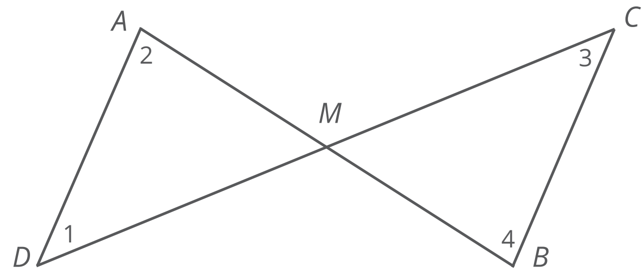


STATEMENT	REASON
1. ______________	1. ______________
2. ______________	2. ______________
3. ______________	3. ______________

1.42 Given: $\overline{AD} \parallel \overline{BC}$

$AD = BC$

Prove: $\triangle ADM \cong \triangle BCM$



STATEMENT	REASON
1. ______________	1. ______________
2. ______________	2. ______________
3. ______________	3. ______________

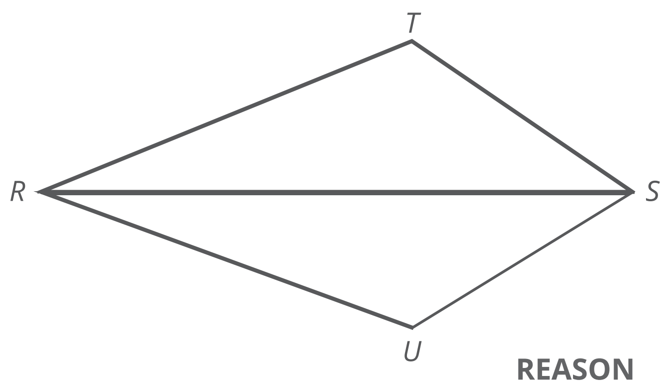
REVIEW THE POSTULATE OF EQUALITY

Reflexive: a = a

1.43 Given: $RT = RU$

$TS = US$

Prove: $\triangle RST \cong \triangle RSU$

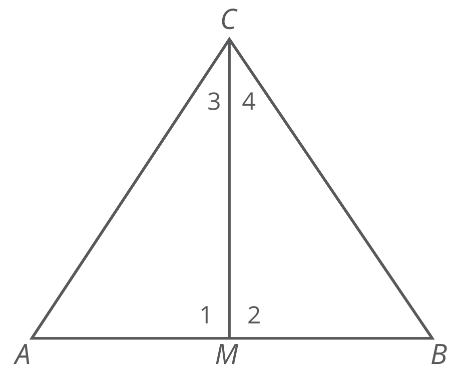


STATEMENT	REASON
1. ______________	1. ______________
2. ______________	2. ______________
3. ______________	3. ______________

1.44 Given: $\overline{CM} \perp \overline{AB}$

$\angle 3 = \angle 4$

Prove: $\triangle AMC \cong \triangle BMC$

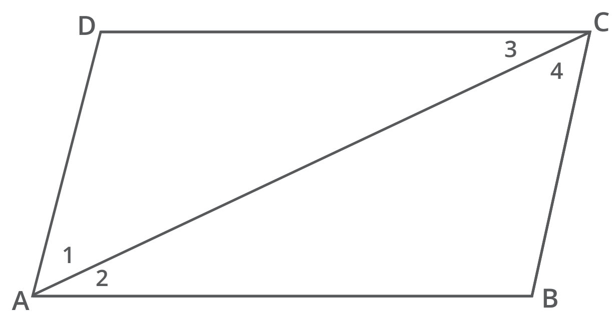


STATEMENT	REASON
1. ______	1. ______
2. ______	2. ______
3. ______	3. ______
4. ______	4. ______

1.45 Given: $\overline{DC} \parallel \overline{AB}$

$\overline{AD} \parallel \overline{BC}$

Prove: $\triangle ACD \cong \triangle CAB$



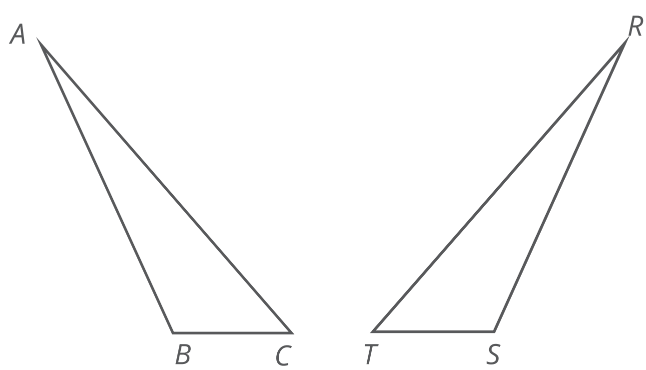
STATEMENT	REASON
1. ______	1. ______
2. ______	2. ______
3. ______	3. ______
4. ______	4. ______

Write the required information.

Given: $\triangle ABC \cong \triangle RST$

If $AB = 6$, $ST = 8$, $AC = 12$,

$\angle A = 40°$, $\angle T = 20°$, then



1.46 BC = ______

1.47 RT = ______

1.48 $\angle C$ = ______

1.49 $\angle B$ = ______

1.50 $\angle S$ = ______

1.51 $\angle R$ = ______

1.52 RS = ______

SELF TEST 1

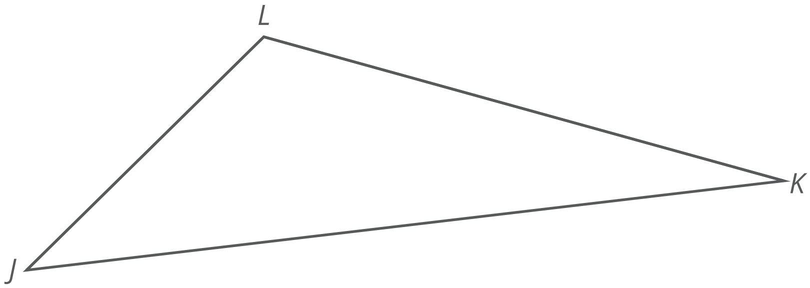
Name the corresponding parts if Δ *RST* ≅ Δ *WXY* (each answer, 3 points)**.**

1.01 $\angle R$ = ____________________ **1.02** $\angle S$ = ____________________

1.03 $\angle T$ = ____________________ **1.04** RS = ____________________

1.05 ST = ____________________ **1.06** RT = ____________________

Answer the following questions about this triangle (each answer, 3 points).



1.07 $\angle J$ is included between a. ______________ and b. ______________

1.08 $\overline{JK}$ is included between a. ______________ and b. ______________

1.09 $\angle$K is included between $\overline{LK}$ and ____________________

1.010 $\angle$L is not an included angle for sides ______________ and ______________

Write the complete statements (each answer, 4 points).

Answer is: If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

1.011 ASA ____

____

____

1.012 HL ____

____

____

1.013 SSS ____

____

____

1.014 LA ______________________________

____

____

1.015 SAS ______________________________

____

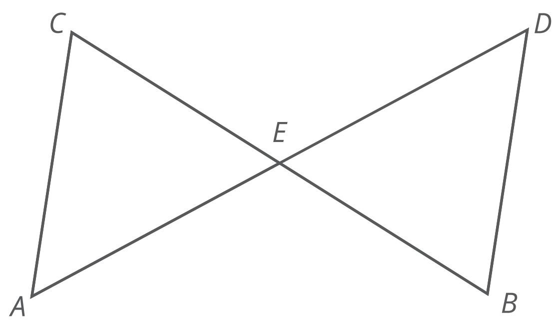
____

Complete the proof (each answer, 4 points).

Given: $\overline{CA} \parallel \overline{DB}$

E is midpoint of $\overline{AD}$

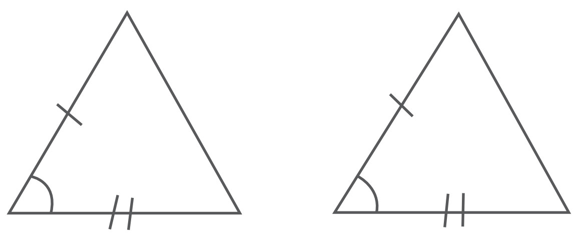
Prove: $\triangle AEC \cong \triangle DEB$



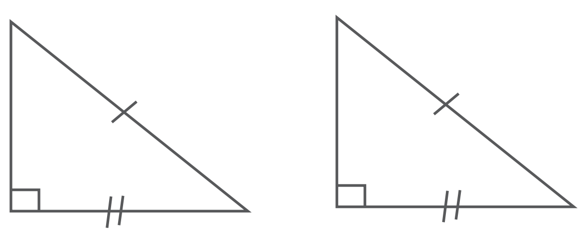
	STATEMENT	REASON
1.016	______________	______________
1.017	______________	______________
1.018	______________	______________
1.019	______________	______________

Write the congruence statement that you would use to show the Δ's ≅ (each answer, 3 points).

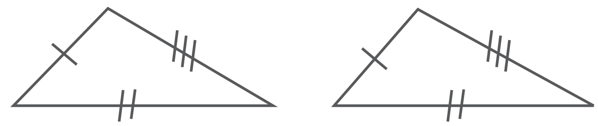
1.020 ____________



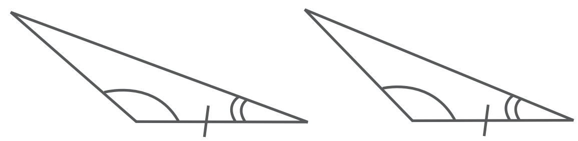
1.021 ____________



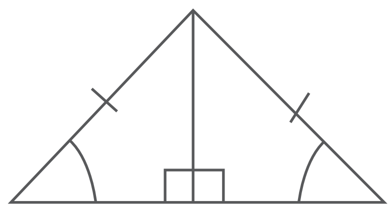
1.022 ______________



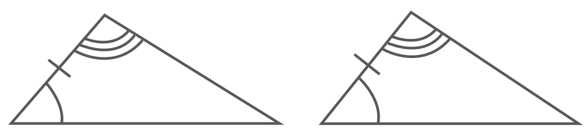
1.023 ______________



1.024 ______________



1.025 ______________



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