MATH
STUDENT BOOK

10th Grade | Unit 4
# MATH 1004

Congruency

## INTRODUCTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. TRIANGLES</td>
<td>5</td>
</tr>
</tbody>
</table>

### 1. TRIANGLES

- Defining Congruent Triangles | 5
- Proving Triangles Congruent | 9
- Proving Right Triangles Congruent | 14
- Self Test 1 | 19

### 2. CORRESPONDING PARTS

- Independent Triangles | 22
- Overlapping Triangles | 28
- Isosceles Triangles | 34
- Self Test 2 | 40

### 3. INEQUALITIES

- Inequalities in One Triangle | 42
- Inequalities in Two Triangles | 50
- Self Test 3 | 54

### 4. QUADRILATERALS

- Parallelograms | 56
- Trapezoids | 70
- Self Test 4 | 74
- Glossary | 76

---

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.
Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. State the definition of congruent triangles.
2. Prove triangles congruent by using SSS, SAS, ASA, and AAS statements.
3. Prove right triangles congruent by using HL, LL, HA, and LA statements.
4. Prove corresponding parts equal when triangles are in normal position.
5. Prove corresponding parts equal when triangles are overlapping.
6. Prove properties of isosceles triangles.
7. Prove inequalities in one triangle.
8. Prove inequalities in two triangles.
9. Identify the properties of parallelograms, rectangles, squares, rhombuses, and trapezoids.
1. TRIANGLES

Most of the material goods we use today are mass produced. Every product is produced by the thousands, and all are exactly alike. They are the same size and the same shape. When your car needs a new part, the mechanic can replace the old part with a new one that is exactly the same as the old one. Figures, whether plane or solid, that have the same size and the same shape are called congruent figures.

Section Objectives

Review these objectives. When you have finished this section, you should be able to:

1. State the definition of congruent triangles.
2. Prove triangles congruent by using SSS, SAS, ASA, and AAS statements.
3. Prove right triangles congruent by using HL, LL, HA, and LA statements.

DEFINING CONGRUENT TRIANGLES

All three triangles shown are congruent. One way of describing the situation is to say any one of these triangles can be moved onto any other one in such a way that it fits exactly. To show this fit we can match the vertices of the triangles. This matching can take place in several ways, but only one way will make one triangle fit exactly over the other.

Model 1:  \(A \rightarrow E\)
\(B \rightarrow F\)
\(C \rightarrow D\)

When the vertices are matched as in Model 1, then \(\triangle ABC\) will fit over \(\triangle EFD\).

Model 2:  \(A \rightarrow G\)
\(B \rightarrow H\)
\(C \rightarrow I\)

When the vertices are matched as in Model 2, then \(\triangle ABC\) will fit over \(\triangle GHI\).

A matching of vertices in this way is called a one-to-one correspondence between the vertices of the two triangles. The angles at the vertices that are matched are called corresponding angles. Three corresponding sides also match.

Model 3:  \(AB \rightarrow GH \rightarrow EF\)
\(BC \rightarrow HI \rightarrow FD\)
\(CA \rightarrow IG \rightarrow DE\)
**DEFINITION**

**One-to-one correspondence:** the situation when each member of a set, such as angles of a triangle, can be paired with one and only one member of another set.

**Corresponding angles:** angles paired with one another in a one-to-one correspondence.

**Corresponding sides:** sides paired with one another in a one-to-one correspondence.

For each part (angle or side) of one triangle, a corresponding part of the other triangle exists. Therefore, we have a one-to-one correspondence between all six parts of one triangle with all six parts of another triangle.

If the one-to-one correspondence of all six parts leads to one triangle fitting over the other exactly, then the triangles are congruent. The symbol for congruent is \(\cong\).

**DEFINITION**

**Congruent Triangles:** If a one-to-one correspondence between the parts of two triangles is such that the corresponding parts are equal, then the triangles are congruent.

To show which parts correspond to each other, we name the triangles in a special way.

**Model 4:**

First write the name of one triangle, then write the vertices of the other triangle so that the corresponding vertices are in matching position in the name.

\[\triangle ABC \cong \triangle RST\]

or

\[\triangle BCA \cong \triangle STR\]

or

\[\triangle CAB \cong \triangle TRS\]

When we draw models of congruent triangles, we often mark pairs of corresponding parts in the same way, to show which parts are equal.

**Model 5:**

The marks \(\|\), \(\|\|\), and \(\|\|\|\) show that \(YX = MN\), \(YG = GN\), and \(XG = GM\).

**Model 6:**

The marks \(\approx\), \(\approx\), and \(\approx\approx\) show that \(\angle Y = \angle N\), \(\angle X = \angle M\), and \(\angle XGY = \angle MGN\).

Two more definitions that we will be using are the definitions for **included angle** and **included side**.
DEFINITION
Included Angle: the angle formed by two sides of a triangle.

Model:

\[ \angle C \text{ is the included angle between sides } \overline{AC} \text{ and } \overline{BC}. \]
\[ \angle B \text{ is the included angle between sides } \overline{AB} \text{ and } \overline{BC}. \]
\[ \angle A \text{ is the included angle between sides } \overline{AB} \text{ and } \overline{AC}. \]

DEFINITION
Included Side: the side of a triangle that is formed by the common side of two angles.

Model:

\[ \overline{AB} \text{ is the included side between } \angle A \text{ and } \angle B. \]
\[ \overline{BC} \text{ is the included side between } \angle B \text{ and } \angle C. \]
\[ \overline{AC} \text{ is included between } \angle A \text{ and } \angle C. \]

Complete the correspondence so a congruence can be established.

1.1  \( A \rightarrow \) __________
1.2  \( B \rightarrow \) __________
1.3  \( C \rightarrow \) __________
1.4  \( E \rightarrow \) __________
1.5  \( D \rightarrow \) __________
1.6  \( O \rightarrow \) __________
Basing your answer on the appearance of the figures, write true or false.

1.7 ________  \( \triangle ROB \cong \triangle PTA \)
1.8 ________  \( \triangle ROB \cong \triangle PAT \)
1.9 ________  \( \triangle RBO \cong \triangle PTA \)
1.10 ________  \( \triangle OBR \cong \triangle APT \)
1.11 ________  \( \triangle DEF \cong \triangle WXY \)
1.12 ________  \( \triangle DFE \cong \triangle YWX \)
1.13 ________  \( \triangle FED \cong \triangle WXY \)
1.14 ________  \( \triangle PAT \cong \triangle WXY \)
1.15 ________  \( \triangle ROB \cong \triangle DEF \)

In the following pairs of congruent triangles, complete the pairs of corresponding parts.

1.16 \( \overline{AB} \leftrightarrow \) ________
1.17 \( \overline{CD} \leftrightarrow \) ________
1.18 \( \overline{AD} \leftrightarrow \) ________
1.19 \( \angle 1 \leftrightarrow \) ________
1.20 \( \angle 3 \leftrightarrow \) ________
1.21 \( \angle B \leftrightarrow \) ________
1.22 \( \overline{RS} \leftrightarrow \) ________
1.23 \( \overline{TS} \leftrightarrow \) ________
1.24 \( \overline{RT} \leftrightarrow \) ________
1.25 \( \angle 5 \leftrightarrow \) ________
1.26 \( \angle 7 \leftrightarrow \) ________
1.27 \( \angle U \leftrightarrow \) ________
PROVING TRIANGLES CONGRUENT

Suppose you take two identical sets of three sticks with the sticks in one set the same length as the sticks in the other set. However you put the sticks together, the two Δ’s formed will be the same size and shape. The two triangles will be congruent. This result suggests the following postulate.

POSTULATE 11

P11: If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

(SSS Postulate)

Postulate 11 states that we only need to show that three sides of one triangle are equal to three sides of the other triangle for the triangles to be congruent. We do not need to know anything about the angles to use this postulate. The following two postulates can be used to prove triangles congruent in other ways.
POSTULATE 12
P12: If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the triangles are congruent.  
(SAS Postulate)

POSTULATE 13
P13: If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.  
(ASA Postulate)

With the ASA postulate we can prove the next congruent triangle statement.

THEOREM 4-1
If two angles and a not-included side of one triangle are equal to the corresponding parts of another triangle, then the triangles are congruent.  
(AAS Theorem)

Note: We will use $\angle A$ for $m \angle A$.

Given: $\angle A = \angle R$
$\angle B = \angle S$
$BC = ST$

To Prove: $\triangle ABC \cong \triangle RST$

Plan: Show that $\angle C = \angle T$ and use ASA.

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle A = \angle R$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$\angle B = \angle S$</td>
<td>2. If 2 $\angle$'s of one $\triangle$ are = to 2 $\angle$'s of another, then the third $\angle$'s are also =.</td>
</tr>
<tr>
<td>$BC = ST$</td>
<td>3. ASA Postulate</td>
</tr>
<tr>
<td>2. $\angle C = \angle T$</td>
<td></td>
</tr>
<tr>
<td>3. $\triangle ABC \cong \triangle RST$</td>
<td></td>
</tr>
</tbody>
</table>

We now have four ways that can be used to prove any two triangles congruent: SSS, SAS, ASA, and AAS. When you use these abbreviations, make sure you understand the complete statement.

Two other statements about sides and angles of triangles are correspondence statements only, not congruence statements. They are AAA and SSA.

$\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$ but $\triangle ABC$ does not fit exactly over $\triangle DEF$. The triangles are the same shape, but different sizes. They are not congruent $\triangle$'s.
The other statement, SSA, is also not a congruence statement.

\[ AB = DE, \ BC = EF, \ \text{and} \ \angle C = \angle F; \text{but} \ \triangle ABC \text{ will not fit} \ \triangle DEF; \text{therefore, the two triangles are not congruent. These two triangles are not even the same shape.} \]

Remember that when you are asked to write a postulate or theorem, you should write the statement or its abbreviation (such as SSS) rather than writing the number of the theorem or postulate. The statements are easier and more important to learn than are their numbers.

**Write the abbreviation of the postulate or theorem that supports the conclusion that \( \triangle WAS \cong \triangle NOT. \)**

**Given:**

1.34 \( \angle A = \angle O, \ WA = NO, \ AS = OT. \)
1.35 \( WA = NO, \ AS = OT, \ SW = TN. \)
1.36 \( \angle A = \angle O, \ \angle W = \angle N, \ SW = TN. \)
1.37 \( WS = NT, \ AS = OT, \ \angle S = \angle T. \)
1.38 \( WA = NO, \ WS = NT, \ \angle W = \angle N. \)
1.39 \( \angle W = \angle N, \ \angle S = \angle T, \ WS = NT. \)
1.40 \( \angle W = \angle N, \ \angle S = \angle T, \ WA = NO. \)
Complete the two-column proofs.

1.41 Given: \( AM = BM \)
\( DM = CM \)
Prove: \( \triangle AMD \cong \triangle BMC \)

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AM = BM )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( DM = CM )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle AMD \cong \angle BMC )</td>
<td>3. ( \angle AMD \cong \angle BMC )</td>
</tr>
</tbody>
</table>

1.42 Given: \( \overline{AD} \parallel \overline{BC} \)
\( AD = BC \)
Prove: \( \triangle ADM \cong \triangle BCM \)

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle DMA \cong \angle BMA )</td>
<td>1. ( \angle DMA \cong \angle BMA )</td>
</tr>
<tr>
<td>2. ( AD = BC )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \triangle ADM \cong \triangle BCM )</td>
<td>3. ( \triangle AD = BC )</td>
</tr>
</tbody>
</table>

**REVIEW THE POSTULATE OF EQUALITY**

Reflexive: \( a = a \)

1.43 Given: \( RT = RU \)
\( TS = US \)
Prove: \( \triangle RST \cong \triangle RSU \)

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( RT = RU )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( TS = US )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \triangle RST \cong \triangle RSU )</td>
<td>3. ( \triangle RST \cong \triangle RSU )</td>
</tr>
</tbody>
</table>
1.44 Given: $\overline{CM} \perp \overline{AB}$
$\angle 3 = \angle 4$
Prove: $\triangle AMC \cong \triangle BMC$

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{CM} \perp \overline{AB}$</td>
<td>1.</td>
</tr>
<tr>
<td>2. $\angle 3 = \angle 4$</td>
<td>2.</td>
</tr>
<tr>
<td>3. $\triangle AMC \cong \triangle BMC$</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
</tbody>
</table>

1.45 Given: $\overline{DC} \parallel \overline{AB}$
$\overline{AD} \parallel \overline{BC}$
Prove: $\triangle ACD \cong \triangle CAB$

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{DC} \parallel \overline{AB}$</td>
<td>1.</td>
</tr>
<tr>
<td>2. $\overline{AD} \parallel \overline{BC}$</td>
<td>2.</td>
</tr>
<tr>
<td>3. $\triangle ACD \cong \triangle CAB$</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
</tbody>
</table>

Write the required information.
Given: $\triangle ABC \cong \triangle RST$
If $AB = 6$, $ST = 8$, $AC = 12$,
$\angle A = 40^\circ$, $\angle T = 20^\circ$, then

1.46 $BC = \underline{\hspace{2cm}}$
1.50 $\angle S = \underline{\hspace{2cm}}$
1.47 $RT = \underline{\hspace{2cm}}$
1.51 $\angle R = \underline{\hspace{2cm}}$
1.48 $\angle C = \underline{\hspace{2cm}}$
1.52 $RS = \underline{\hspace{2cm}}$
1.49 $\angle B = \underline{\hspace{2cm}}$
SELF TEST 1

Name the corresponding parts if $\triangle RST \cong \triangle WXY$ (each answer, 3 points).

1.01 $\angle R =$ ____________________________  
1.02 $\angle S =$ ____________________________  
1.03 $\angle T =$ ____________________________  
1.04 $RS =$ ____________________________  
1.05 $ST =$ ____________________________  
1.06 $RT =$ ____________________________

Answer the following questions about this triangle (each answer, 3 points).

1.07 $\angle J$ is included between a. ___________ and b. ___________

1.08 $\overline{JK}$ is included between a. ___________ and b. ___________

1.09 $\angle K$ is included between $\overline{LK}$ and ___________________________

1.10 $\angle L$ is not an included angle for sides ___________________ and ___________________

Write the complete statements (each answer, 4 points).

Answer is: If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

1.011 ASA _______________________________________

1.012 HL _______________________________________

1.013 SSS _______________________________________
Complete the proof (each answer, 4 points).

Given: \( \overline{CA} \parallel \overline{DB} \)
\( E \) is midpoint of \( \overline{AD} \)
Prove: \( \triangle AEC \cong \triangle DEB \)

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.016</td>
<td></td>
</tr>
<tr>
<td>1.017</td>
<td></td>
</tr>
<tr>
<td>1.018</td>
<td></td>
</tr>
<tr>
<td>1.019</td>
<td></td>
</tr>
</tbody>
</table>

Write the congruence statement that you would use to show the \( \Delta \)'s \( \cong \) (each answer, 3 points).

1.020

1.021