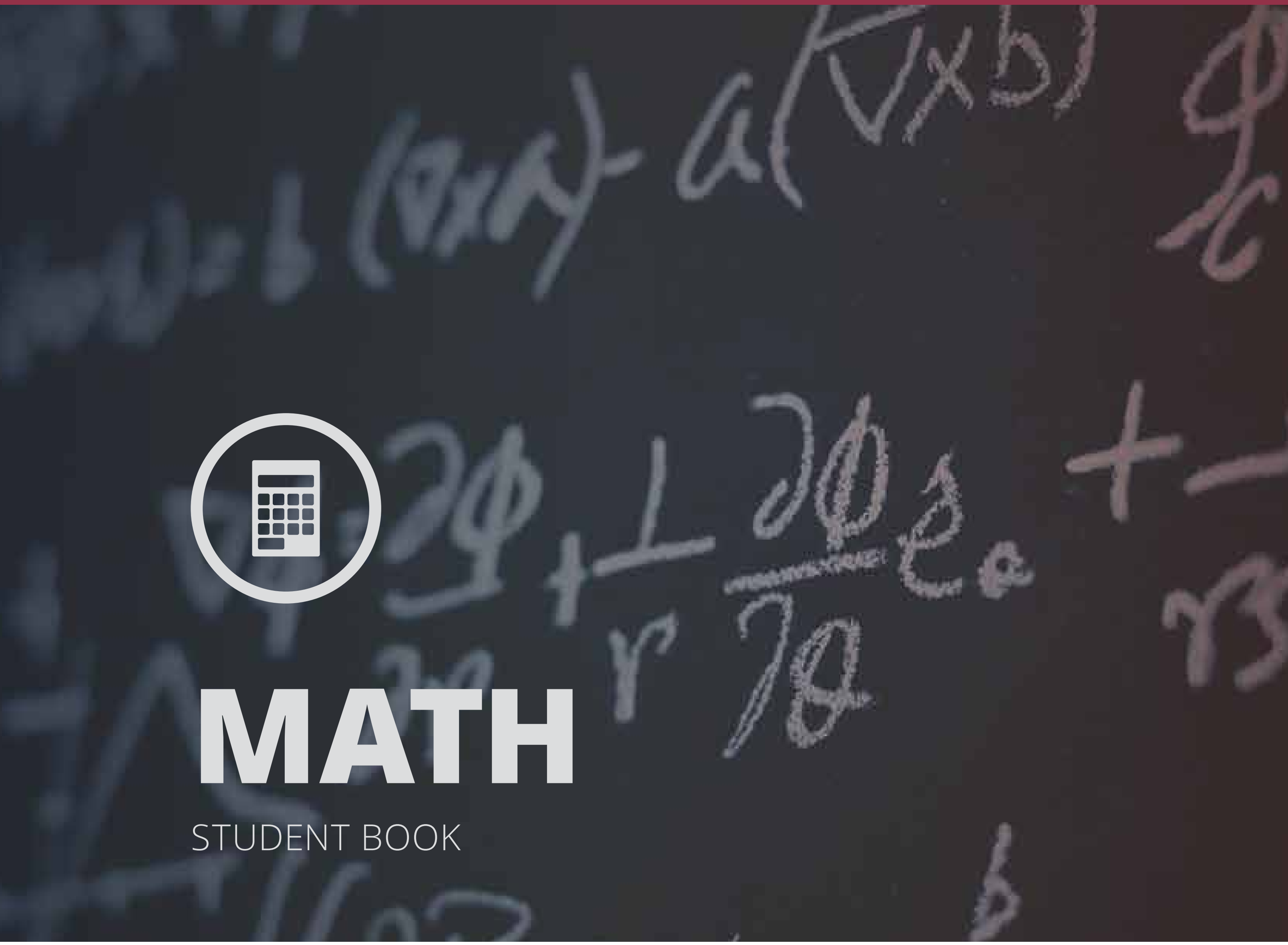
LIFEPAC®



MATH

STUDENT BOOK

10th Grade | Unit 5

Alpha Omega
PUBLICATIONS

MATH 1005
Similar Polygons

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Author:
Milton R. Christen, M.A.

Editor-in-Chief:
Richard W. Wheeler, M.A.Ed.

Editor:
Robin Hintze Kreutzberg, M.B.A.

Consulting Editor:
Robert L. Zenor, M.A., M.S.

Revision Editor:
Alan Christopherson, M.S.

Alpha Omega
PUBLICATIONS

804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

Similar Polygons

Introduction

In our study of congruent triangles, we learned that congruent triangles have the same size and the same shape. We are now going to study objects that have the same shape, but not necessarily the same size.

We have all looked at photographs. The photograph shows a smaller or larger version of the object that was photographed. The picture is the same shape but a different size.

We have all taken trips and used a road map to help get us from here to there. A map is a smaller version of the real thing.

Some of you may have built model airplanes or may have a model train. These models are scaled-down versions of a real item. The model is the same shape as the real thing but the size is different.

These examples are all practical examples of similarly shaped objects. In mathematics we also study similar shapes. In this LIFEPAC® we shall learn and use some principles of algebra in our study of similarity and its application to the right triangle.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Write ratios in simplest form.
2. Name the properties of proportions.
3. Solve proportion problems.
4. Identify similar polygons by using definitions, postulates, and theorems.
5. Use theorems about special segments in similar triangles.
6. Solve problems about similar right triangles.
7. Use the Pythagorean Theorem.
8. Solve triangle problems using trigonometry.
9. Find measurements indirectly.

1. PRINCIPLES OF ALGEBRA

Before we learn and use some of the properties of similar polygons, we should review some basic principles from algebra.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Express ratios in simplest form.
2. Name the properties of proportion.
3. Solve proportion problems.

RATIOS AND PROPORTIONS

Two ideas from algebra that we need to review at this time are ratios and proportions.

> **DEFINITION**
>
> **Ratio:** the comparison of two numbers by division. The quotient is the ratio of the two numbers.

The ratio of 3 to 15 is $\frac{1}{5}$. The ratio of 8 to 2 is $\frac{4}{1}$. The ratio of a to b is $\frac{a}{b}$. Notice the quotients, $\frac{1}{5}, \frac{4}{1}, \frac{a}{b}$, are written as fractions. We can arrive at the ratio by dividing the "to" number into the "of" number. Keep in mind that you are not finding the ratio of one object to another, but rather the ratio of two numbers that are measures of the object in the same unit.

Rather than go through a division process to find a ratio, we can set up the two numbers as a fraction and reduce the fraction.

The "of" number will be the numerator (top) and the "to" number will be the denominator (bottom).

Ratio is simply a number. No units are connected to a ratio.

Model 1: Find the ratio of 6 to 8.

$\frac{6}{8} = \frac{3}{4}$

Model 2: If AB = 6 inches and CD = 18 inches, find the ratio of AB to CD.

$\frac{AB}{CD} = \frac{6}{18} = \frac{1}{3}$

Model 3: If $\angle A = 35°$, $\angle B = 50°$, find the ratio of $\angle A$ to $\angle B$.

$\frac{A}{B} = \frac{35}{50} = \frac{7}{10}$

Model 4: If the side of one triangle is 2 feet and the side of another triangle is 18 inches, find the ratio of the small side to the large side.

$\frac{18}{24} = \frac{3}{4}$ (2 feet was changed to 24 inches)

$\frac{1\frac{1}{2}}{2} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$ (18 inches was changed to $1\frac{1}{2}$ feet)

The ratio of two numbers can also be written in the form a:b. This form is useful when we are comparing three or more numbers.

Model 5: The ratio of 3 to 4 to 5 can be written as 3:4:5.

This ratio means that the ratio of the first to the second is $\frac{3}{4}$, the ratio of the second to the third is $\frac{4}{5}$, and the ratio of the first to the third is $\frac{3}{5}$.

The following sets of numbers all have the ratio 3:4:5.

{12, 16, 20}	$\frac{12}{16} = \frac{3}{4}, \frac{16}{20} = \frac{4}{5}, \frac{12}{20} = \frac{3}{5}$
$\{3x, 4x, 5x\}$	$\frac{3x}{4x} = \frac{3}{4}, \frac{4x}{5x} = \frac{4}{5}, \frac{3x}{5x} = \frac{3}{5}$
$\{15x^2, 20x^2, 25x^2\}$	$\frac{15x^2}{20x^2} = \frac{3}{4}, \frac{20x^2}{25x^2} = \frac{4}{5}, \frac{15x^2}{25x^2} = \frac{3}{5}$

Express each ratio in simplest form.

1.1 $\frac{6}{12}$ = ________________

1.2 $\frac{6}{9}$ = ________________

1.3 $\frac{18}{29}$ = ________________

1.4 $\frac{36}{72}$ = ________________

1.5 21:28 = ________________

1.6 25:45 = ________________

1.7 10:20 = ________________

1.8 15:25:35 = ________________

If a = 3 and b = 5, find each ratio.

1.9 $\frac{a}{b}$ = ________________

1.10 $\frac{b}{a}$ = ________________

1.11 $\frac{a}{(a + b)}$ = ________________

1.12 $\frac{b}{(a + b)}$ = ________________

If c = 4 and d = 5, find each ratio.

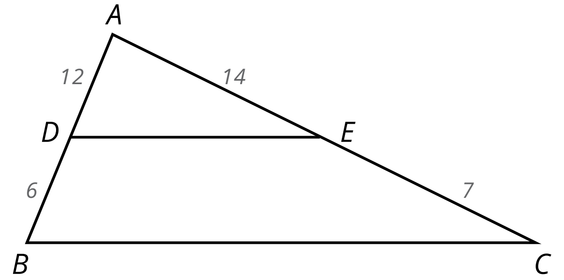
1.13 c to d = ________________

1.14 d to c = ________________

1.15 $\frac{d}{(c + d)}$ = ________________

1.16 $\frac{(d - c)}{(d + c)}$ = ________________

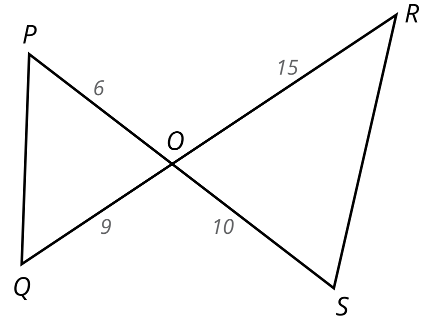
Use the following figures to find each ratio in simplest form.



1.17 $\frac{AD}{DB}$ = __________ **1.18** $\frac{AD}{AB}$ = __________

1.19 $\frac{DB}{AB}$ = __________ **1.20** $\frac{AE}{AC}$ = __________

1.21 $\frac{AC}{AE}$ = __________ **1.22** $\frac{EC}{AC}$ = __________



1.23 $\frac{PO}{OS}$ = __________ **1.24** $\frac{PO}{PS}$ = __________

1.25 $\frac{OS}{PS}$ = __________ **1.26** $\frac{OR}{QO}$ = __________

1.27 $\frac{OR}{QR}$ = __________ **1.28** $\frac{QR}{PS}$ = __________

Work the following problems.

1.29 The measures of the angles of a triangle are in the ratio of 1:2:3. Find the measure of each angle. (Hint: The sum of the angles of a triangle = 180°.)

1.30 The distance from *A* to *B* is 60 feet. The distance from *B* to *C* is 10 yards. The distance from *C* to *D* is 20 inches. Find the ratio of *AB*:*BC*:*CD*.

> **DEFINITION**
>
> **Proportion:** an equation that states that two ratios are equal.

The proportion $\frac{a}{b} = \frac{c}{d}$ tells us that the ratio a to b and the ratio c to d are equal ratios. The proportion can be read, a is to b as c is to d; or, the quotient of a and b equals the quotient of c and d. The proportion can also be written $a{:}b = c{:}d$.

Each of the four numbers a, b, c, and d is called a **term** of the proportion: a is the first term, b is the second term, c is the third term, and d is the fourth term.

The first and the fourth terms are known as the **extremes** of the proportion. The second and third terms are called the **means**.

The fact that more than two ratios are equal is often expressed in the form of an **extended proportion**.

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$ is an extended proportion stating that all four ratios are the same number. Another extended proportion is $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$.

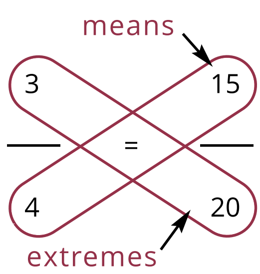
We can pick any two of the ratios and form a regular proportion: $\frac{3}{6} = \frac{6}{12}$, $\frac{7}{14} = \frac{2}{4}$.

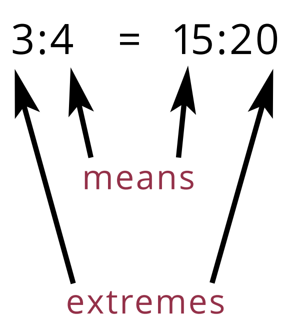
Since a proportion is an equation, we can use the properties of equality to transform a proportion to another form. For example, $\frac{a}{b} = \frac{c}{d}$ can be written as $ad = bc$ by using the multiplication property of equality (multiply each side of the equation by bd).

$$\frac{a}{b} = \frac{c}{d}$$

$$(bd)\frac{a}{b} = (bd)\frac{c}{d}$$

$$ad = bc$$





Name the means and the extremes in these proportions.

1.31 $\frac{3}{4} = \frac{15}{20}$ means: ________________ extremes: ________________

1.32 $\frac{5}{7} = \frac{20}{28}$ means: ________________ extremes: ________________

1.33 $\frac{6}{11} = \frac{x}{y}$ means: ________________ extremes: ________________

1.34 3:9 = 2:6 means: ________________ extremes: ________________

1.35 1:2 = 4:8 means: ________________ extremes: ________________

1.36 $x{:}y = 3{:}7$ means: ________________ extremes: ________________

Find the value of *x* in each of these proportions.

1.37 $\frac{x}{25} = \frac{2}{5}$ $x =$ ____________

1.38 $\frac{x}{6} = \frac{3}{2}$ $x =$ ____________

1.39 $\frac{9}{x} = \frac{3}{12}$ $x =$ ____________

1.40 $\frac{10}{7} = \frac{x}{5}$ $x =$ ____________

1.41 $9{:}x = x{:}4$ $x =$ ____________

1.42 $\frac{1}{2}{:}x = \frac{2}{3} : \frac{3}{4}$ $x =$ ____________

1.43 $\frac{(x+3)}{6} = \frac{5}{4}$ $x =$ ____________

1.44 $\frac{(x+1)}{(x+2)} = \frac{2}{3}$ $x =$ ____________

1.45 $\frac{3}{2} = \frac{x}{4}$ $x =$ ____________

Find the ratio of *x* to *y*.

1.46 $2x = 3y$ ______________

1.47 $5x = 7y$ ______________

1.48 $\frac{x}{3} = \frac{y}{2}$ ______________

1.49 $2x - 3y = 0$ ______________

1.50 $x - 5y = 0$ ______________

Along with our knowledge of complementary and supplementary angles, ratios can help us solve geometry problems.

Model: Two complementary angles have measures in the ratio of 4 to 5. Find the measure of each angle.

Solution: Let $4x$ = measure of the smaller angle.
$5x$ = measure of the larger angle.

Then $4x + 5x = 90°$ (angles are complementary)

$9x = 90°$

$x = 10°$

$4x = 40°$

$5x = 50°$

Solve the following problems. Show your work and circle your answer.

1.51 Two complementary angles have measures in the ratio of 1 to 5. Find the measure of each angle.

1.52 The ratio of the measures of two supplementary angles is 3:7. Find the measure of each angle.

1.53 A 30-inch segment is cut into two parts whose lengths have the ratio 3 to 5. Find the length of each part.

1.54 The perimeter of a triangle is 48 inches and the sides are in the ratio of 3:4:5. Find the length of each side.

1.55 A triangle has a perimeter of 18 inches. If one side has a length of 8 inches, find the other two sides if their lengths are in the ratio of $\frac{2}{3}$.

PROPERTIES OF PROPORTIONS

You will often wish to change a proportion into some equivalent equation. Although we can make this change by using basic properties of algebra, we will save time and steps by using some special properties of proportions. When we use these properties for a reason on a proof, we shall simply write POP (Property of Proportion).

In these statements the variables used represent nonzero numbers.

CROSS PRODUCT PROPERTY

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Model 1:

$$\frac{3}{5} = \frac{15}{25}$$
$$3 \cdot 25 = 5 \cdot 15$$
$$75 = 75$$

Model 2:

$$\frac{6}{11} = \frac{12}{22}$$
$$6 \cdot 22 = 11 \cdot 12$$
$$132 = 132$$

Model 3:

$$\frac{x}{10} = \frac{3}{5}$$
$$5x = 30$$
$$x = 6$$

EQUIVALENT FORMS PROPERTY

$\frac{a}{b} = \frac{c}{d}$, $\frac{a}{c} = \frac{b}{d}$, $\frac{d}{b} = \frac{c}{a}$, $\frac{b}{a} = \frac{d}{c}$ are equivalent proportions. The cross product of each proportion gives the same equation, $ad = bc$.

Model 1:

$$\frac{2}{4} = \frac{5}{10}$$
$$20 = 20$$

Model 2:

$$\frac{10}{4} = \frac{5}{2}$$
$$20 = 20$$

Model 3:

$$\frac{2}{5} = \frac{4}{10}$$
$$20 = 20$$

Model 4:

$$\frac{4}{2} = \frac{10}{5}$$
$$20 = 20$$

DENOMINATOR SUM PROPERTY

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{(a+b)}{b} = \frac{(c+d)}{d}$.

We can add 1 to both sides of the equation:

$$\frac{a}{b} = \frac{c}{d}$$
$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$
$$\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$$
$$\frac{(a+b)}{b} = \frac{(c+d)}{d}$$

Model 1:

$$\frac{2}{4} = \frac{5}{10}$$
$$\frac{(2+4)}{4} = \frac{(5+10)}{10}$$
$$\frac{6}{4} = \frac{15}{10}$$

Model 2:

$$\frac{3}{9} = \frac{1}{3}$$
$$\frac{(3+9)}{9} = \frac{(1+3)}{3}$$
$$\frac{12}{9} = \frac{4}{3}$$

Model 3:

$$\frac{2}{8} = \frac{3}{12}$$
$$\frac{(2+8)}{8} = \frac{(3+12)}{12}$$
$$\frac{10}{8} = \frac{15}{12}$$

DENOMINATOR DIFFERENCE PROPERTY

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{(a-b)}{b} = \frac{(c-d)}{d}$.

We can subtract 1 from both sides of the equation:

$$\frac{a}{b} = \frac{c}{d}$$
$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$
$$\frac{a}{b} - \frac{b}{b} = \frac{c}{d} - \frac{d}{d}$$
$$\frac{(a-b)}{b} = \frac{(c-d)}{d}$$

Model 1:

$$\frac{8}{3} = \frac{16}{6}$$
$$\frac{(8-3)}{3} = \frac{(16-6)}{6}$$
$$\frac{5}{3} = \frac{10}{6}$$

Model 2:

$$\frac{12}{3} = \frac{4}{1}$$
$$\frac{(12-3)}{3} = \frac{(4-1)}{1}$$
$$\frac{9}{3} = \frac{3}{1}$$

Model 3:

$$\frac{6}{8} = \frac{3}{4}$$
$$\frac{(6-8)}{8} = \frac{(3-4)}{4}$$
$$-\frac{2}{8} = -\frac{1}{4}$$

NUMERATOR-DENOMINATOR SUM PROPERTY

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \ldots$, then $\frac{(a + c + e + g + \ldots)}{(b + d + f + h + \ldots)} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ldots$

Let $\frac{a}{b} = x, \frac{c}{d} = x, \frac{e}{f} = x, \ldots$

then $a = bx, c = dx, e = fx, \ldots$

so
$$\frac{(a + c + e + \ldots)}{(b + d + f + \ldots)} = \frac{(bx + dx + fx + \ldots)}{(b + d + f + \ldots)}$$
$$= x\frac{(b + d + f + \ldots)}{(b + d + f + \ldots)}$$
$$= x$$
$$= \frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ldots$$

Model 1: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

$\frac{(1 + 2 + 3 + 4)}{(2 + 4 + 6 + 8)} = \frac{1}{2}$

$\frac{10}{20} = \frac{1}{2}$

Model 2: $\frac{3}{5} = \frac{6}{10} = \frac{9}{15}$

$\frac{(3 + 6 + 9)}{(5 + 10 + 15)} = \frac{6}{10}$

$\frac{18}{30} = \frac{6}{10}$

Name the POP illustrated.

1.56 If $\frac{a}{2} = \frac{b}{5}$, then $\frac{(a + 2)}{2} = \frac{(b + 5)}{5}$. ______________________

1.57 If $\frac{(a + 1)}{3} = \frac{b}{5}$, then $5(a + 1) = 3b$. ______________________

1.58 If $\frac{x}{y} = \frac{r}{s} = \frac{t}{u}$, then $\frac{(x + r + t)}{(y + s + u)} = \frac{x}{y}$. ______________________

1.59 If $\frac{x}{4} = \frac{y}{3}$, then $\frac{x}{y} = \frac{4}{3}$. ______________________

1.60 If $\frac{(x + 3)}{2} = \frac{(y + 6)}{3}$, then $\frac{(x + 1)}{2} = \frac{(y + 3)}{3}$. ______________________

1.61 If $\frac{r}{x} = \frac{s}{t}$, then $sx = rt$. ______________________

SELF TEST 1

Complete the following statements (each answer, 3 points)**.**

1.01 A ratio compares two numbers by ______________________ .

1.02 The ratio of 3 to 4 is written as ______________________ .

1.03 Another way of writing the ratio of 3 to 4 is ______________________ .

1.04 In writing the ratio of measurement numbers, the units must be the __________________ .

1.05 The ratio of 3 to 4 and the ratio of 4 to 3 (are/are not) ______________ the same number.

1.06 A proportion is an equation that states two ______________________ are equal.

1.07 "If $\frac{2}{3} = \frac{x}{y}$, then $2y = 3x$" is an example of the ______________________ POP.

1.08 "If $\frac{x}{y} = \frac{2}{3} = \frac{a}{b}$, then $\frac{(x + 2 + a)}{(y + 3 + b)} = \frac{2}{3}$" is an example of the ______________________ POP.

1.09 In the proportion $\frac{a}{b} = \frac{p}{q}$, the third term is ______________________ .

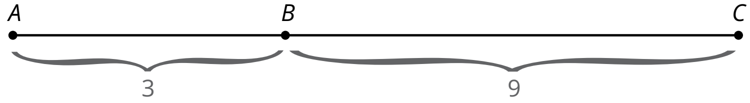
1.010 In the proportion $\frac{5}{6} = \frac{10}{12}$, the means are a. ______________ and b. ______________ .

Express each ratio in simplest form (each answer, 2 points)**.**

1.011 Ratio of 6 feet to 3 feet. ____________________________

1.012 Ratio of 7 yards to 6 feet. ____________________________

1.013 Ratio of 12 to 100. ____________________________



1.014 Ratio of *AB* to *BC*. ____________________________

1.015 Ratio of *BC* to *AC*. ____________________________

Find *x* in the following proportions (each answer, 2 points)**.**

1.016 $\frac{x}{7} = \frac{3}{5}$ $x =$ ____________

1.017 $3:8 = x:32$ $x =$ ____________

1.018 $\frac{5}{2x} = \frac{25}{4}$ $x =$ ____________

1.019 $\frac{x}{3} = \frac{x+2}{5}$ $x =$ ____________

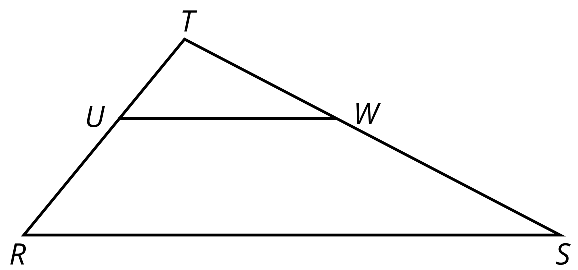
1.020 $\frac{16}{x} = \frac{x}{4}$ $x =$ ____________

1.021 $\frac{x+2}{2} = \frac{6+2}{2}$ $x =$ ____________

Find the required numbers (each answer, 2 points)**.**

Given: $\frac{TU}{TR} = \frac{UW}{RS} = \frac{WT}{ST}$

$TU = 3$
$RS = 15$
$TW = 4$
$TR = 9$



1.022 $UW =$ ____________

1.023 $ST =$ ____________

1.024 $WS =$ ____________

1.025 $UR =$ ____________

54/67 **SCORE** ____________ **TEACHER** ____________ ____________
initials date

Alpha Omega
PUBLICATIONS

804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

800-622-3070
www.aop.com

MAT1005 – Apr '15 Printing

ISBN 978-0-86717-635-3

9 780867 176353