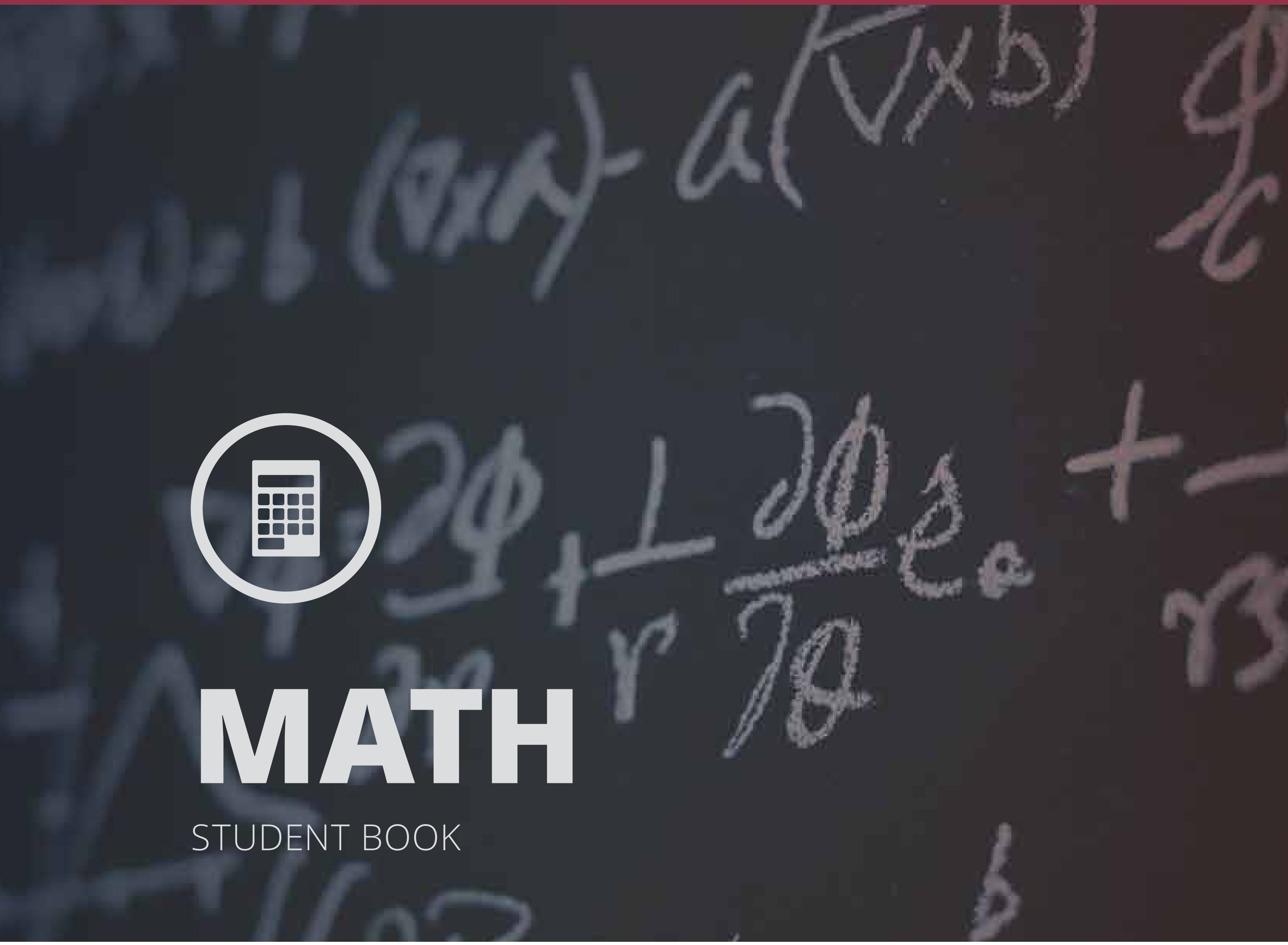
LIFEPAC®



MATH

STUDENT BOOK

10th Grade | Unit 8

Alpha Omega
PUBLICATIONS

MATH 1008
Area and Volume

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Area and Volume

Introduction

In other mathematics LIFEPACs, you have learned how to find the area and volume of many geometric shapes. You were given a formula or a rule and some numbers to substitute into it. You then did the computations and came out with an answer.

In this LIFEPAC®, you will study area and volume as a part of our deductive system. You will learn why the formulas for area and volume are written as they are rather than simply using them. We shall define some terms, state some postulates, and prove theorems about area and volume. Some of the theorems will be the familiar formulas you have used in the past.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Define area.
2. Calculate the area of certain polygons.
3. Compare areas of similar polygons.
4. Calculate the circumference of a circle.
5. Calculate the area of a circle.
6. Calculate the area of a sector of a circle.
7. Calculate the area of a segment of a circle.
8. Calculate the surface areas of certain solid figures.
9. Calculate the volumes of certain solid figures.

1. POLYGONS

In this section we shall derive the formulas for the areas of some quadrilaterals, triangles, and regular polygons. We shall also compare the areas of similar polygons using proportions.

Section Objectives

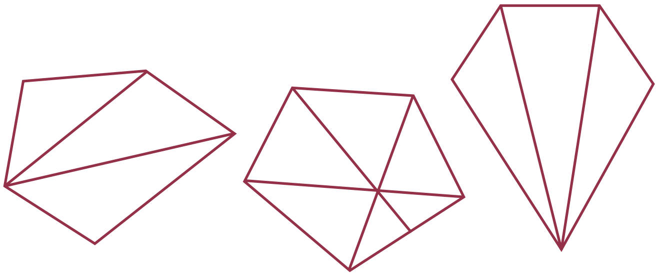
Review these objectives. When you have completed this section, you should be able to:

1. Define area.
2. Calculate the area of certain polygons.
3. Compare areas of similar polygons.

AREA CONCEPTS

When we speak of the area of figures, we are referring to the measure of what can go inside the figure. This "inside" of a figure is called a *region*. A *triangular region* is the union of a triangle and its interior. A *polygonal region* is the union of the nonoverlapping triangular regions. Every polygon can be broken into triangular regions in different ways.

Region



DEFINITIONS

Area the measure of the space inside a geometric figure.

Region the space inside and including the sides of a figure.

You will now learn three postulates that have to do with area and regions.

POSTULATE 17

P17: For every polygonal region there exists one unique positive number that is called the area of that region.

Postulate 17 gives us a number that can be used to represent area.

POSTULATE 18

P18: If two triangles are congruent, then their areas are the same number.

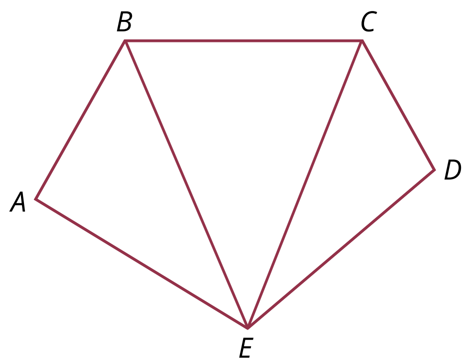
Postulate 18 seems reasonable, since congruent triangles have the same shape and the same size and since area is a number that measures the size of the triangle.

POSTULATE 19:

P19: The area of a polygonal region is the sum of the areas of the triangular regions that form the polygonal region.

(Area Addition Postulate)

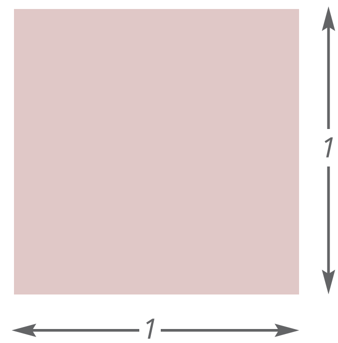
Postulate 19 gives us a way of adding areas of triangles to find the area of any polygon. This postulate is similar both to the angle addition theorem that allows us to add angle measures and to the definition of betweenness in which we add lengths of segments.



Area *ABCDE* =
Area *AEB* + Area *BEC* + Area *CED*.

To make use of the idea of area in the physical world, we need to agree on a unit for measuring area. The *square unit* has been developed in the following manner.

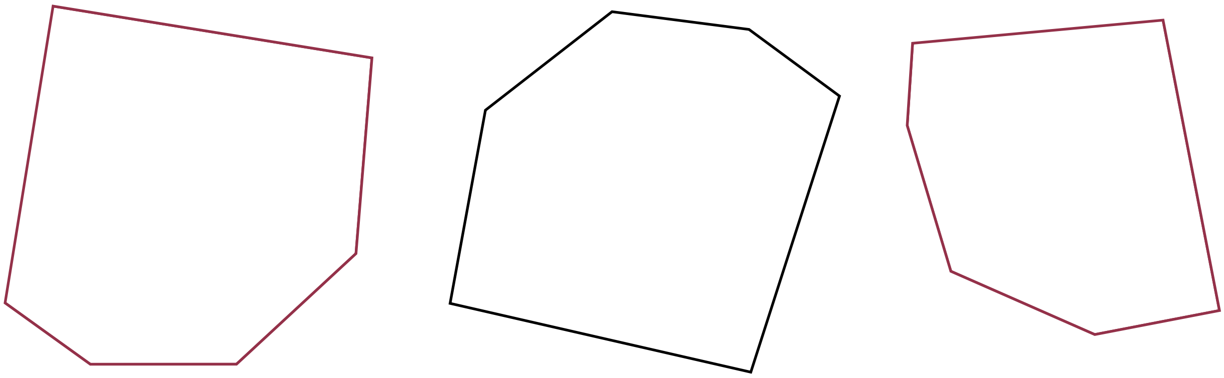
Consider a square one unit on each side. We shall agree that the area of this square is one square unit. When we find the area of a region, we are looking for the number of these square units that will fit onto the surface of that region. The unit of measurement used could be inches, feet, yards, or any other unit of length. The corresponding unit for area would be square inch, square foot, and square yard.



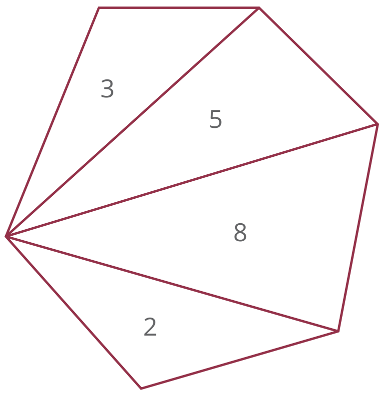
1 square unit

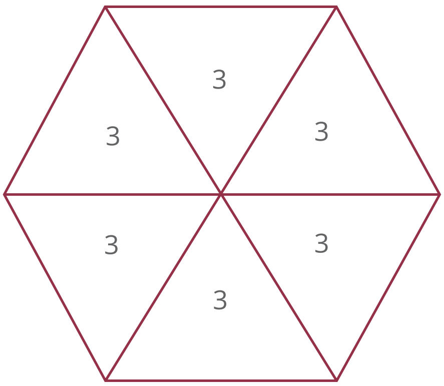
Complete the following activities.

1.1 Divide the three similar polygons into triangular regions in three different ways.



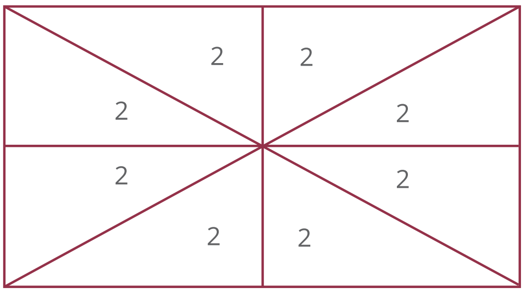
1.2 If the area of each of the triangular regions is as shown, what is the area (A) of each polygon?

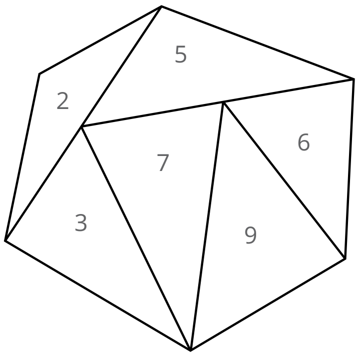




a. A = ____________________

b. A = ____________________





c. A = ____________________

d. A = ____________________

1.3 If triangle *RST* is congruent to triangle *WXY* and the area of triangle *WXY* is 20 square inches, what is the area of triangle *RST*? ______________________

1.4 Which three of the following numbers cannot represent the area of a polygon? __________

a. 68　　b. 40　　c. -5　　d. $\sqrt{30}$

e. 0　　f. -200

1.5 If two triangles have the same area, must they be congruent? ______________________

1.6 Can a rectangle and a triangle have the same area? ______________________

1.7 When a diagonal is drawn in a rectangle, what is true of the areas of the two triangles into which it divides the rectangle? Why? ______________________

____

1.8 Do the diagonals of a rhombus divide it into four triangles of equal area?______________

1.9 If two equilateral triangles have equal perimeters, must they also have equal areas?

____

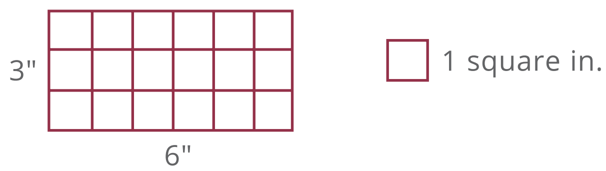
1.10 If two polygons have the same area, must they have the same number of sides? __________

RECTANGLE

Suppose we take a rectangle with length of six inches and width of three inches.

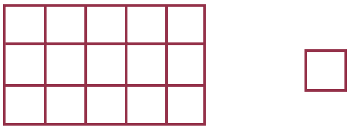
We can place 18 square inches inside this region, as the figure shows. Therefore, the area of the rectangle is 18 sq. in.

Model 1:



Suppose we take a rectangle that measures 5 ft. by 3 ft.

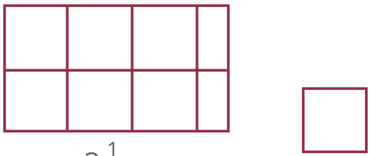
Model 2:



We can put 15 sq. ft. in the rectangle. The area of the rectangle would be 15 sq. ft.

If a rectangle were $3\frac{1}{2}$ cm long and 2 cm wide, its area would be 7 sq. cm.

Model 3:



We can calculate the area number in each case by multiplying the length of the base of the rectangle by the length of the altitude to that base.

Model 1: $6 \times 3 = 18$

Model 2: $3 \times 5 = 15$

Model 3: $3\frac{1}{2} \times 2 = 7$

This fact suggests the next postulate.

POSTULATE 20:

P20: The area of a rectangle is the product of the length of a base and the length of the altitude to that base:

$A = bh$

Any side of a rectangle can be considered to be the base. Then each side adjacent to that base is an altitude to that base. Another way of saying Postulate 20 is that the area is the product of the length and the width.

A useful corollary to Postulate 20 tells how to find the area of a square.

Corollary: The area of a square is the square of the length of its side: $A = s^2$.

Since a square is a rectangle with base and altitude equal, we can replace s for b and h in the formula for area of a rectangle.

$A = s \cdot s$
$A = s^2$

When using an area formula in an application problem, you may use any practical unit of length. However, within any particular problem all dimensions must be in the same unit of length.

Model 1: Find the area of a rectangle with length of 2 yds. and width of 3 feet.

Solution: Let the linear unit be feet. Therefore, the length is 2 yds. × 3 = 6 ft.

$A = bh$

$A = 6(3)$

$A = 18$ sq. ft.

Model 2: Find the area of a rectangle if its base is 2 ft. and its height is 18 inches.

Solution: Let the linear unit be inches. Therefore, the base is 2 ft. × 12 = 24 inches.

$A = bh$

$A = 24(18)$

$A = 432$ sq. in.

Alternatively, let the linear unit be feet. Then the height is 18 inches divided by 12 = $1\frac{1}{2}$ ft.

$A = bh$

$A = 2(1\frac{1}{2})$

$A = 3$ sq. ft.

Complete the following activities.

1.11 Find the area of a rectangle with base of 12 inches and height of 8 inches.

____

1.12 Find the area of a square with side of 7 feet. ______________________________

1.13 Find the area of a rectangle with length of 6 inches and width of 2 feet.

a. In sq. ft.: ________________________ b. In sq. in.: ________________________

1.14 Find the length of a rectangle with area of 16 sq. cm and width of 8 cm.

____

1.15 Find the side of a square with area of 25 sq. ft. ______________________________

1.16 Find the number of square inches in one square foot. __________________________

1.17 The area of a desk top is $8\frac{3}{4}$ square feet. If the length is $3\frac{1}{2}$ ft., find the width.

____

1.18 The Smiths plan to recarpet their family room, which measures 15 ft. by 20 ft. How many square yards of carpet are needed?

____

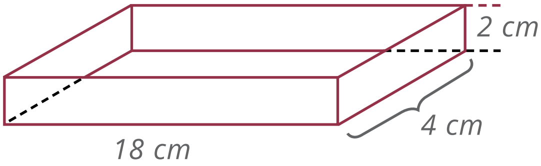
1.19 The roof of a cabin is to be shingled at a cost of $70 a square. (A square, in shingling, is a region with an area of 100 sq. ft.) Find the cost of shingling the roof shown.

____

1.20 A wooden fence 6 ft. high and 300 ft. long is to be painted. How many gallons of paint are needed if one gallon covers 400 sq. ft.?

____

1.21 A piece of sheet metal is cut and bent to form a box, as shown. The box has no top.



Find the area of:

a. the bottom: _______________________________________

b. the two longer sides: _______________________________________

c. the two shorter sides: _______________________________________

1.22 A 3-ft.-wide sidewalk surrounds a rectangular plot that measures 20 ft. by 30 ft. Find the area of the sidewalk. _____________________________________

1.23 A rectangle is twice as long as it is wide. If its area is 50 sq. yds., find the length and the width.

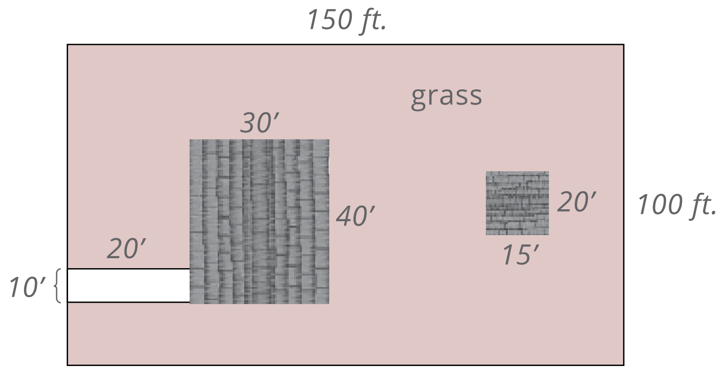
____

1.24 Some pleated draperies must be twice as wide as the window they cover. How many square yards of material are needed to cover a window 2 ft. wide and 3 ft. long?

____

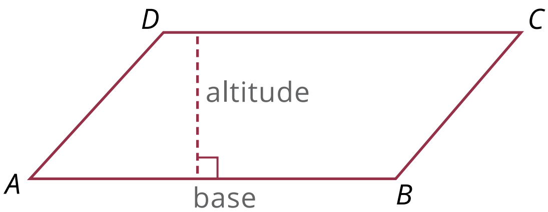
1.25 Find the area of grass in the home landscape diagram below.

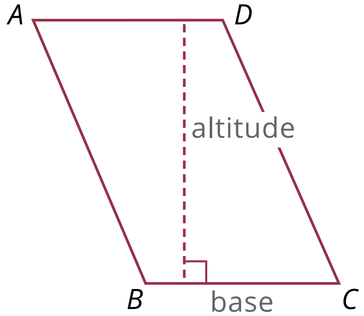
____



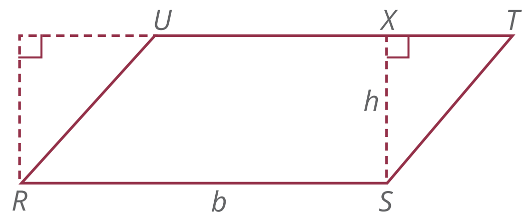
PARALLELOGRAM

Any side of a parallelogram can be called its *base*. The *altitude* to that base is the length of the perpendicular segment between the *D* base and its opposite side.





> **THEOREM 8-1**
>
> The area of a parallelogram is the product of any base and the altitude to that base:
>
> $A = bh$



Given: $\square RSTU$ with $RS = b$ = base
$\overline{SX} \perp \overline{UT}$ and $SX = h$ = altitude

To Prove: Area of $\square RSTU = bh$

Outline of Proof: Move $\triangle STX$ to the other end of *RSTU* forming a rectangle with base = *b* and altitude = *h*. Thus, the area of the rectangle is *bh*. The area of the parallelogram equals the area of the rectangle; the area of $\square RSTU = bh$.

Find the area of a parallelogram if a base and corresponding altitude have the indicated lengths.

1.26 Base $3\frac{1}{2}$ feet, altitude $\frac{3}{4}$ feet Area = ______________________________

1.27 Base 8 inches, altitude 4 inches Area = ______________________________

1.28 Base $1\frac{1}{2}$ feet, altitude 6 inches Area = ______________________________

1.29 Base *x* yards, altitude *y* feet Area = ______________________________

Find the area of $\square ABCD$ given m $\angle A = 30°$ and the following measures.

1.30 $AB = 10$ in. $AD = 6$ in. Area = ______________________________

1.31 $AB = 6$ ft. $AX = 3\sqrt{3}$ ft. Area = ______________________________

1.32 $AD = 4\sqrt{3}$ in. $AB = 8$ in. Area = ______________________________

1.33 $AX = 3$ ft. $AB = 4\sqrt{2}$ ft. Area = ______________________________w

SELF TEST 1

Match the formula with the polygon (each answer, 2 points)**.**

1.01 ________ Area of rectangle
1.02 ________ Area of parallelogram
1.03 ________ Area of trapezoid
1.04 ________ Area of square
1.05 ________ Area of rhombus
1.06 ________ Area of regular polygon
1.07 ________ Area of triangle
1.08 ________ Area of equilateral triangle

a. $A = \frac{1}{2}bh$
b. $A = \frac{1}{2}h(b_1 + b_2)$
c. $A = \frac{1}{2}ap$
d. $A = \frac{1}{4}s^2\sqrt{3}$
e. $A = bh$
f. $A = \frac{1}{2}d_1d_2$
g. $A = s^2$

Complete the following items (each answer, 4 points)**.**

1.09 Find the area of a regular pentagon with side equal to 3 and apothem equal to K.

____

1.010 Find the area of a regular hexagon with a 48-inch perimeter.

____

1.011 Find the area of a triangle with base of 10 inches and altitude to the base of 16 inches.

____

1.012 Find the area of a parallelogram with sides of 6 and 12 and an angle of 60°.

____

1.013 Find the area of a trapezoid with bases of 8 and 16 and a height of 10.

____

1.014 Find the area of an equilateral triangle with a side of 6 inches.

____

1.015 Two polygons are similar with the longest side of one 8 and the longest side of the other 10. Find the ratio of the areas.

____

1.016 The sides of a rhombus with angle of 60° are 6 inches. Find the area of the rhombus.

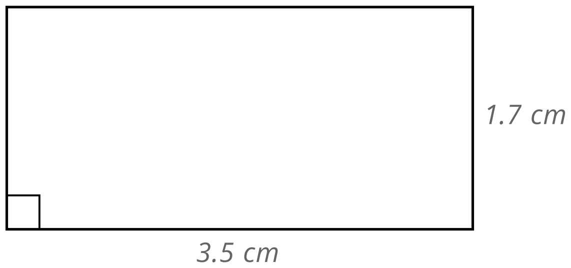
____

1.017 An isosceles trapezoid has bases of 4 and 10. If the base angle is 45°, find the area.

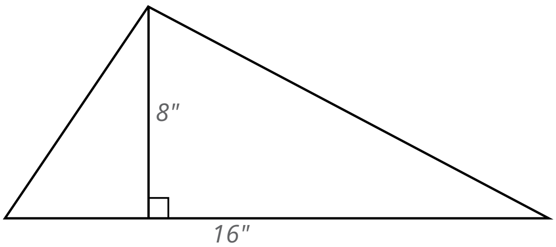
____

Find the area of the following figures (each answer, 4 points)**.**

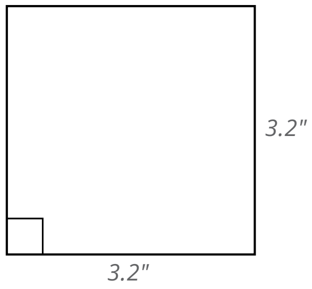
1.018 *A* = ________________________



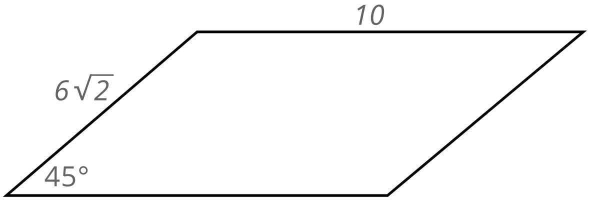
1.019 *A* = ________________________



1.020 *A* = ________________________



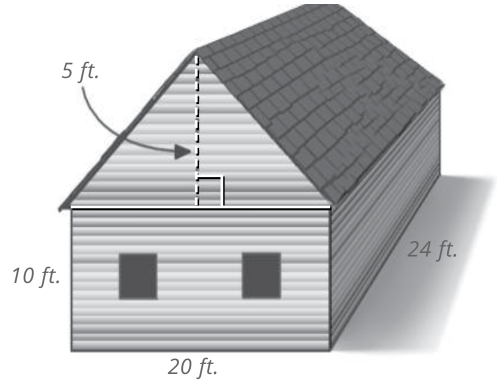
1.021 *A* = ________________________



Solve the following problem (5 points)**.**

1.022 Find the number of gallons of paint needed to cover the sides of the building shown with two coats of paint if one gallon covers 350 square feet. Disregard windows and doors. Round to the nearest gallon.

____



59/73 **SCORE** __________ **TEACHER** __________ __________
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