



MATH

STUDENT BOOK

▶ **10th Grade | Unit 9**

MATH 1009

Coordinate Geometry

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LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Coordinate Geometry

Introduction

Studying geometry with the use of coordinates is a relatively new approach. Geometry has been known and used even before the time of Christ. However, only in recent times (the 17th century) did Rene Descartes (1596-1650) publish the first book on analytical geometry. He showed how a systematic use of coordinates helped to attack problems. In coordinate geometry, geometric figures are studied by means of algebraic equations. The equations that are used involve numbers, called coordinates, which serve to locate points.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAAC. When you have finished this LIFEPAAC, you should be able to:

1. Plot points on the coordinate axes.
2. Identify a figure as having point, line, or plane symmetry.
3. Graph equations.
4. Graph inequalities.
5. Solve problems using the distance formula.
6. Find the equation of a circle.
7. Solve problems using the midpoint formula.
8. Find the slope of a line.
9. Identify parallel and perpendicular lines using slope.
10. Write the equation of a line.
11. Write geometric proofs by coordinate methods.

1. ORDERED PAIRS

To begin our study of coordinate geometry, we must set up a method of picturing points, lines, circles, and other figures in a plane.

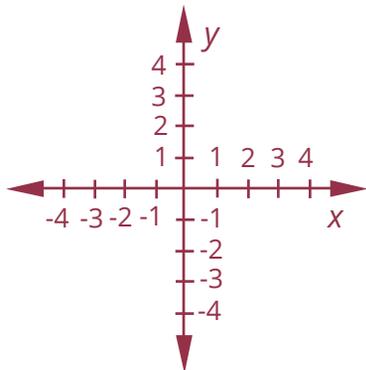
Section Objectives

Review these objectives. When you have finished this section, you should be able to:

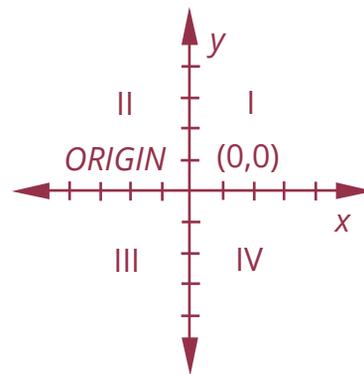
1. Plot points on the coordinate axes.
2. Identify a figure as having point, line, or plane symmetry.
3. Graph equations.
4. Graph inequalities.

POINTS IN A PLANE

To set up a system so that we might picture figures in a plane will require two number lines. We place them so that they are perpendicular to each other and intersect at their zero points. We usually name the horizontal line the x -axis and the vertical line the y -axis.



The two lines, called *coordinate axes*, separate the plane into four regions, called *quadrants*, which are numbered as shown. The point where the two lines intersect is called the *origin*, and its location is given by the pair of numbers $(0, 0)$. The positive numbers are to the right and above the origin. The negative numbers are to the left and below the origin.



DEFINITIONS

x-axis: the horizontal line.

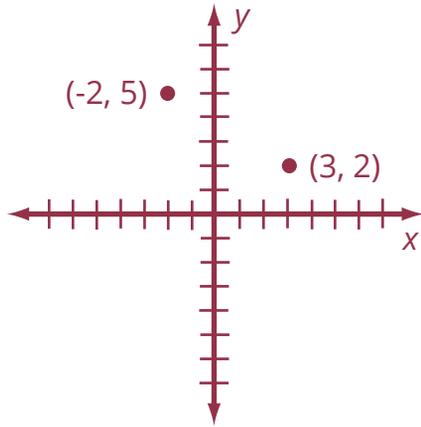
y-axis: the vertical line.

coordinate axes: the x -axis and y -axis together.

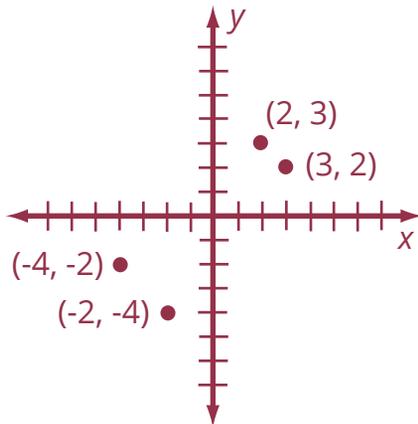
origin: the intersection of the coordinate axes at $(0, 0)$.

To locate a point on the coordinate axes, we need to know its x -coordinate and its y -coordinate: its distance from the origin along each number line. These numbers are given as an ordered pair, (x, y) .

The point $(3, 2)$ is located 3 units to the right of the origin and 2 units above the origin. The point $(-2, 5)$ is located 2 units to the left of the origin and 5 units above the origin.



The order in which the coordinates are given is important: $(3, 2)$ and $(2, 3)$ are different points, as are $(-4, -2)$ and $(-2, -4)$. Remember that the first number given in the pair is the x -coordinate and the second number is the y -coordinate.



Model 1: Locate the following points.

- | | |
|------------|-------------|
| $A(2, 4)$ | $D(-4, -1)$ |
| $B(-3, 7)$ | $E(4, -3)$ |
| $C(0, 4)$ | $F(-3, 0)$ |

Solution: Start at the origin and move 2 units to the right along the x axis,

then 4 units up parallel to the y -axis. This location is point A .

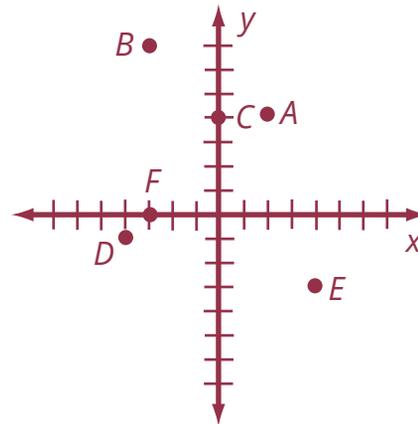
Start at the origin and move 3 units to the left (-3) , then 7 units up to locate point B .

Start at the origin; do not move right or left; then move 4 units up to locate point C .

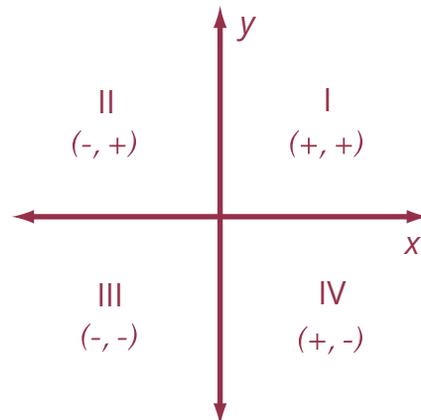
Start at the origin and move 4 units to the left; then move 1 unit down to locate point D .

Start at the origin and move 4 right; then move 3 down to locate point E .

Start at origin and move 3 left; then do not move up or down to locate point F .



Notice that any point that has both coordinates positive would be in Quadrant I. If the first



coordinate is negative and the second is positive, then the point will be in the second quadrant. A point in the third quadrant would have both coordinates negative. A point with the first

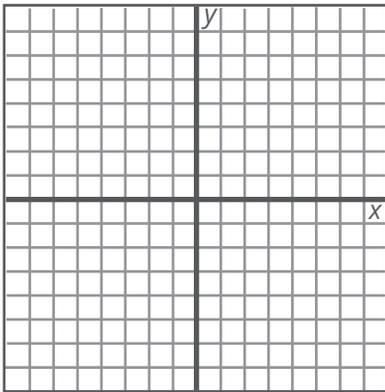
coordinate positive and the second negative will be in Quadrant IV.

If the first coordinate is zero, the point lies on the y -axis. If the second coordinate is zero the point is on the x -axis.

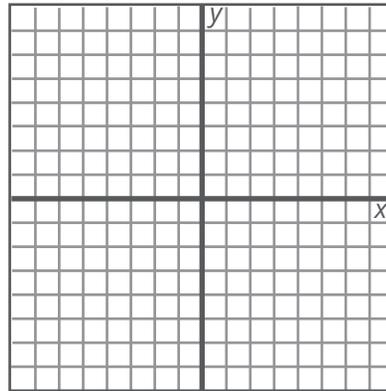


Graph the points using one set of axes. Label the points.

- 1.1** $A(1, 1)$, $B(-3, 0)$,
 $C(-4, -1)$, $D(3, -2)$

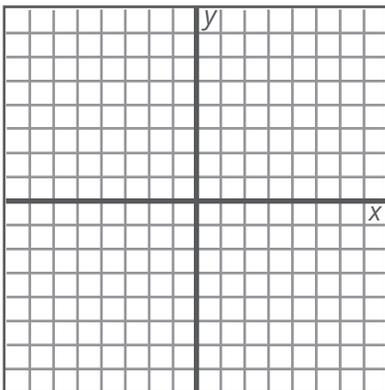


- 1.2** $R(2, 0)$, $S(2, -3)$
 $T(-2, 3)$, $U(0, -4)$

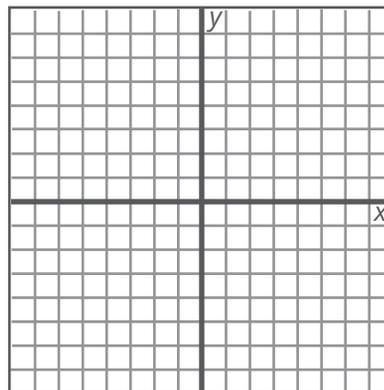


Complete the following activities.

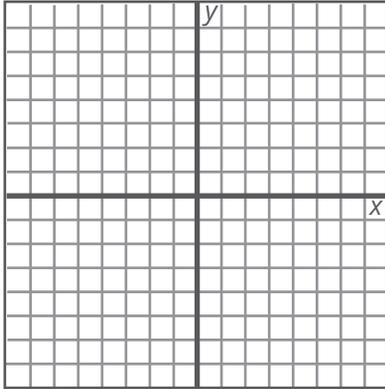
- 1.3** Graph these points on one set of axes: $A(0, 0)$, $B(3, 4)$ and $C(4, 2)$. Connect the points making segments. What geometric figure is formed?



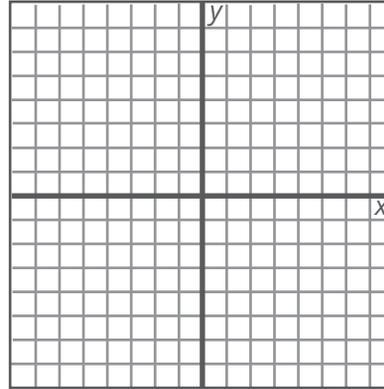
- 1.4** Graph these points on the same axes: $M(-3, 4)$, $N(4, 4)$, $O(4, -2)$, and $P(-3, -2)$. Connect the points to form quadrilateral $MNOP$. What special figure is formed?



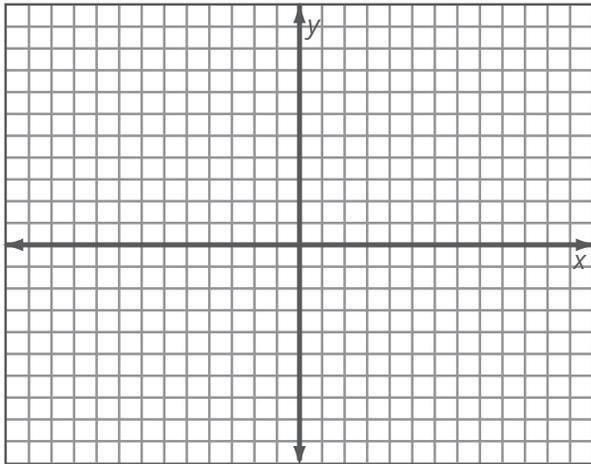
- 1.5** Graph these points on one set of axes: $R(-1, 2)$, $S(3, 2)$, $T(5, -2)$, and $U(-2, -2)$. Connect the points to form quadrilateral $RSTU$. What special quadrilateral is formed?



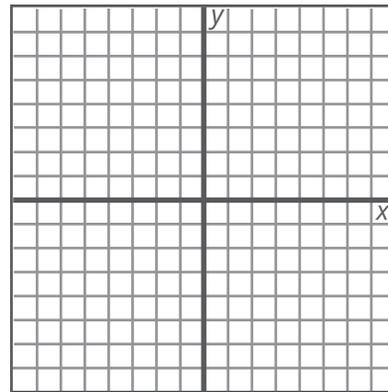
- 1.6** Graph these points on one set of axes: $A(0, 0)$, $B(0, 6)$, $C(-6, 6)$, and $D(-6, 0)$. Connect the points. What special quadrilateral is formed?



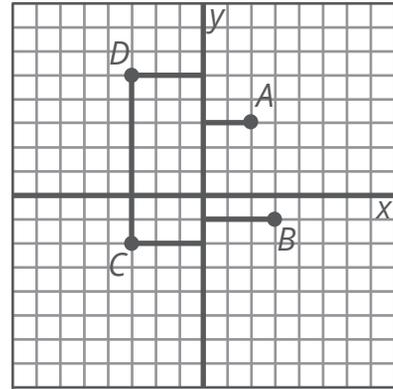
- 1.7** Graph these points on one set of axes: $A(4, -6)$, $B(4, 3)$, $C(8, 3)$, $D(8, 5)$, $E(4, 5)$, $F(4, 10)$, $G(2, 10)$, $H(2, 5)$, $I(-2, 5)$, $J(-2, 3)$, $K(2, 3)$, and $L(2, -6)$. Connect these points in alphabetical order.



- 1.8** Graph these points on one set of axes: and connect them in the order given: $(-2, -1)$, $(-5, -1)$, $(0, 4)$, $(5, -1)$, $(2, -1)$, $(2, 1)$, $(-2, 1)$, $(-2, -1)$, $(0, 5)$, $(2, -1)$, $(0, 1)$, $(-2, -1)$.

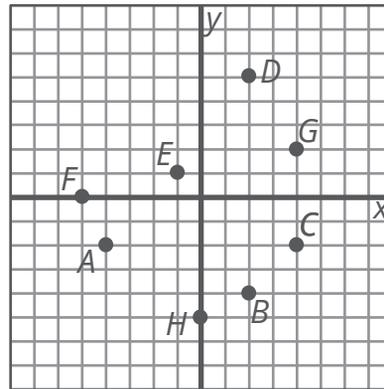


To find the ordered pair when we have the graph, we count to the axes. From the point count units perpendicular to the x -axis to find the x -coordinate. Also from the point count units perpendicular to the y -axis to find the y -coordinate. Point A has x -coordinate of 2 and y -coordinate of 3. Therefore, the ordered pair that is associated with point A is $(2, 3)$. The coordinates of B are $(3, -1)$; of C , $(-3, -2)$; and of D , $(-3, 5)$.



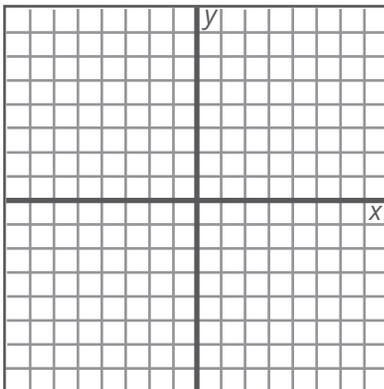
Find the coordinates of the following points.

- 1.9 A (_____)
 B (_____)
 C (_____)
 D (_____)
 E (_____)
 F (_____)
 G (_____)
 H (_____)

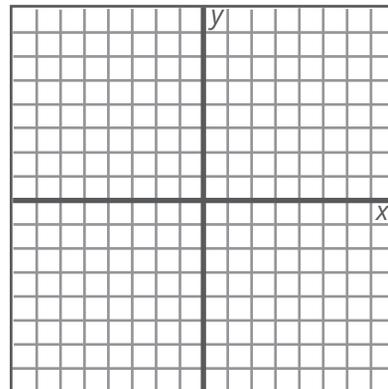


Draw a graph and write the answers.

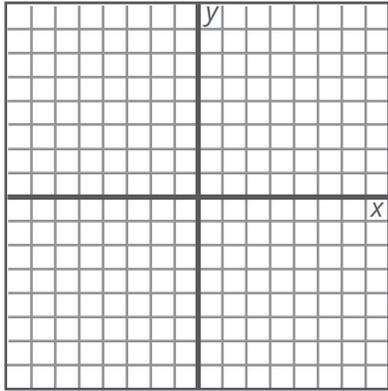
- 1.10 Three of the vertices of a square are the points whose coordinates are $(2, 3)$, $(2, 6)$, and $(5, 3)$. What are the coordinates of the fourth vertex?



- 1.11 Three of the vertices of a rectangle have coordinates $(-1, -1)$, $(6, -1)$, and $(-1, 2)$. What are the coordinates of the fourth vertex?

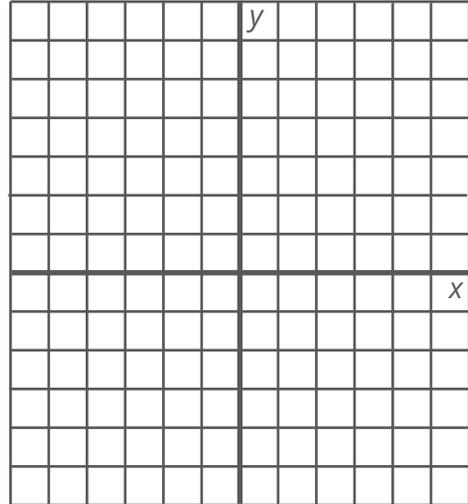


- 1.12** The longer base of an isosceles trapezoid joins points $(-3, -2)$ and $(7, -2)$. One end point of the shorter base is $(-1, 4)$. What are the coordinates of the other end point?

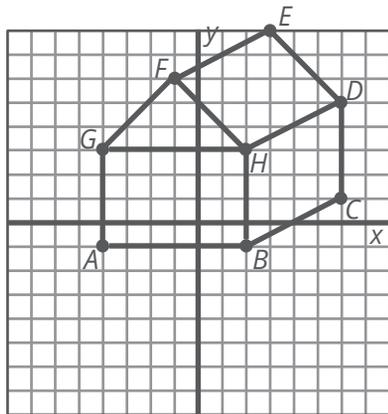


- 1.13** Two vertices of a square are $(3, -2)$ and $(3, 6)$. What are the coordinates of *two* pairs of points for the other vertices?

- a. _____
b. _____

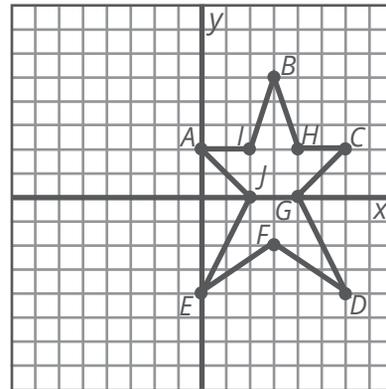


- 1.14** Write the coordinates of all the points in this figure.



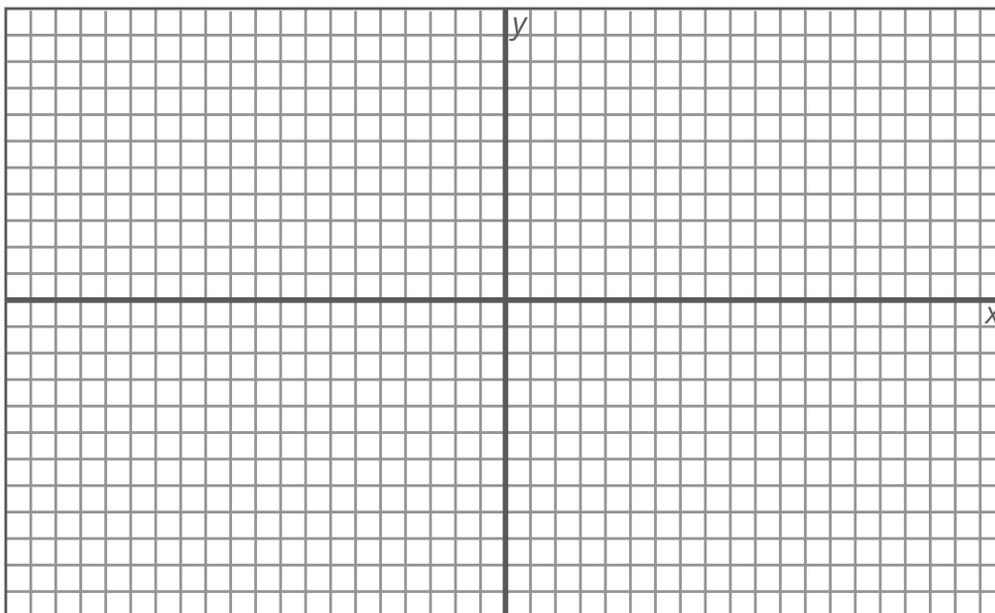
- A (_____)
B (_____)
C (_____)
D (_____)
E (_____)
F (_____)
G (_____)
H (_____)

- 1.15** Write the coordinates of all the points in this figure.



- A (_____) B (_____)
C (_____) D (_____)
E (_____) F (_____)
G (_____) H (_____)
I (_____) J (_____)

1.16 Make your own segment figure and write the coordinates of the points.



TEACHER CHECK

initials _____

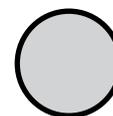
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SYMMETRY

People enjoy looking at certain objects partly because they have some sort of balance to them. A well-formed tree is balanced about its trunk. A church window has a certain balance to it. A snowflake and a butterfly are balanced.

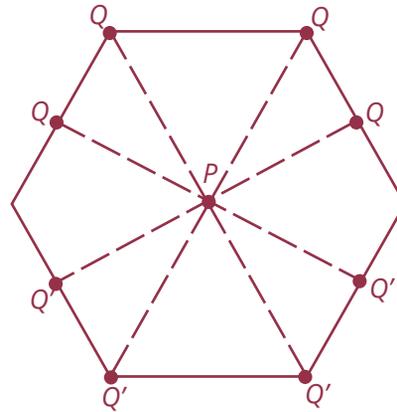
Some geometric figures are more pleasing to look at than others because they, too, have a special kind of balance to them.

An isosceles triangle is more pleasing to look at than a scalene triangle. A rectangle or a square is easier to look at than just any quadrilateral. Regular polygons are also more pleasing. Probably the most visually pleasing of all shapes is the circle. Isaiah 40:22, as you may recall, refers to God sitting upon the circle of the earth to judge it.

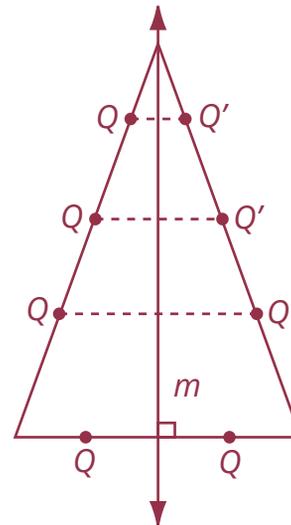


Although people speak of physical objects as being *symmetrical* when they appear to be well-balanced, mathematicians give the idea of *symmetry* exact meaning in three different cases.

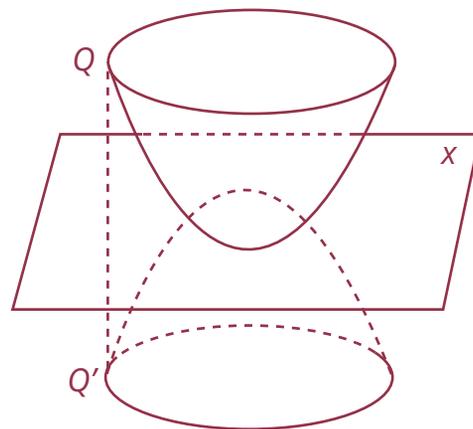
1. A figure has symmetry with respect to a point P if for every point of the figure a partner point Q' exists such that P is the midpoint of $\overline{QQ'}$. The regular hexagon shown has *point symmetry*. The point of symmetry is the center of the polygon, P .



2. A figure has symmetry with respect to a line m if for every point Q of the figure a partner point Q' exists such that m is the perpendicular bisector of $\overline{QQ'}$. The isosceles triangle shown has *line symmetry*. The line of symmetry is the altitude to its base.



3. A figure has symmetry with respect to a plane X if for every point Q of the figure a partner point Q' exists such that X is the perpendicular bisector of $\overline{QQ'}$. The solid figure shown has *plane symmetry*. A plane of symmetry is X .



Some figures have more than one line of symmetry or more than one plane of symmetry. Some figures may have all three kinds of symmetry, but other figures may not be symmetrical at all.



Name the kind or kinds of symmetry the following figures have: point, line, plane, or none.

1.17 _____

A

1.18 _____

B

1.19 _____

C

1.20 _____

D

1.21 _____

E

1.22 _____

F

1.23 _____

H

1.24 _____

2

1.25 _____

3

1.26 _____

O

1.27 _____

8

1.28 _____

100

1.29 _____

7

SELF TEST 1

Write the coordinates of these points (each answer, 2 points).

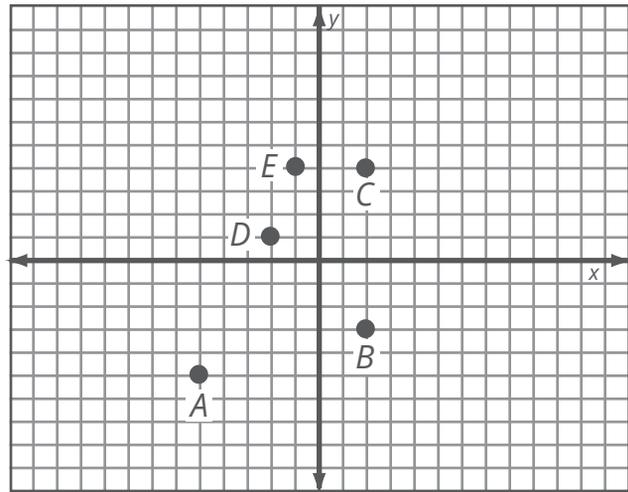
1.01 A (_____)

B (_____)

C (_____)

D (_____)

E (_____)

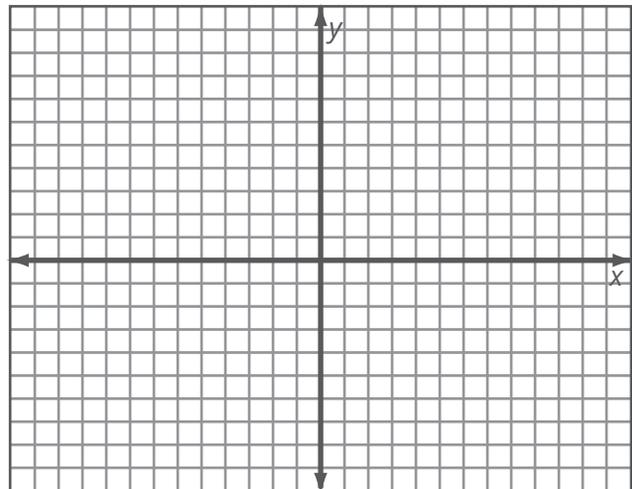


Graph these points on the axes given (each answer, 2 points).

1.02 R (3, 2) U (-2, -4)

S (5, 0) V (6, -3)

T (0, -4)



1.03 In which quadrant are these points located? (each answer, 2 points).

a. (2, 3) _____

d. (-2, 5) _____

b. (4, -2) _____

e. (5, 1) _____

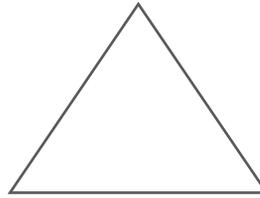
c. (-3, -2) _____

Tell the kind of symmetry these figures have: point, line, plane, or none (each answer, 2 points).

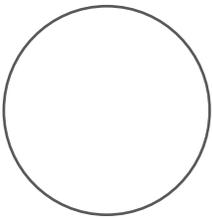
1.04 _____



1.05 _____



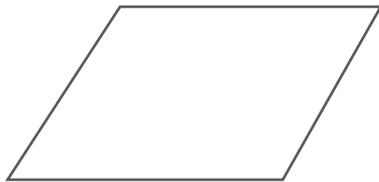
1.06 _____



1.07 _____

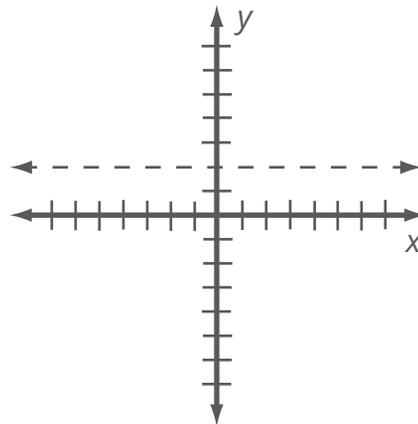


1.08 _____



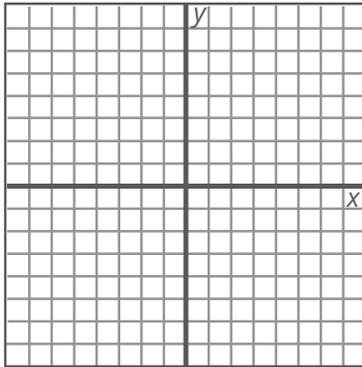
Complete these items (each answer, 3 points).

1.09 Points A and A' have symmetry with respect to the line two units above, and parallel to, the x -axis. Graph the points for A' when A is $(1, 1)$, $(-3, 0)$, $(3, 4)$, and $(-2, -1)$.

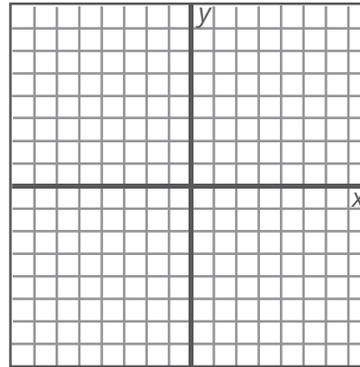


1.010 Points B and B' have symmetry with respect to P . Find the coordinates of P when B is $(2, 8)$ and B' is $(2, 2)$. _____

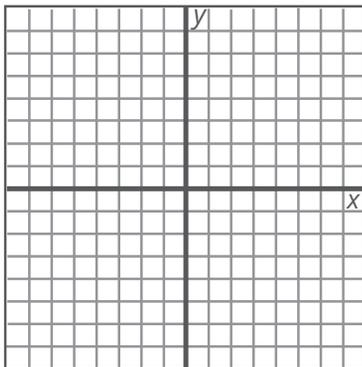
1.016 $\{(x, y): y \leq -4\}$



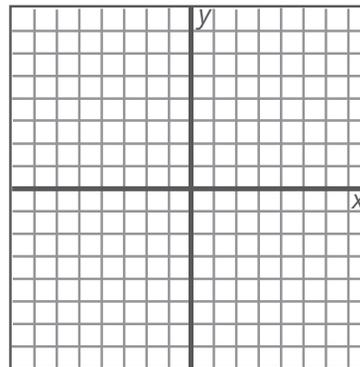
1.017 $\{(x, y): x - y \geq 8\}$



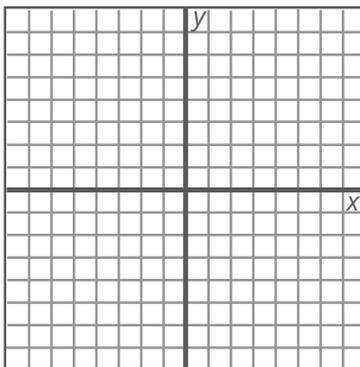
1.018 $\{(x, y): x \geq 2\} \cap \{(x, y): y \geq 2\}$



1.019 $\{(x, y): x \leq 3\} \cup \{(x, y): y \leq -3\}$



1.020 $\{(x, y): x = 4\} \cap \{(x, y): y = 4\}$



<div style="border: 1px solid black; padding: 5px; display: inline-block;"> 67 <hr style="border: 0; border-top: 1px solid black; margin: 0;"/> 83 </div>	SCORE _____	TEACHER _____	initials _____ date _____
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