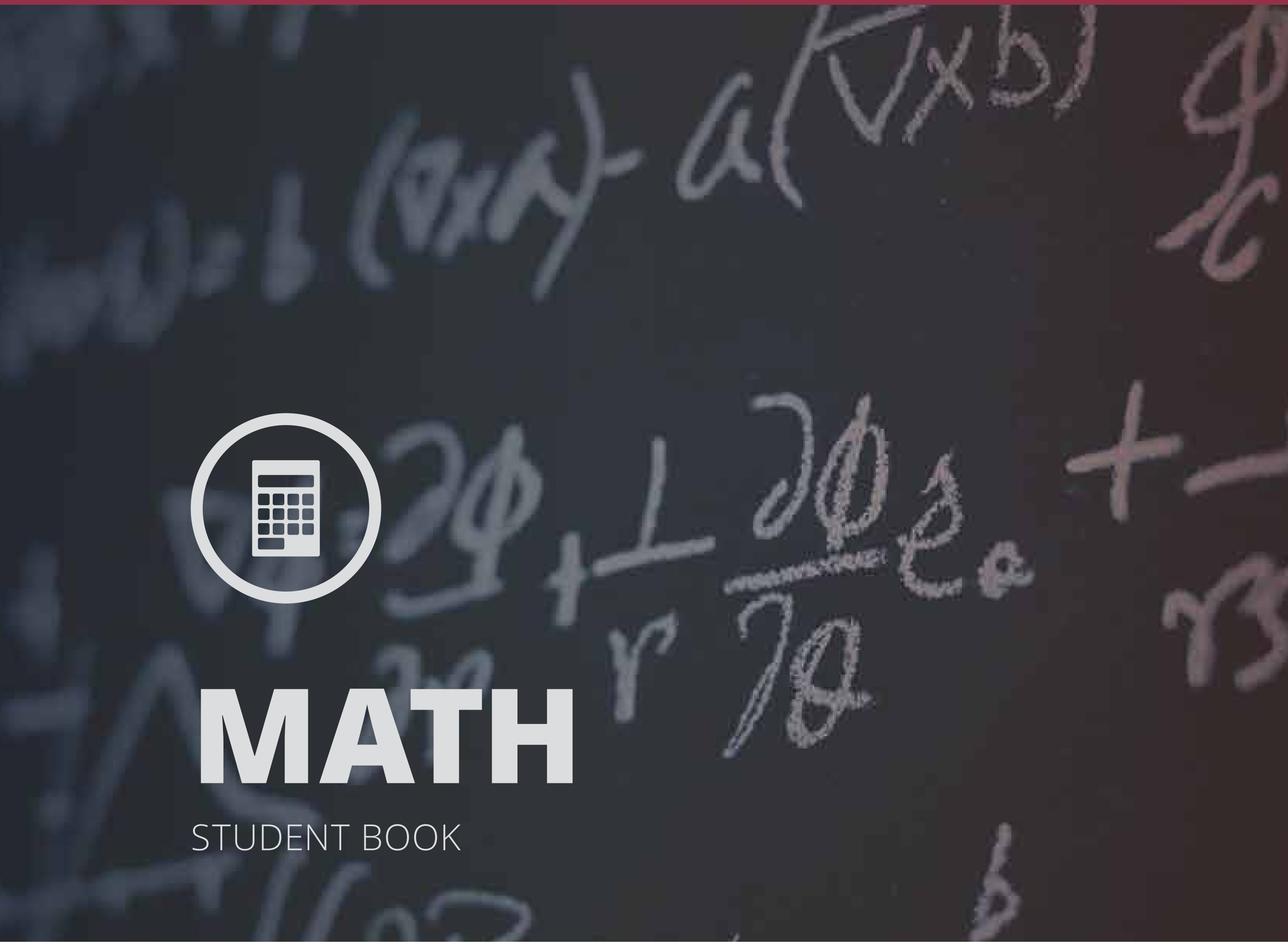
LIFEPAC®



MATH

STUDENT BOOK

10th Grade | Unit 10

Alpha Omega
PUBLICATIONS

MATH 1010

Geometry Review

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Geometry Review

Introduction

This LIFEPAC® contains the definitions, postulates, theorems, and other information that you have studied in the previous nine LIFEPACs. Review activities for you to complete are included in each section. If you have trouble completing any of the review activities, you should go back to the original LIFEPAC and restudy the sections that you are not sure of. When you have completed this review, you will be ready to take the final examination for geometry.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Name, sketch, and label geometric figures.
2. Write and identify conditional sentences and their converse, inverse, and contrapositive.
3. Make and use truth tables.
4. Calculate linear and angle measures.
5. Prove geometric theorems using definitions, postulates, and properties.
6. Identify congruent figures and apply the properties of congruence.
7. Identify similar figures and apply the properties of similarity.
8. Identify and sketch parts of circles.
9. Construct geometric figures using only a compass and straightedge.
10. Sketch and name figures that meet locus conditions.
11. Calculate area and volume of geometric figures.
12. Sketch geometric figures on the coordinate axes.
13. Use algebraic notation to solve geometric distance, slope, and midpoint problems.

1. GEOMETRY, PROOF, AND ANGLES

This section contains the review of geometry as a mathematical system, proof of theorems, and basic angle relationships. Do not hesitate to go back and restudy at any time!

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Name, sketch, and label geometric figures.
2. Write and identify conditional sentences and their converse, inverse, and contrapositive.
3. Make and use truth tables.
4. Calculate linear and angle measures.
5. Prove geometric theorems using definitions, postulates, and properties.

GEOMETRY AS A SYSTEM

A mathematical system is a logical study of shape, arrangement, and quantity. Algebra, geometry, trigonometry, and calculus are examples of mathematical systems. Geometry is the logical study of the shape and size of things. The word comes from Greek and means *earth measurement*.

Any mathematical system contains four items:

1. Basic undefined terms;
2. All other terms, carefully defined;
3. Postulates; and
4. Theorems.

The basic undefined terms in geometry are

1. point,
2. line, and
3. plane.

Other fundamental terms are carefully defined in the Definitions box.

DEFINITIONS

Space: the set of all points.

Collinear points: a set of two or more points all on the same line.

Coplanar points: a set of points all on the same plane.

Betweenness of points: point B is *between* points A and C if A, B, and C are collinear and $AB + BC = AC$.

Line segment: the set of two different points and all points between them.

Midpoint of a segment: the point on a segment that divides the segment into two segments of equal length.

Bisector of a segment: a line or segment that intersects the first segment at its midpoint.

Ray: the set of all points $\overline{AB}$ and all points P, such that B is between A and P.

Opposite rays: two rays with a common endpoint that form a line.

Postulate: a statement accepted without proof.

Theorem: a general statement that can be proved.

Postulates will be numbered consecutively throughout this LIFEPAC. If you wish to review a postulate in more detail, it will be found in the Math LIFEPAC with the same name as the section name in this review LIFEPAC. The first five basic postulates are listed here.

POSTULATES

P1: A line contains at least two points; a plane contains at least three points not on one line; space contains at least four points not all in one plane.

P2: Through any two different points exactly one line exists.

P3: Through any three points not on one line exactly one plane exists.

P4: If two points lie in a plane, the line containing them lies in that plane.

P5: If two planes intersect, then their intersection is a line.

Theorems will be numbered exactly as they were in the first nine LIFEPACs of the Math 1000 series. You may wish to review the proof of each theorem as it is presented again.

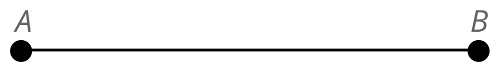
THEOREMS

1–1 If two lines intersect, then their intersection is exactly one point.

1–2 Exactly one plane contains a given line and a given point not on the line.

1–3 If two lines intersect, then exactly one plane contains both lines.

Name the following geometric figures.

1.1 

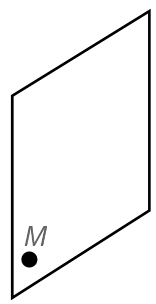
____

1.2 

____

1.3 

____

1.4 

____

1.5 

____

Complete the following activities.

1.6 The set of all points is called ______________________ .

1.7 The endpoint of ray $\overrightarrow{RS}$ is point ______________________ .

1.8 If S is between R and T, then $RS + ST =$ ______________________ .

1.9 A plane contains at least ________________ points.

1.10 A line contains at least ________________ points.

1.11 A postulate is accepted without ______________________ .

1.12 If two lines intersect, their intersection is exactly ________________ point(s).

1.13 Space contains at least ________________ points.

1.14 How many lines are determined by four points, no three of which are collinear?

____

1.15 Two opposite rays form a ______________________ .

Sketch and label the following conditions.

1.16 Collinear points B, U, N, T

1.17 Segment $\overline{WX}$ with midpoint M

1.18 Lines m and n, both in plane T, intersecting at point P

1.19 Opposite rays $\overrightarrow{OP}$ and $\overrightarrow{OG}$

1.20 Plane A and Plane B intersecting in line $\overleftrightarrow{PQ}$

PROOF

One of the main items of our geometric system are statements that we call theorems. Theorems are statements that we can prove to be true. We prove theorems true by using logical thinking and deductive reasoning.

DEFINITIONS

Statement: a sentence that is either true or false but not both.

Conjunction: a statement formed by combining two statements with the word *and*.

Disjunction: a statement formed by combining two statements with the word *or*.

Negation: if p is a statement, the new statement, not p, is called the negation of p.

Conditional: a statement formed from two statements by connecting them in the form *if* ________ , *then* ________ .

Hypothesis: the *if* clause in a conditional statement.

Conclusion: the *then* clause in a conditional statement.

Converse: a statement formed by interchanging the hypothesis and the conclusion in a conditional statement.

Inverse: a statement formed by negating both the hypothesis and the conclusion of a conditional statement.

Contrapositive: a statement formed by exchanging the hypothesis and conclusion and negating both of them.

Inductive reasoning: the process of making a general conclusion based on specific examples.

Deductive reasoning: the process of making a conclusion by fitting a specific example to a general statement.

Truth table: an arrangement of truth values to determine when a statement is true or false.

Two-column proof: a formal proof of a theorem composed of six standard parts.

Indirect proof: a proof of a theorem by indirect means.

The compiled truth table for use in this geometry course is shown here for your reference.

COMPILED TRUTH TABLE

STATEMENT		NEGATION		CONDITIONAL	CONVERSE	INVERSE	CONTRAPOSITIVE
p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

The six parts of a two-column proof are listed in order:

Statement: a full written statement of the theorem.

Figure: a lettered figure drawn to illustrate the given conditions of the statement.

Given: the given conditions of the statement expressed in terms of the letter and numerals used in the figure.

To Prove: the part of the statement that requires proof expressed in terms of the letters and numerals that are used in the figure.

Plan of Proof: a brief description of the plan you are going to use in the proof.

Proof: the actual proof; a series of numbered statements in one column with a like-numbered column next to it for the reasons.

The normal method of an indirect proof is to follow the three steps outlined here.

1. Suppose the negative of the conclusion is true.
2. Reason from your assumed statement until you reach a contradiction of a known fact.
3. Point out why the assumed statement must be false and that the desired conclusion must be true.

Identify the following statements as conjunction, disjunction, negation, or conditional, and tell if the statement is true or false.

1.21 If three sides of one triangle are equal to three sides of another triangle then the triangles are congruent.

____________________________________ (T/F) __________

1.22 A triangle has three sides and a pentagon has five sides.

____________________________________ (T/F) __________

1.23 It is false that $3 + 2 \neq 5$.

____________________________________ (T/F) __________

1.24 The sum of the angles of a triangle equals 180°, or a right triangle has two right angles.

____________________________________ (T/F) __________

1.25 If a triangle has at least two sides equal, then it is an isosceles triangle.

____________________________________ (T/F) __________

Write the converse, inverse, and contrapositive of the following theorems, and tell if the new statements are true or false.

1.26 If two lines are parallel, then the alternate interior angles are equal.

a. Converse: ____

____ (T/F) ____________

b. Inverse: ____

____ (T/F) ____________

c. Contrapositive: ____

____ (T/F) ____________

1.27 If two lines intersect, then the vertical angles formed are equal.

a. Converse: ____

____ (T/F) ____________

b. Inverse: ____

____ (T/F) ____________

c. Contrapositive: ____

____ (T/F) ____________

1.28 The diagonals of a parallelogram bisect each other.

a. Converse: ____

____ (T/F) ____________

b. Inverse: ____

____ (T/F) ____________

c. Contrapositive: ____

____ (T/F) ____________

1.29 Base angles of isosceles triangles are equal.

a. Converse: ____

______________________________________ (T/F) ______________

b. Inverse: ____

______________________________________ (T/F) ______________

c. Contrapositive: ____

______________________________________ (T/F) ______________

1.30 If two lines are perpendicular they meet to form right angles.

a. Converse: ____

______________________________________ (T/F) ______________

b. Inverse: ____

______________________________________ (T/F) ______________

c. Contrapositive: ____

______________________________________ (T/F) ______________

Sketch a figure to represent the following theorems.

1.31 If two lines are perpendicular, then they form right angles.

1.32 In a plane, if two lines are perpendicular to a third line, then they are parallel to each other.

1.33 If two legs of one right triangle are equal to the legs of another right triangle, then the triangles are congruent.

1.34 If two lines are parallel, then the alternate interior angles are equal.

1.35 If two adjacent acute angles have their exterior sides in perpendicular lines, then the angles are complementary.

Complete the following truth tables.

1.36

p	$\sim p$
T	a. ____________
F	b. ____________

1.37

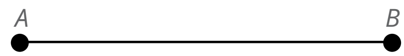
p	q	$p \rightarrow q$
T	F	a. ____________
F	F	b. ____________
T	T	c. ____________
F	T	d. ____________

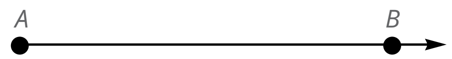
1.38

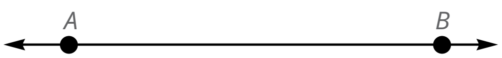
p	q	$q \rightarrow p$
T	T	a. ____________
T	F	b. ____________
F	F	c. ____________
F	T	d. ____________

SELF TEST 1

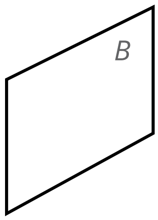
Match the following items (each answer, 2 points).

1.01 ________ 

1.02 ________ 

1.03 ________ 

1.04 ________ 

1.05 ________ 

a. plane *B*
b. line *AB*
c. plane *A*
d. point *A*
e. segment *AB*
f. ray *AB*

Complete the following truth tables (each answer, 2 points).

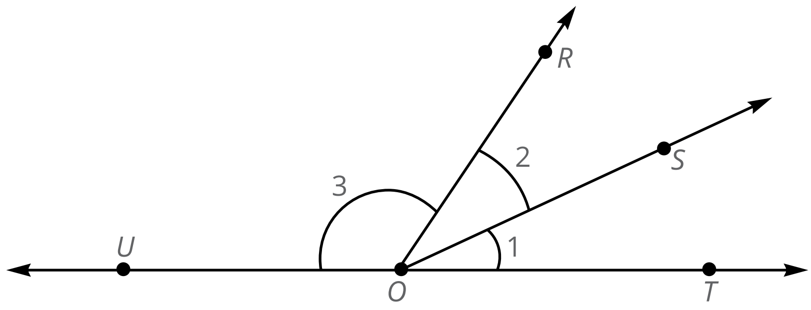
1.06

p	*q*	$p \to q$
T	F	a. ____________
F	F	b. ____________
T	T	c. ____________
F	T	d. ____________

1.07

p	*q*	*p* and *q*
T	T	a. ____________
F	F	b. ____________
T	F	c. ____________
F	T	d. ____________

Use this diagram to find the required measures (each answer, 3 points).



1.08 m $\angle 1 = 30°$ m $\angle 2 = 30°$ m $\angle 3 =$ __________

1.09 m $\angle 2 = 20°$ m $\angle 3 = 130°$ m $\angle 1 =$ __________

1.010 m $\angle 1 = 40°$ m $\angle 3 = 110°$ m $\angle 2 =$ __________

1.011 m $\angle 1 = 45°$ m $\angle 2 = 45°$ m $\angle 3 =$ __________

1.012 m $\angle 2 = 15°$ m $\angle 3 = 118°$ m $\angle 1 =$ __________

Write the correct letter of the answer on the blank (each answer, 2 points).

1.013 The sum of the interior angles of a triangle is ____________________ .
a. 1,800° b. 360° c. 180° d. 3,240°

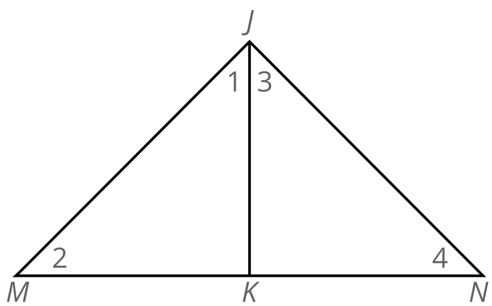
1.014 The sum of the interior angles of a quadrilateral is ____________________ .
a. 540° b. 720° c. 360° d. 180°

1.015 The sum of the interior angles of a pentagon is ____________________ .
a. 540° b. 720° c. 360° d. 180°

1.016 The sum of the interior angles of a 20-gon is ____________________ .
a. 1,800° b. 360° c. 180° d. 3,240°

Complete the following proofs (each proof, 6 points).

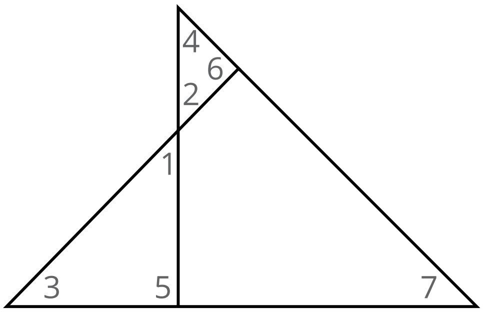
1.017 Given: $\overline{JK} \perp \overline{MN}$
To Prove: $\angle 1$, $\angle 2$ are complementary



STATEMENT	REASON

1.018 Given: m $\angle 5$ = m $\angle 6$

To Prove: m $\angle 3$ = m $\angle 4$

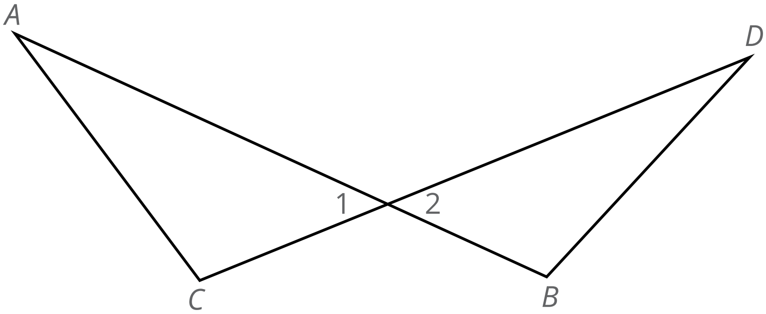


STATEMENT	REASON

1.019 Given: $\overline{AC} \perp \overline{CD}$

$\overline{DB} \perp \overline{AB}$

To Prove: m $\angle A$ = m $\angle D$



STATEMENT	REASON

54/67 **SCORE** ______ **TEACHER** ______ initials ______ date

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