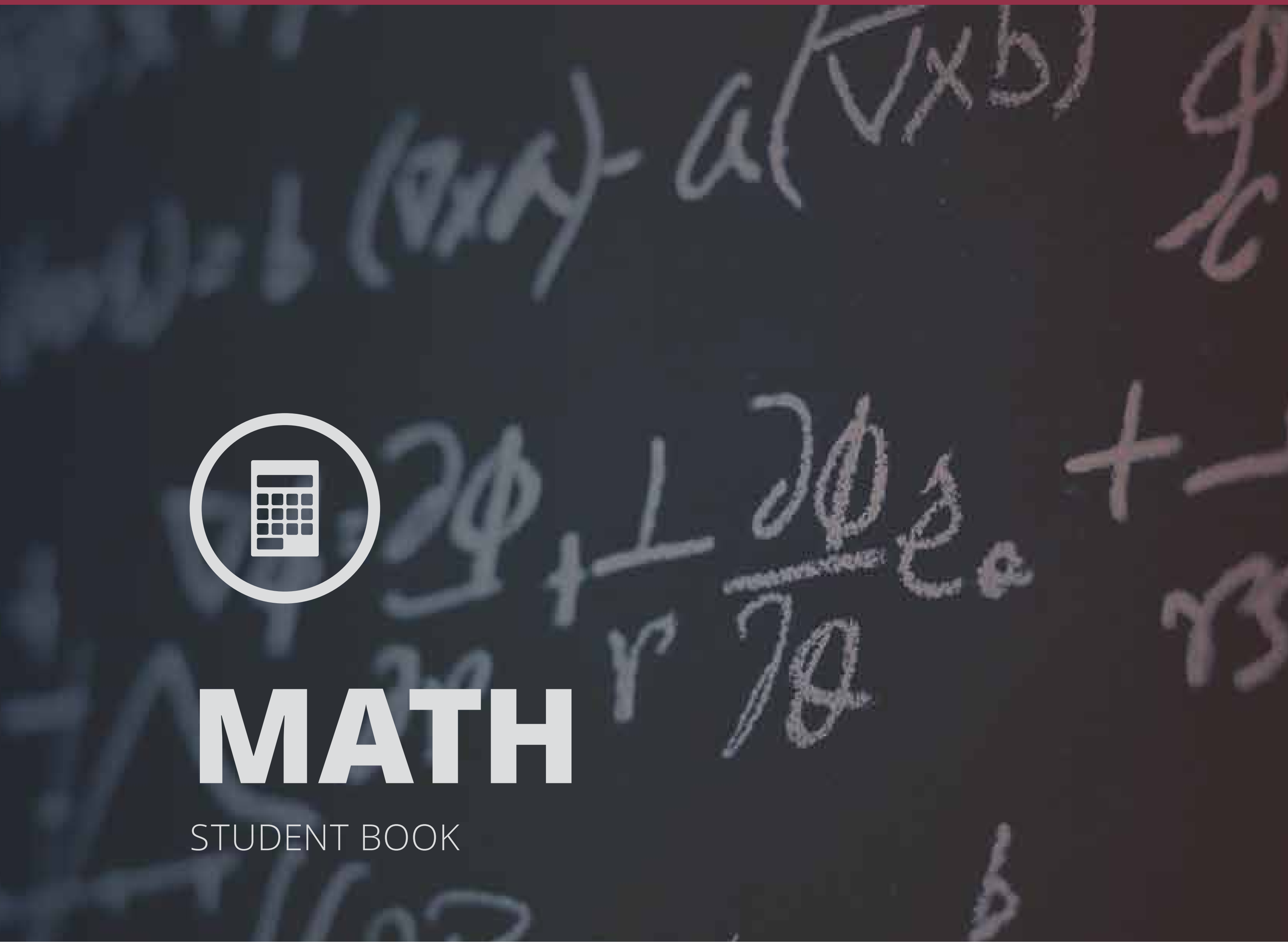
LIFEPAC®



MATH

STUDENT BOOK

11th Grade | Unit 1

Alpha Omega
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MATH 1101

SETS, STRUCTURE, AND FUNCTIONS

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Sets, Structure, and Functions

Introduction

The main theme in the study of math is commonly understood to be the function. Importance of the function becomes more evident as you continue your study in math. Many skills have to be mastered in order to study functions effectively. In addition to the concept of functions, this LIFEPAC® contains a study of sets and the algebra of sets, the properties of the real number system, the skills involved with combining algebraic terms, and the use of the exponent.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Combine sets through intersection and union.
2. Identify subsets.
3. Perform multiple operations of an expression in correct order.
4. Identify operational axioms when simplifying expressions.
5. Identify a function and its range and domain.
6. Evaluate functions.
7. Find the inverse of a function.
8. Express repetitive factors in exponential notation.
9. Evaluate numbers raised to a power.
10. Express fractions as numbers with negative exponents.
11. Simplify exponential notation.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. SETS

Sets, a concept as old as man, has a definite place in the study of math. From earliest times men have thought in terms of sets or collections of objects. We can think of such things as "sets of dishes," "members of a particular organization," or "the set of planets," as being natural in our environment. In math, a set, symbolized { }, is a well-defined collection of objects. We need to define the properties of sets as they apply to math so that we may use these properties to justify math operations.

Natural numbers are counting numbers from +1 to infinity.

Whole numbers are all the natural numbers plus zero; whole numbers range from 0 to infinity.

Integers are all positive and negative numbers and zero; integers range from negative infinity to positive infinity.

Remember?

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Combine sets through intersection and union.
2. Identify subsets.

PROPERTIES

In this section you will be concerned with the definitions, concepts, and operations of sets.

PROPERTIES OF SETS

A. Sets are made up of *members* or **elements**. The symbol used is $\in$.

B. A **finite set** is a set in which the number of elements is bounded by an interval. An **infinite set** is a set that is limitless.

C. Sets may be designated in one of two ways: the *list* method or the *rule* method. In the *list* method, elements of the set are *listed*.

A. Model: If A is the set of whole numbers, then $7 \in A$ and is read, "Seven is a member of Set A."

B. Model 1: The set of states in the United States is finite and numbers 50.

Model 2: The set of whole numbers is infinite.

C. Model 1: If Set A is the set of even whole numbers less than 10, then the *list* method would be $A = \{0, 2, 4, 6, 8\}$.

Model 2: The *rule* method uses set-builder notation. $A = \{x \mid x$ is a whole number $< 10\}$. This notation is read, "A is the set of numbers such that x is a whole number less than 10." Note that 10 is *not* an element of the set.

Using the list method, write the following sets.

1.1 The odd integers between 2 and 15. ____________________

1.2 The even whole numbers less than 8. ____________________

1.3 The perfect square integers between 1 and 80 inclusive. ____________________

____

1.4 The last names of all your teachers. ____________________

____

1.5 Every two-digit number whose units' digit is three times its tens' digit. ____________________

____

Using the rule method, write the following sets.

1.6 $\{2, 4, 6, 8, 10, 12\}$ ____________________

1.7 $\{1, 8, 27, 64\}$ ____________________

1.8 $\{a, l, g, e, b, r\}$ ____________________

1.9 $\{2, 4, 6\}$ ____________________

1.10 $\{0\}$ ____________________

PROPERTIES OF SETS

D. Two sets are **equal** if the elements are identical.

E. Two sets are **equivalent** if they contain the same number of elements.

F. If every member of Set A is also a member of Set B, then A is a *subset* of B. This property in symbolic form is $A \subset B$.

G. An **empty set** is a set whose elements number zero. The symbol for an empty set is Ø or { }.

H. The *empty* set is a subset of every set. This property in symbolic form is $Ø \subset A$.

D. Model: If $A = \{X, Y, Z\}$ and $B = \{Z, X, Y\}$, then $A = B$.

E. Model: If $A = \{X, Y, Z\}$ and $B = \{P, Q, R\}$, then A is *equivalent* to B.

F. Model: If $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \subset B$.

Replace the question mark with the symbol = (equal to) or ≠ (not equal to) to make the expression true.

1.11 $\{2, 4, 6\}$? $\{6, 4, 2\}$ ____________

1.12 $\{X, Y, Z\}$? $\{A, B, C\}$ ____________

1.13 $\{0\}$? $\{\ \}$ ____________

1.14 {1, 2, 3, 4, 5} ? {whole numbers between 1 and 5 inclusive} ____________

1.15 {Adams, Polk, Harrison} ? {all Presidents of the United States} ____________

Write the number of elements in each of the following sets.

1.16 $\{A, B, C\}$ ____________

1.17 $\{0\}$ ____________

1.18 $\{\ \}$ ____________

1.19 {Students in your room} ____________

1.20 {Whole numbers between 3 and 15} ____________

Complete the missing part (?) that will make the statement true.

1.21 $8 \in \{3, 5, ?\}$ ____________

1.22 $\{6, 2\} \subset \{1, 2, 3, A, ?\}$ ____________

1.23 $\{1\}$? $\{5, 4, 3, 2, 1\}$ ____________

1.24 $\{\ \}$? $\{0\}$ ____________

1.25 $\{\ \}$? $\varnothing$ ____________

Write the answers for the following three groups of problems.

Given that $A = \{a, b, c\}$

1.26 List all of the subsets of A that have exactly one element. ______________________________

1.27 List all of the subsets of A that have exactly two elements. ______________________________

1.28 List all of the subsets of A that have exactly three elements. ______________________________

1.29 How many subsets does Set A have including Ø? ______________________________

Given that $B = \{1, 2, 3, 4\}$

1.30 List all of the subsets of B that have exactly one element. ______________________________

1.31 List all of the subsets of B that have exactly two elements. ______________________________

1.32 List all of the subsets of B that have exactly three elements. ______________________________

1.33 List all of the subsets of B that have exactly four elements. ______________________________

1.34 How many subsets does Set B have including Ø? ______________________________

Given that $C = \{M, N, O, P, Q\}$

1.35 How many subsets does Set C have including Ø? ______________________________

1.36 Write a formula for the number of subsets of any set; identify any symbols you use.

____

OPERATIONS

In arithmetic the operations, adding and multiplying, were defined and developed. Similarly, in the algebra of sets, we define two other operations. They are **union** and **intersection**. The algebra of sets is built around these two operations and around the properties just defined.

OPERATIONS OF SETS

A. The *union* of two sets, A and B, is a set whose elements are those that appear in *both* Set A and Set B, without repetition. The symbol for union is $\cup$.

B. The *intersection* of two sets, A and B, is a set whose elements are those that are *common* to A and B. The symbol for intersection is $\cap$.

A. Model 1: If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cup B = \{1, 2, 3, 4\}$. ($A \cup B$ is read “the union of A and B.”) Elements 2 and 3 are not repeated.

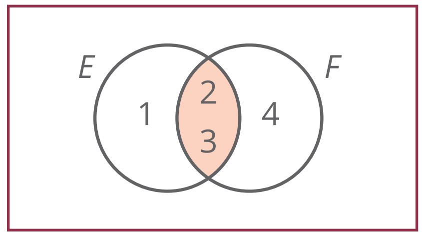
$A \cup B$ is commonly described as the elements of Set A *or* Set B in the sense that the elements of $A \cup B$—1, 2, 3, and 4—are found either in Set A or in Set B. The word *or* is used here not in the sense of choice.

Model 2: If $C = \{a, b, c, d\}$ and $D = \{a, c\}$, then $C \cup D = \{a, b, c, d\}$.

Notice that in Model 2 all the elements of D are in C. Therefore, the following property is evident:

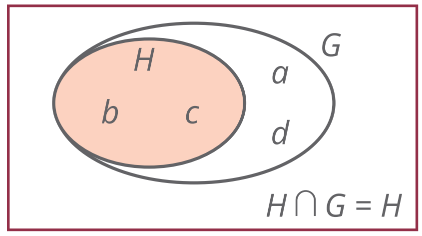
If $D \subset C$, then $C \cup D = C$.

B. Model 1: If $E = \{1, 2, 3\}$ and $F = \{2, 3, 4\}$, then $E \cap F = \{2, 3\}$. ($E \cap F$ is read "the intersection of E and F.")



$E \cap F$ may be described also as the elements of Set E *and* Set F in the sense that the elements of $E \cap F$—2 and 3—are found in both Set E *and* Set F. The word *and* is used here in the sense of common occurrence.

Model 2: If $G = \{a, b, c, d\}$ and $H = \{b, c\}$, then $G \cap H = \{b, c\}$.



Notice that in Model 2,

if $H \subset G$, then $H \cap G = H$.

Write the following sets.

Given $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 4, 6\}$ $C = \{1, 3, 5\}$

1.37 $A \cup B$ ____________________

1.38 $A \cup C$ ____________________

1.39 $B \cup C$ ____________________

1.40 $A \cap B$ ____________________

1.41 $A \cap C$ ____________________

1.42 $B \cap C$ ____________________

1.43 $A \cup (B \cup C)$ ____________________

1.44 $A \cap (B \cap C)$ ____________________

1.45 $A \cup (B \cap C)$ ____________________

1.46 $A \cap (B \cup C)$ ____________________

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