LIFEPAC®

MATH

STUDENT BOOK

11th Grade | Unit 4

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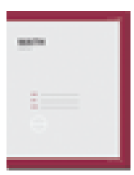
MATH 1104
POLYNOMIALS



LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Polynomials

Introduction

In this LIFEPAC® you will be studying products and factoring, as well as operations with polynomials and the patterns of variations.

You may have already studied multiplication of polynomials and factoring of polynomials. In this LIFEPAC you will review and learn some new factoring techniques.

Addition, subraction, and division will be reviewed and a new synthetic divison will be introduced. Learning the operational aspects of polynomials will be useful in sloving equations.

Direct variation and inverse variation are important relationships between variables that are found in many natural laws. These relationships are often used alone or in combination to solve practical problems through the equations they represent.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Multiply polynomials.
2. Identify special products and factor completely.
3. Perform addition, subraction, and division with polynomials.
4. Use direct, inverse, joint, and combined variations in writing equations and solving problems.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. PRODUCTS AND FACTORING

This LIFEPAC section will explore topics that you have studied before. Multiplying polynomials, recognizing special products, and factoring trinomials and special products were studied earlier.

New material included features a special product not previously covered and the factors of this product. The polynomial factors may surprise you.

You will need expertise in this section to complete later work with equations and word problems.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Multiply polynomials.
2. Identify special products and to factor completely.

MULTIPLYING WITH MONOMIALS

Certain laws of exponents must be used in operations with polynomials.

LAWS OF EXPONENTS

A. $a^m a^n = a^{m+n}$

B. $(a^m)^n = a^{mn}$

C. $(ab)^m = a^m b^m$

Models: A. $x^3 \cdot x^5 = x^8$

B. $(x^3)^5 = x^{15}$

C. $(x^2y^4)^3 = (x^2)^3(y^4)^3$ or x^6y^{12}

The distributive property is the basis for multiplying monomials with polynomials.

DISTRIBUTIVE PROPERTY

$a(b + c) = ab + ac$

Model: $3x(a + y) = 3ax + 3xy$

The associative and commutative properties are used along with the distributive property and the laws of exponents to multiply.

ASSOCIATIVE PROPERTY

$a(bc) = (ab)c$

COMMUTATIVE PROPERTY

$ab = ba$

Model:

$$\begin{aligned} 6x^2y(3xy^3 + 14x^2y^2) &= (6x^2y)(3xy^3) + (6x^2y)(14x^2y^2) \\ &= 6 \bullet 3 \bullet x^2 \bullet x \bullet y \bullet y^3 + 6 \bullet 14 \bullet x^2 \bullet x^2 \bullet y \bullet y^2 \\ &= 18x^3y^4 + 84x^4y^3 \end{aligned}$$

Find the indicated products.

1.1 $x^2 \bullet x^5$ ____________________

1.2 $a^4 \bullet a^6$ ____________________

1.3 $(a^4)^5$ ____________________

1.4 $(x^6)^4$ ____________________

1.5 $(x^2y)^3$ ____________________

1.6 $(2a^2b)^4$ ____________________

1.7 $-2x^2y^3 \bullet 14x^2y^3$ ____________________

1.8 $(3x^2y^6)^7$ ____________________

1.9 $a^2b(3a^2 + 4ab^2)$ ____________________

1.10 $3a^n(a^n + a^{n-1})$ ____________________

1.11 $6xy\,(\frac{1}{2}x^2 - \frac{1}{3}xy + \frac{1}{6}y^2)$ ____________________

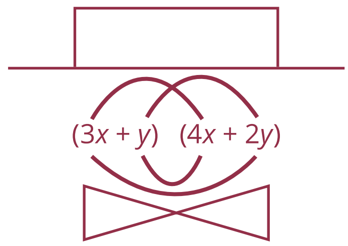
MULTIPLYING POLYNOMIALS BY POLYNOMIALS

Multiplying two polynomials together involves the distributive, associative, and commutative properties reviewed in the previous section as well as some memory of products.

$$\text{Model:} \quad \begin{aligned} (3x + y)(4x + 2y) &= (3x + y)4x + (3x + y)2y \\ &= 12x^2 + \underbrace{4xy + 6xy} + 2y^2 \\ &= 12x^2 + \quad 10xy \quad + 2y^2 \end{aligned}$$

Using either the “foil” method or remembering the unforgettable face of George P. Trinomial can help with finding the product. You should not have to write anything but the answer in finding this product.

The “foil” method involves using the letters in the word to remember that the product consists of the product of the two **f**irst terms $(3x)(4x)$, plus the product of the **o**uter terms $(3x)(2y)$ combined with the product of the **i**nner terms $(y)(4x)$, and finally the product of the two **l**ast terms $(y)(2y)$.



George P. Trinomial

George P. Trinomial is the name of the face you see.

Of course, the crossed eyebrows, nose, and chin help direct us to the products.

In certain polynomials you may group the terms and treat your selected binomials as monomials. With practice these products can be done mentally.

$$\text{Model:} \quad \begin{aligned} (a + b - 3)(a + b + 5) &= [(a + b) - 3][(a + b) + 5] \\ \text{let } x &= a + b \\ (x - 3)(x + 5) &= x^2 + 2x - 15 \end{aligned}$$

Now replace x with $a + b$:

$$\begin{aligned} &= (a + b)^2 + 2(a + b) - 15 \\ &= a^2 + 2ab + b^2 + 2a + 2b - 15 \end{aligned}$$

Find the indicated products mentally when possible.

1.12 $(a + 3)(a - 2)$ ____________________________________

1.13 $(3xy - 1)(4xy + 2)$ ____________________________________

1.14 $(2x - 3y)(4x - y)$ ____________________________________

1.15 $(ab - 9)(ab + 8)$ ____________________________________

1.16 $(m^3n + 8)(m^3n - 5)$ ____________________________________

1.17 $(3 - c^2d)(4 - 4c^2d)$ ____________________________________

1.18 $(1 - 7x)(1 + 9x)$ ____________________________________

1.19 $(3m^3 - \frac{1}{2}y)(3m^3 - \frac{1}{2}y)$ ____________________________________

1.20 $(a + b)(a^2 - ab + b^2)$ ____________________________________

1.21 $(a - b)(a^2 + ab + b^2)$ ____________________________________

Treat selected binomials as monomials to find these products mentally.

1.22 $(x + y + 3)(x + y - 4)$ ______________________

1.23 $(a + b - c)(a + b + c)$ ______________________

1.24 $(x^2 + 2x - 1)(x^2 + 2x + 5)$ ______________________

1.25 $[4 - (3c - 1)][6 - (3c - 1)]$ ______________________

1.26 $(3x - 4y - 5z)(4x + 4y + 5z)$ ______________________

USING SPECIAL PRODUCTS

Although binomial products can always be found mentally as in the previous lesson, some special products require less mental energy if you can notice the patterns.

A **perfect square trinomial** is the name given to trinomials that result from squaring a binomial.

DEFINITION

Perfect square trinomial: the square of a binomial.

Model 1: $(a + b)^2 = a^2 + 2ab + b^2$

Model 2: $(2x - 4y^2)^2 = 4x^2 - 16xy^2 + 16y^4$

Note that the first term in the first trinomial is a^2, which is the square of a. The middle term is twice the product of the two terms in the binomial, $2ab$. The third term in the trinomial is the square of the second term in the binomial. Is $4x^2$ the same as $(2x)^2$? Is $-16xy^2$ the same as $2(2x)(-4y^2)$? Is $16y^4$ the same as $(-4y^2)^2$?

Another special product is the difference of two perfect squares. This product is the result of multiplying the sum of two numbers and the difference of those same two numbers.

Model 3: $(a + b)(a - b) = a^2 - b^2$

Model 4: $(3x^2 + 4y)(3x^2 - 4y) = 9x^4 - 16y^2$

Most surprising is the product of $(a + b)$ and $(a^2 - ab + b^2)$.

Model 5:

$$\begin{array}{r} a^2 - ab + b^2 \\ a + b \\ \hline a^3 - a^2b + ab^2 \\ + a^2b - ab^2 + b^3 \\ \hline a^3 \qquad\qquad + b^3 \end{array}$$

The product of $(a - b)$ and $(a^2 + ab + b^2)$ is $a^3 - b^3$.

To find this product mentally you must notice that the binomial factor is a difference when the product is the difference of two perfect cubes; it is a sum when the product is the sum of two perfect cubes. Another observation is that the trinomial is not a perfect square trinomial, simply because it lacks the factor of 2 in its middle term.

Model 6: $(a + b)(a^2 - ab + b^2) = a^3 + b^3$

Model 7: $(2a - 3b)(4a^2 + 6ab - 9b^2) = 8a^3 + 27b^3$

Find the indicated products.

1.27 $(w + x)(w + x)$ ______

1.28 $(y - x)(y - x)$ ______

1.29 $(wx - y)^2$ ______

1.30 $(x + 2y)^2$ ______

1.31 $(4x^3 + 7y^3z^4)^2$ ______

1.32 $(10^x + 4^y)^2$ ______

1.33 $(2x - \frac{1}{2})^2$ ______

1.34 $(0.1x + 0.4y)^2$ ______

1.35 $(2a^x + \frac{1}{2}b^x)^2$ ______

1.36 $(3x^{2a} - 4y^a z^{3a})^2$ ______

1.37 $(x + 2)(x - 2)$ ______

1.38 $(4a + x)(a - x)$ ______

1.39 $(y + \frac{1}{3})(y - \frac{1}{3})$ ______

1.40 $(b^2 + 8)(b^2 - 8)$ ______

1.41 $(5c + \frac{2}{3})(5c - \frac{2}{3})$ ______

1.42 $(x + 0.2)(x - 0.2)$ ______

1.43 $(y^n - 5)(y^n + 5)$ ______

1.44 $(a^x - b^y)(a^x + b^y)$ ______

1.45 $(4y^a - 6x^b)(4y^a + 6x^b)$ ______

1.46 $(a + b - c)(a + b + c)$ ______

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