LIFEPAC®

MATH

STUDENT BOOK

11th Grade | Unit 5

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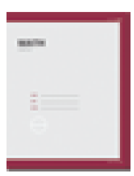
MATH 1105
ALGEBRAIC FRACTIONS



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Algebraic Fractions

Introduction

In this LIFEPAC® you will study algebraic fractions. You have previously studied fractions in arithmetic and in your first year of algebra; therefore, you should already be acquainted with many of the ideas presented here.

Reducing fractions, along with adding, subtracting, multiplying, and dividing with algebraic fractions, lead to the solving of certain equations that involve fractions. In this LIFEPAC you will put the finishing touches on most of your previous work, and you will see how all the things you have learned fit together to help in solutions of word problems that you might have considered impossible before.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Use positive, negative, and zero integers as exponents.
2. Simplify algebraic fractions by reducing them to lowest terms.
3. Multiply and divide with algebraic fractions.
4. Use the lowest common denominator to find sums and differences of algebraic fractions.
5. Solve equations containing fractions and fractional equations.
6. Solve motion problems, mixture problems, and work problems.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. MULTIPLYING AND DIVIDING WITH FRACTIONS

Exponents may be negative integers or zero as well as the familiar positive integers. The laws of exponents are important when dealing with fractions.

An exponent is a small digit written to the upper right of a number, indicating how many times the number is to be used as a factor.

Remember?

Algebraic fractions are fractions that contain variables in either the numerator or denominator or both. Basically, algebraic fractions are treated in the same way that arithmetic fractions are. Of course, the expressions will be more complex and will require careful work.

DEFINITION

Algebraic fraction: a fraction with a variable in the numerator or the denominator.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Use positive, negative, and zero integers as exponents.
2. Simplify algebraic fractions by reducing them to lowest terms.
3. Multiply and divide with algebraic fractions.

ZERO AND NEGATIVE EXPONENTS

Originally exponents were used to tell us the number of times a factor occurs in certain expressions.

Model 1: $10^4 = 10 \cdot 10 \cdot 10 \cdot 10$

Certain laws of exponents were listed in Math LIFEPAC 1104.

LAWS OF EXPONENTS

A. $a^m a^n = a^{m+n}$

B. $(a^m)^n = a^{mn}$

C. $(ab)^m = a^m b^m$

When dividing with numbers that have the same base, we simply subtract the exponent of the divisor (denominator) from the exponent of the dividend (numerator).

D. $\frac{a^m}{a^n} = a^{m-n}$

Model 2: $\frac{a^{16}}{a^{14}} = a^{16-14}$ or a^2

Sometimes we end up with a negative exponent or with zero as an exponent.

Model 3: $\frac{x^8}{x^{10}} = x^{8-10}$ or x^{-2}

Model 4: $\frac{x^4}{x^4} = x^{4-4}$ or x^0

Explaining the negative exponents and the 0 exponent as the number of factors occurring does not make sense. How could x^{-2} be a number with x as a factor -2 times? How could x^0 be a number with x as a factor zero times? Negative and zero exponents are defined by the next law of exponents.

E. $a^m = \frac{1}{a^{-m}}$ when $m < 0$; and $a^0 = 1$.

If $m < 0$, then $\frac{1}{a^{-m}}$ will be a fraction with 1 as the numerator and with a positive exponent in the denominator.

Model 1: $x^{-2} = \frac{1}{x^{-(-2)}}$ or $\frac{1}{x^2}$

Model 2: $\frac{x^4}{x^4} = x^0$ or 1

In an algebraic expression, you may sometimes wish to replace the negative exponent by the reciprocal to obtain positive exponents only. A principle of quotients can be applied.

PRINCIPLE OF QUOTIENTS

$\frac{xy}{ab} = \frac{x}{a} \cdot \frac{y}{b}$ $\quad a \neq 0, b \neq 0$

Model 1:
$$\frac{ab^{-2}c^4de^0}{a^2b^4c^{-2}e^{-3}} = \frac{a}{a^2} \bullet \frac{b^{-2}}{b^4} \bullet \frac{c^4}{c^{-2}} \bullet d \bullet \frac{1}{e^{-3}}$$
$$= \frac{a}{a^2} \bullet \frac{1}{b^2b^4} \bullet \frac{c^4c^2}{1} \bullet \frac{d}{1} \bullet \frac{1 \bullet e^3}{1}$$
$$= \frac{c^6de^3}{ab^6}$$

Exponents may be variables. If they are variables, they will be treated as any other exponents.

Model 2:
$$\frac{x^{3a}}{x^{-2a}} = x^{3a-(-2a)} = x^{5a}$$

Model 3:
$$\left(\frac{2x^ay^{-b}}{3y^{2b}}\right)^d = \left(\frac{2x^ay^{-3b}}{3}\right)^d$$
$$= \frac{2^dx^{ad}y^{-3bd}}{3^d}$$
$$= \frac{2^dx^{ad}}{3^dy^{3bd}}$$

State the value of each expression.

1.1 7^0 ________________

1.2 $5^0 \bullet 2$ ________________

1.3 2^{-2} ________________

1.4 $\frac{4}{2x^0}$ ________________

1.5 $2^{-4} \bullet 18^0$ ________________

1.6 $(12a)^0$ ________________

1.7 $(2 \bullet 5)^{-2}$ ________________

1.8 $2^{-2} + 5^{-2}$ ________________

1.9 $3^0 \bullet 10^{-4}$ ________________

1.10 $\frac{1}{2^0 + x^0}$ ________________

Express with positive exponents.

1.11 a^{-4} ________________

1.12 $\left(\frac{1}{2}\right)^{-1}$ ________________

1.13 $3a^2b^{-1}$ ________________

1.14 $\left(\frac{3a^2}{5}\right)^{-3}$ ________________

1.15 $\frac{x^{-1}y^{-2}}{z^{-3}}$ ________________

1.16 $(6a^2)^{-4}$ ________________

1.17 $\frac{2a^{-4}}{5b^{-2}}$ ________________

1.18 $3(x + y)^{-2}$ ________________

1.19 $\frac{a^{6n}}{a^{3n}}$ ________________

1.20 $(a^4b^5)^n$ ________________

1.21 $(5a^{3n})^3$ ________________

1.22 $\frac{b^{x+5}c}{b^xc}$ ________________

1.23 $\left(\frac{x^{n+5}}{y}\right)^2$ ________________

Find the value when $x = 2$ and $y = 3$.

1.24 y^0 ______________________ **1.25** $2x^0y^{-2}$ ______________________

1.26 $x^{-2}y^{-2}$ ______________________ **1.27** $(\frac{2}{3} + 5y)^0$ ______________________

1.28 $3x^{-1} + 2y^{-1}$ ______________________ **1.29** $x^{-3}y^{-3}$ ______________________

1.30 $x^{-3} + y^{-3}$ ______________________ **1.31** $(x + y)^{-3}$ ______________________

REDUCING RATIONAL EXPRESSIONS

A fraction is the quotient of two numbers. In arithmetic, these two numbers were generally integers.

Models: $\frac{2}{3}, \frac{7}{8}, \frac{4}{6}, \frac{9}{12}, \frac{11}{7}, \frac{100}{12}$

You have already learned that any fraction with a common factor for the numerator and denominator can be reduced.

Model 1: $\frac{4}{6} = \frac{2}{3}$ The common factor for 4 and 6 is 2.

Model 2: $\frac{9}{12} = \frac{3}{4}$ The common factor for 9 and 12 is 3.

Model 3: $\frac{100}{12} = \frac{25}{3}$ The common factor for 100 and 12 is 4.

Of course, as you saw in the models, the numerator and denominator are each divided by the largest common factor to give us the reduced or simplified fraction.

You have learned how to factor polynomials. When the numerator or the denominator of a fraction, or both, contain variables, we can simplify these fractions by dividing the numerator and denominator by the largest common factor of each.

Model 1: $\frac{2x + 4}{8} = \frac{\overset{1}{\cancel{2}}(x + 2)}{\underset{4}{\cancel{8}}}$ or $\frac{x + 2}{4}$

Model 2: $\frac{x^2 - x - 12}{3x + 9} = \frac{\overset{1}{\cancel{(x + 3)}}(x - 4)}{3\underset{1}{\cancel{(x + 3)}}}$ or $\frac{x - 4}{3}$

Model 3: $\frac{3x^3 - 81y^3}{2x^2 - 18y^2} = \frac{3\overset{1}{\cancel{(x - 3y)}}(x^2 + 3xy + 9y^2)}{2(x + 3y)\underset{1}{\cancel{(x - 3y)}}} = \frac{3(x^2 + 3xy + 9y^2)}{2(x + 3y)}$

In Model 2 and Model 3, we have variables in the denominator. Whenever this condition occurs, we must identify the **exclusions**, both for the original denominators and for the denominators that result from working a problem or reducing a fraction. Exclusions are values of the variable that would make the denominator equal to zero. You may recall that division by 0 is not considered in our number system because the division would result in an undefined numeral.

DEFINITION

Exclusion: a value of a variable in a denominator that would make the denominator equal zero.

Model 1: $10 \div 0$ What number multiplied by 0 will result in 10? No such number exists and therefore division by 0 is undefined.

Model 2: $\frac{x^2 - x - 12}{3x + 9}$

x cannot equal -3; in notation form, $x \neq -3$.

Model 3: $\frac{3x^3 - 81y^3}{2x^2 - 18y^2} = \frac{3(x - 3y)(x^2 + 3xy + 9y^2)}{2(x + 3y)(x - 3y)} = \frac{3(x^2 + 3xy + 9y^2)}{2(x + 3y)}$

$x \neq 3y$ for the original denominator,
$x \neq -3y$ for the final denominator;
or in terms of y, $y \neq \frac{x}{3}$ and $y \neq -\frac{x}{3}$.

Reduce these fractions.

1.32 $\frac{3}{18}$ ______________________

1.33 $\frac{15}{45}$ ______________________

1.34 $\frac{14}{18}$ ______________________

1.35 $\frac{56}{64}$ ______________________

1.36 $\frac{34}{119}$ ______________________

1.37 $\frac{180}{216}$ ______________________

1.38 $\frac{3{,}500}{5{,}000}$ ______________________

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