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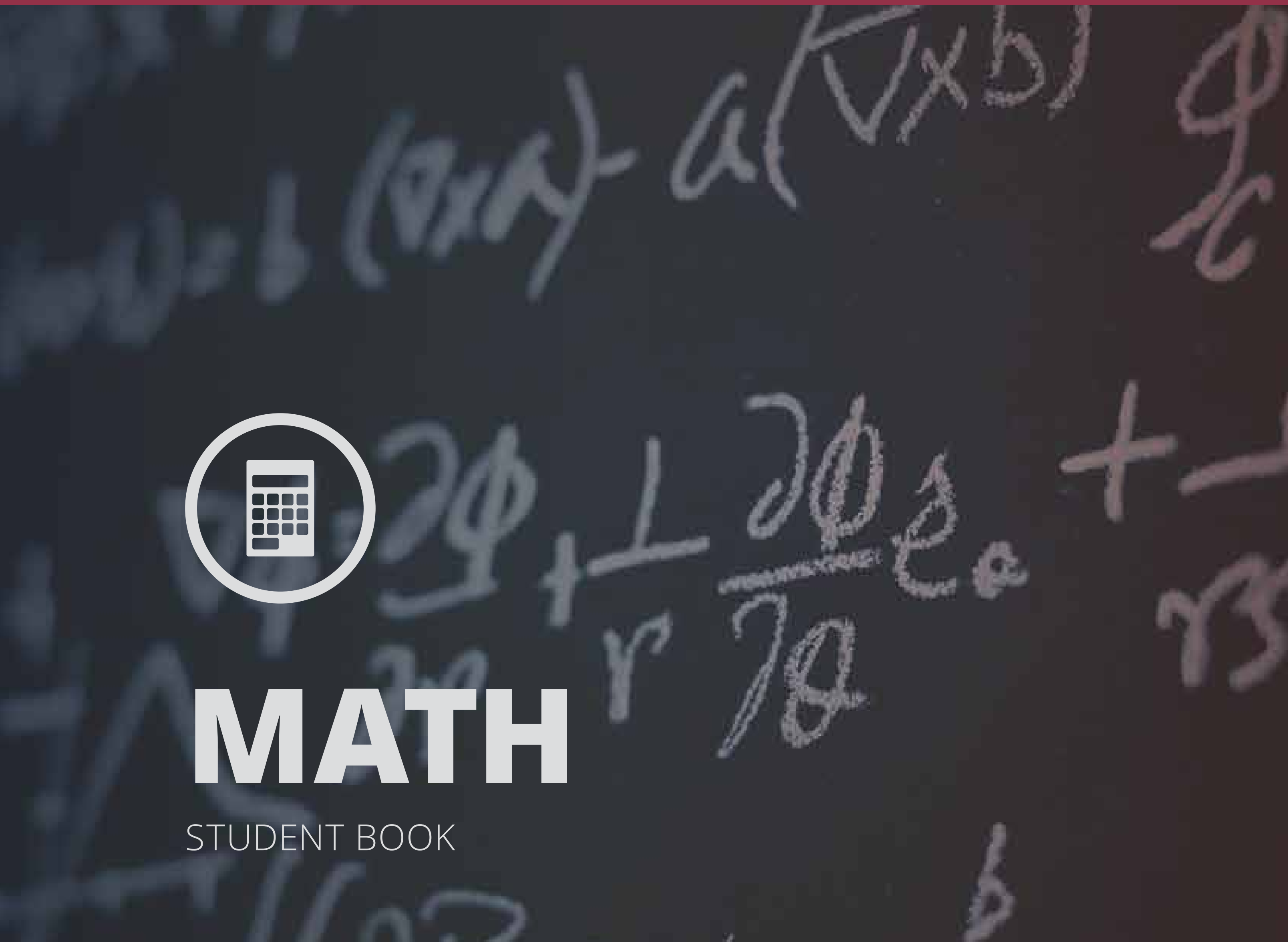

MATH

STUDENT BOOK

11th Grade | Unit 8

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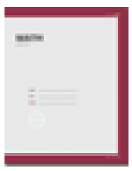
MATH 1108

EXPONENTIAL FUNCTIONS



LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Exponential Functions

Introduction

In this LIFEPAC® you will study three important concepts in math: exponential functions, logarithmic functions, and matrices. You have perhaps heard of the exponential rate of growth that is associated with exponential functions. Closely related to exponential functions are logarithmic functions. Logarithms were developed by John Napier in the seventeenth century. At that time, many discoveries in astronomy were being made with the aid of the telescope and logarithmic calculations. Today we have the computer to perform calculations with the aid of matrices, which are rectangular arrays of numbers. You will study both operations with matrices and applications of matrices.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Evaluate and simplify math expressions using positive integer exponents.
2. Evaluate and simplify math expressions and solve exponential equations using radicals and integral and rational exponents.
3. Graph exponential functions.
4. Express logarithmic relations as exponential relations and exponential relations as logarithmic relations.
5. Evaluate and simplify math expressions using logarithms.
6. Find logarithms and antilogarithms in logarithm tables.
7. Use scientific notation in logarithmic calculations.
8. Perform calculations and solve equations using logarithms.
9. Graph logarithmic functions.
10. Solve selected problems using exponential functions, logarithmic functions, logarithm tables, and theorems about logarithms.
11. Solve systems of linear equations using matrices.
12. Add and multiply matrices.
13. Apply and interpret matrix operations with respect to business applications.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. EXPONENTIAL FUNCTIONS

Exponential functions are used in a wide range of applications. Some of these applications occur in business, chemistry, biology, and statistics. In business, the exponential function $y = e^x$ ($e \doteq 2.71828$, the **natural number**) is used in calculating the earnings of money invested at continuous compound interest. In chemistry, $y = e^{-x}$ is used in calculating the half-life of radioactive material. In biology, $y = e^x$ is used to calculate the amount of bacteria present after a period of growth. In statistical studies, used in almost every career area, $y = e^{x^2}$ is the equation of a **normal curve**, which is the standard arrangement of data for many experiments or surveys. For many years most areas of physical science have used the exponential function $y = 10^x$, which is the basis for common logarithms.

Your study of exponential functions will begin with a definition of exponents. You should then be ready to solve and graph exponential equations and observe some applications of exponents.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Evaluate and simplify math expressions using positive integer exponents.
2. Evaluate and simplify math expressions and solve exponential equations using radicals and integral and rational exponents.
3. Graph exponential functions.

EXPONENTS

Rules for exponents will be helpful in your study of this LIFEPAC. Fractional exponents (exponents that are fractions) will also be presented.

RULES FOR EXPONENTS

You no doubt remember the following definition of exponents.

DEFINITION

When x is a positive integer such as 1, 2, 3, ... $\quad b^x = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{x}$

Models: $b^1 = b$

$b^2 = b \cdot b$

$b^3 = b \cdot b \cdot b$

In the expression b^x, b is called the *base* and x is called the *exponent*.

Remember?

To expand the use of exponents, mathematicians have agreed upon the following definitions, which are also related to exponents.

DEFINITIONS

A. If $b \neq 0$, $b^0 = 1$.

B. If x is an integer and $b \neq 0$, then $b^{-x} = \frac{1}{b^x}$ and $\frac{1}{b^{-x}} = b^x$.

A. Models: $2^0 = 1$

$5^0 = 1$

B. Models: $b^{-2} = \frac{1}{b^2}$ if $b \neq 0$

$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

$\frac{1}{2^{-4}} = 2^4$

With these definitions we can prove the following rules (theorems) about exponents.

If x and y are integers, then

1. $b^x \cdot b^y = b^{x+y}$ (To multiply with like bases, add the exponents.)
2. $(b^x)^y = b^{x \cdot y}$ (To raise a base to another power, multiply the exponents.)
3. $\frac{b^x}{b^y} = b^{x-y}$, $b \neq 0$ (To divide with like bases, subtract the exponents.)
4. $(b \cdot a)^x = b^x \cdot a^x$ (To raise a product to a power, raise each factor to the power.)

The following models show how to use the definitions and theorems of this section to evaluate or simplify expressions. Answers are expressed with positive exponents.

Model 1: $(4 + 6)^0 = 10^0 = 1$

Model 2: $5 \cdot 4^0 = 5 \cdot 1 = 5$

Model 3: $2^{-1} + 4^{-2} = \frac{1}{2^1} + \frac{1}{4^2} = \frac{8}{16} + \frac{1}{16} = \frac{9}{16}$

Model 4: $\dfrac{1}{2^{-3}} = \dfrac{1}{\frac{1}{2^3}} = 2^3 = 8$

Model 5: $\left(\dfrac{1}{3}\right)^{-1} = \dfrac{1}{\frac{1}{3}} = 3$

Model 6: $(b \bullet a)^{-1} = \dfrac{1}{(b \bullet a)^1} = \dfrac{1}{ba}$

Model 7: $4x^{-5} = 4 \bullet \dfrac{1}{x^5} = \dfrac{4}{x^5}$

Model 8: $(2^x)^{-2} = \dfrac{1}{(2^x)^2} = \dfrac{1}{2^{2x}} = \dfrac{1}{4^x}$

Model 9: $\dfrac{b^{-4}}{b^{-5}} = b^{-4-(-5)} = b^1$

Model 10: Evaluate the exponential function for $y = 3^x$.

x	1	2	3	0	-1	-2	-3
y							

Graph it for $x = 1, 2, 3, 0$.

Let $x = 1$, $y = 3^1 = 3$

$x = 2$, $y = 3^2 = 9$

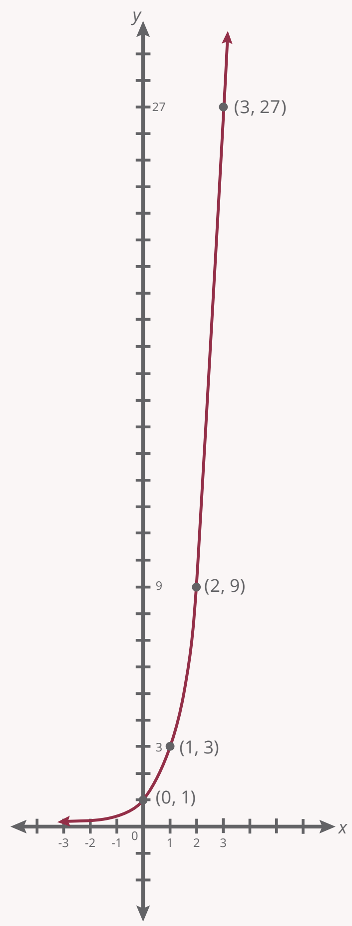
$x = 3$, $y = 3^3 = 27$

$x = 0$, $y = 3^0 = 1$

$x = -1$, $y = 3^{-1} = \frac{1}{3} \doteq 0.3$

$x = -2$, $y = 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \doteq 0.1$

$x = -3$, $y = 3^{-3} = \frac{1}{3^3} = \frac{1}{27} \doteq 0.04$



For $x = -1, -2, -3$, the graph approaches the x-axis from above. To graph this function accurately requires more paper or a change in scale.

Express answers with positive exponents.

1.1 $2^0 + 5^0 =$ ______

1.2 $(2 + 5)^0 =$ ______

1.3 $6 \cdot 5^0 =$ ______

1.4 $3^{-2} =$ ______

1.5 $\frac{1}{3^{-2}} =$ ______

1.6 $(2 + 3)^{-2} =$ ______

1.7 $2^{-3} + 4^{-2} =$ ______

1.8 $4^2 \cdot 2^{-3} =$ ______

1.9 $(2 \cdot 3)^{-2} =$ ______

1.10 $\left(\frac{1}{2}\right)^{-1} =$ ______

1.11 $m \cdot n^{-1} =$ ______

1.12 $(m \cdot n)^{-1} =$ ______

1.13 $2x^{-3} =$ ______

1.14 $(2x)^{-3} =$ ______

1.15 $\frac{b^{-2}}{b^{-3}} =$ ______

1.16 $\frac{a^8}{a^6} =$ ______

1.17 $b^5 \cdot b^8 =$ ______

1.18 $(b^4)^2 =$ ______

1.19 $\frac{2ab^{-1}}{3e^{-3}d} =$ ______

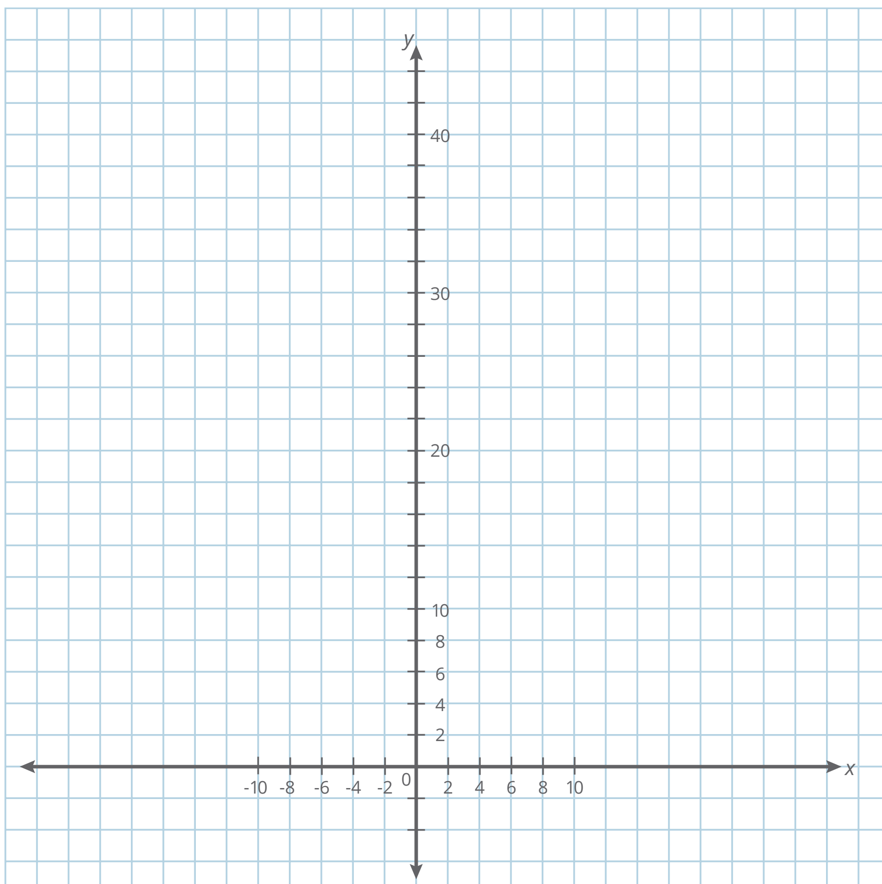
1.20 $\frac{(2x)^{-2}}{(3x)^{-2}} =$ ______

1.21 $\frac{2x^{-2}}{3x^{-2}} =$ ______

Evaluate the exponential function $y = 2^x$ for the values given. Graph.

1.22

x	y
-3	
-2	
-1	
0	
1	
2	
3	
4	



FRACTIONAL EXPONENTS

You probably know the definition of $\sqrt[n]{a}$. Just to review, consider the following definition of a root.

DEFINITION

$\sqrt[n]{a} = x$ if and only if $x^n = a$. $(n \neq 0)$

If two values for x exist, then $\sqrt[n]{a}$ is the positive value. $\sqrt[n]{a}$ is read "the nth root of a."

Model: $\sqrt[3]{8} = x$ such that $x^3 = 8$. By trial and error and knowledge of multiplication, we know that $2^3 = 8$. So $\sqrt[3]{8} = 2$.

To extend the usefulness of exponents, the following definition for fractional exponents is presented.

DEFINITION

$a^{\frac{1}{n}} = \sqrt[n]{a}$ $(n \neq 0, n = \text{integer})$

Model: $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

From the preceding definitions, the following properties hold for fractional exponents and for roots.

PROPERTIES

$\left(a^{\frac{1}{n}}\right)^n = a^{\frac{n}{n}} = a^1$

$\left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}} =$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$a^x b^x = (ab)^x$; $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Definitions A and B and Theorems 1, 2, 3, and 4, shown in the section "Rules for Exponents," hold for fractional exponents also.

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