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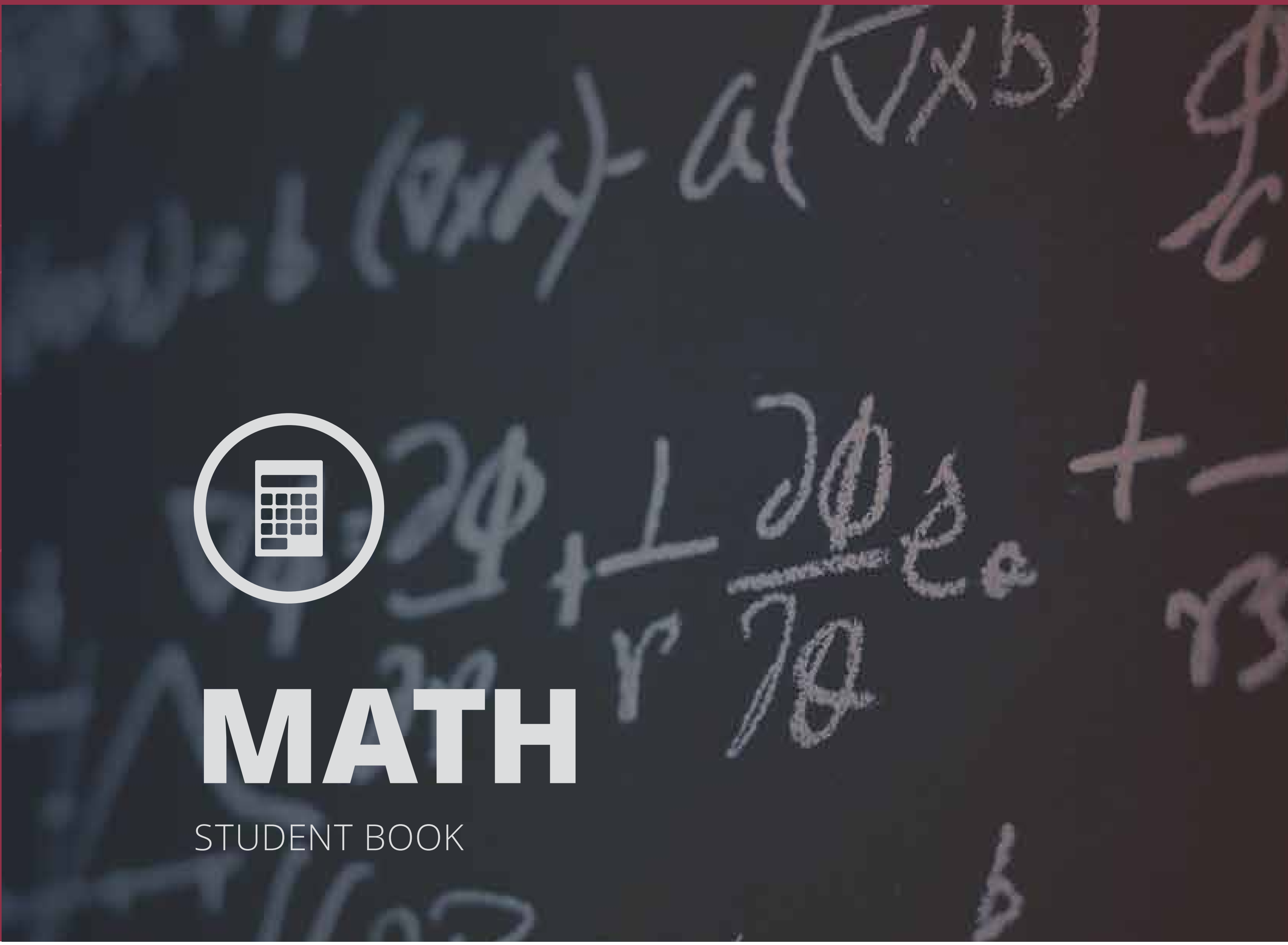

MATH

STUDENT BOOK

12th Grade | Unit 1

Alpha Omega
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MATH 1201
RELATIONS AND FUNCTIONS

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Relations and Functions

Introduction

In this first unit of Pre-Calculus, you will start by learning about Relations and Functions; their similarities, differences, and relation with one another. The second half of the unit focuses on the Algebra of Functions, and you will notice the algebraic functions that you are familiar with from previous math courses can be applied to functions, just like numbers and expressions. Much of the material in this unit will serve as a base for topics covered in later units. Read carefully, take your time, and enjoy! Welcome to Pre-Calculus.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Identify elements of sets.
2. Understand how to make an ordered pair from a set.
3. Identify relations between ordered pairs.
4. Locate ordered pairs in the Cartesian Plane.
5. Solve for the domain and range of ordered pairs.
6. Determine if a relation is a function.
7. Distinguish between linear and quadratic functions.
8. Write equations for linear and quadratic functions.
9. Recognize function notation.
10. Utilize function notation to solve for dependent variable values.
11. Identify equal functions.
12. Apply arithmetic operations to equal functions.
13. Define function composition.
14. Combine functions via composition.
15. Distinguish between zero, constant, and identity functions.
16. Find the inverse of a function.
17. Determine whether or not the inverse of a function is a function.
18. Use composition of functions to verify that two functions are inverses of each other.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. ORDERED-PAIR NUMBERS

ORDERED-PAIR NUMBERS: RELATIONS

You have learned many useful concepts and facts in previous courses. We learn facts and then principles which we can apply in various situations whether the circumstances of the situation are familiar or unknown. “Do not lie” and “do not kill” are rules to guide decision-making under different circumstances. Math also has rules to guide your problem solving.

For example, you have learned facts about the function which you will now learn to apply to new problems. In this unit, you will investigate more advanced concepts of the function. You will also learn their application in practical terms by recognizing the cost of food at the supermarket is a function of its weight, volume, or some other measure. Thus, there is a correspondence between a measurement and the price of a product according to a specific rule. This is just one example of how a function is used in everyday life. The concept of a function involves the correspondence between two sets.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Identify elements of sets.
2. Understand how to make an ordered pair from a set.
3. Identify relations between ordered pairs.
4. Locate ordered pairs in the Cartesian Plane.
5. Solve for the domain and range of ordered pairs.

Vocabulary

Study these words to enhance your learning success in this section.

domain . . . . The set of all first numbers of an ordered pair.

element . . . . An object in a set.

function . . . . Any relation in which no two ordered pairs have the same first element.

ordered pair . . . . A pair of numbers that are grouped together.

range . . . . The set of all second elements of a relation.

relation . . . . Any set of ordered-pair numbers.

R* × *R . . . . All the possible combinations of real numbers denoting ordered pairs.

set . . . . A group or collection of objects.

subset . . . . A set within a set; all the elements of one set are also contained in another set.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

SETS AND ORDERED PAIRS

In order to understand **functions** and function notation, a brief review of **sets** and ordered pairs is presented.

This symbol, $\in$, means "is an **element** of."

To name a set: Use a capital letter followed by an equal sign. After the equal sign use braces, and inside the braces write either a description of the elements in the set or a listing of the elements in the set. Make sure to put commas between each element within this set.

For example, S = {Sam, Jill, Kim, Fred} or
S = {the names of four students}.

And we can say Fred $\in S$, or Jill $\in S$, and so on. (read: "Fred is an element of set S.")

An **ordered pair** is a pair of numbers that are grouped together.

To write an ordered pair: The two numbers are written within a set of parentheses and separated by a comma.

For example, **(6, 2)** is an ordered pair.

The order is designated by the first element 6 and the second element 2.

Another example of an ordered pair is **(2, 6)**.

This ordered pair is not the same as **(6, 2)** because of the different ordering. In this ordered pair, 2 is the first element and 6 is the second element. This makes a *BIG* difference.

ORDERED PAIR RELATIONS

Sets of ordered pairs can represent relations or functions. Some relations and functions are defined by rules of correspondence.

A **relation** is any set of ordered pairs. The members of each pair are related to each other in some way.

Here are the weights of the students recorded by the school nurse.

Student	1	2	3	4
Weight	150	130	100	160

The pairing of the student number and his corresponding weight is a relation and can be written:

$$A = \{(1, 150), (2, 130), (3, 100), (4, 160)\}$$

These data are written as a set of ordered pairs where each element of the set is an ordered pair.

The set of all first numbers of each ordered pair is called the **domain** of the relation.

In the previous example, the first element of each ordered pair is the student number.

Therefore, the domain of A = {1, 2, 3, 4}.

If all the domain values are the same you need to write the value only once.

The set of second numbers of each ordered pair is called the **range** of the relation.

In the previous example, the second element of each ordered pair is the student's weight.

Therefore, the range of A = {150, 130, 100, 160}.

If all the range values are the same, you write the value only once.

For example, suppose that all the students had the same weight of 130 lbs. Then the range of A = {130}.

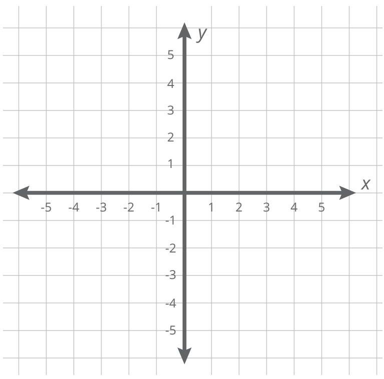
ORDERED PAIRS IN THE CARTESIAN PLANE

Now let's look at set R where R is the set of real numbers. You can create **the relation of ordered pairs of real numbers that is a subset of $R \times R$**. This new subset will be called a relation on set R. Since we are talking about subsets, remember any set B is a **subset** of C if all the elements in B are also elements in C.

For example, let B = {2, 4, 6} and let C = {1, 2, 3, 4, 5, 6, 7, 8}.

B is a subset of C because all the elements in B, namely 2, 4, 6, are also elements in C.

To visualize $R \times R$, think of a horizontal number line extending infinitely in both directions, crossed by a vertical number line extending infinitely in both directions. This is also known as a **Cartesian** plane.



The expression $R \times R$ is read "R cross R," and it contains all the possible combinations of real numbers denoting ordered pairs.

Each positive and negative real number on the horizontal number line can be paired with each positive and negative real number on the vertical number line to give all possible combinations of two real numbers. Any relation that consists of ordered pairs of real numbers, therefore, is a subset of $R \times R$.

In mathematics, many relations can be specified or defined by a rule (which is usually some type of equation) which allows you to determine the element or elements of the range paired with each element of the domain. If, for example, an ordered pair specified by (x, y) is described by an equation $y = x + 2$, you can determine more than one value for the range (i.e., y) by changing the values selected for the domain x. The values selected for x may be arbitrary, but the values for y depend on the values selected for x. Therefore, x is called the **independent** variable and y is called the **dependent** variable.

For example, the solution set over $R \times R$ of an open sentence such as $x - y = 4$ is a relation L that you can define as follows:

$L = \{(x, y): (x, y) \in R \times R \text{ and } x - y = 4\}$, which is read:

"the set L equals the set of ordered pairs (x, y) such that (x, y) is an element of $R \times R$ and $x - y = 4$" **or**

$L = \{(x, y): y = x - 4, x \in R\}$, which is read:

"the set L equals the set of ordered pairs (x, y) such that $y = x - 4$ and x is an element of R."

The **domain** of the relation L is the set of real numbers.

Domain of data = {Real numbers}

The **range** of the relation L is the set of numbers $x - 4$, where x is any real number.

Range of data = $\{x - 4; \text{ where } x \in R\}$

SOLVING FOR DOMAIN AND RANGE

Whenever an open sentence specifies a relation whose domain and range are not explicitly stated, we agree to include in the domain and range those real numbers and *only* those real numbers for which the open sentence is true.

STUDY THESE EXAMPLES:

1. Determine (**a**) the domain and (**b**) the range of

$$H = \{(x, y): y = \sqrt{1 - x^2}\}.$$

Solution:

a. An ordered pair is a real number value; therefore, each value of y is to be a real number. Therefore, we have to determine whether there are any values of x that will make the value of y be something other than a real number. In this case, if the radicand is negative then y is not a real number. Therefore, each value of $1 - x^2$ must be a non-negative number.

Since $x^2 = |x|^2$, $1 - x^2 \geq 0$ means $1 - |x|^2 \geq 0$ (by substitution).

$|x|^2 \leq 1$ (by subtraction and division).

Then $|x| \leq 1$ (by taking the square root).

Therefore, the domain of H is $\{x: -1 \leq x \leq 1\}$.

b. Since $|x| \leq 1$, this implies $0 \leq x^2 \leq 1$. Then $0 \leq 1 - x^2 \leq 1$.

Therefore, the range of H is $\{y: 0 \leq y \leq 1\}$.

2. State the domain of

$$F = \{(x, y): y = \frac{3x}{(x - 2)(x + 5)}\}.$$

Solution:

Since each value of y is to be a real number, $(x - 2)(x + 5) \neq 0$.

If $(x - 2)(x + 5) = 0$, then y would be undefined because whenever a fraction has a denominator of zero it is undefined. Now, $(x - 2)(x + 5) = 0$ when $x = 2$ or $x = -5$. Therefore, the domain of F consists of all the real numbers except 2 and -5.

SUMMARY

1. If the relation is a fraction, then you must exclude all the domain values that make the denominator zero.
2. If the relation is a square root radical, then you must exclude all the domain values that make the radicand negative.

Match the following.

1.1 ____________ the set of all first numbers of each ordered pair

1.2 ____________ an object in a set

1.3 ____________ any relation in which no two ordered pairs have the same first element

1.4 ____________ a pair of numbers that are grouped together

1.5 ____________ the set of all second elements of a relation

1.6 ____________ any set of ordered-pair numbers

1.7 ____________ all the possible combinations of real numbers denoting ordered pairs

1.8 ____________ a group or collection of objects

1.9 ____________ a set within a set; all the elements of one set are also contained in another set

a. range
b. $R \times R$
c. relation
d. ordered pair
e. function
f. element
g. set
h. domain
i. subset

Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

Complete the following activities.

1.10 Write the domain of the following relation in list form.
{(5, 0), (6, 1), (7, 2)}

Domain = ______

a. {0, 1, 2}
b. {5, 6, 7}
c. {0, 1, 2, 5, 6, 7}

1.11 Write the range of the following relation in list form.
{(5, 0), (6, 1), (7, 2)}

Range = ______

a. {0, 1, 2}
b. {5, 6, 7}
c. {0, 1, 2, 5, 6, 7}

1.12 Write the domain of the following relation in list form.
$\{(6, \sqrt{2}), (7, \sqrt{3}), (8, \sqrt{4}), (9, \sqrt{5})\}$

Domain = ______

a. {6, 7, 8, 9}
b. $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$
c. $\{6, 7, 8, 9, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$

1.13 Write the range of the following relation in list form.
$\{(6, \sqrt{2}), (7, \sqrt{3}), (8, \sqrt{4}), (9, \sqrt{5})\}$

Range = _______

a. $\{6, 7, 8, 9\}$
b. $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$
c. $\{6, 7, 8, 9, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$

1.14 The domain of the following relation has how many elements? _______
$\{(\frac{1}{2}, \frac{\pi}{6}), (\frac{1}{2}, \frac{\pi}{4}), (\frac{1}{2}, \frac{\pi}{3}), (\frac{1}{2}, \frac{\pi}{2})\}$

a. 0
b. 1
c. 4

1.15 The range of the following relation has how many elements? _______
$\{(\frac{1}{2}, \frac{\pi}{6}), (\frac{1}{2}, \frac{\pi}{4}), (\frac{1}{2}, \frac{\pi}{3}), (\frac{1}{2}, \frac{\pi}{2})\}$

a. 0
b. 1
c. 4

1.16 Write the domain of the following relation in list form.
$\{(6.2, 0.3), (7.3, 0.3), (8.4, 0.3), (9.5, 0.3)\}$

Domain = _______

a. $\{6, 7, 8, 9\}$
b. $\{0.3\}$
c. $\{6.2, 7.3, 8.4, 9.5\}$

1.17 Write the range of the following relation in list form.
$\{(6.2, 0.3), (7.3, 0.3), (8.4, 0.3), (9.5, 0.3)\}$

Range = _______

a. $\{0.3\}$
b. $\{6.2, 7.3, 8.4, 9.5\}$
c. $\{1\}$

1.18 Write a relation in ordered-pair form for six different packages of fruit.
The domain of the relation is $D = \{10, 15, 20, 30, 60, 90\}$, where the elements represent the weights of the packages.
The range of the relation is $R = \{2, 3, 4, 6, 8, 10\}$, where the elements represent the cost respectively of each package in dollars.

Q = _______

a. $\{(10, 2), (15, 3), (20, 4), (30, 6), (60, 8), (90, 10)\}$
b. $\{(2, 10), (3, 15), (4, 20), (6, 30), (8, 60), (10, 90)\}$
c. $\{(2, 3), (4, 6), (8, 10), (10, 15), (20, 30), (60, 90)\}$

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