LIFEPAC®

MATH

STUDENT BOOK

12th Grade | Unit 2

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MATH 1202
FUNCTIONS

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Functions

Introduction

This LIFEPAC® reviews various types of functions, including linear, quadratic, polynomial, exponential, and logarithmic functions. You will learn how to graph each of these types of functions and solve equations and inequalities in which they appear. You will also learn about complex numbers and their importance in solving polynomial equations.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Identify and solve polynomial, exponential, and logarithmic equations.
2. Graph polynomial, exponential, and logarithmic equations.
3. Recognize various transformations to basic polynomial, exponential, and logarithmic functions and draw appropriate graphs.
4. Use synthetic division to find the roots of a polynomial equation, as well as upper and lower limits on those roots.
5. Geometrically represent basic arithmetic operations of complex numbers on the complex plane.
6. Convert between rectangular and polar forms of complex numbers.
7. Use the complex conjugate to simplify fractions and find the modulus of a complex number.
8. Use interval notation to express the solution to polynomial and rational inequalities in mathematical and real-world contexts.
9. Identify and graph the greatest integer function.
10. Combine functions using basic arithmetic operations.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. LINEAR FUNCTIONS

GRAPHS OF LINEAR FUNCTIONS

As we continue studying the concept of functions, we will first look at the polynomial functions of degrees one and two. After that, we will study higher-degree polynomial functions. Finally, we will study several special functions including the greatest integer function, the exponential function, the logarithmic function, and combinations of these three functions.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Identify polynomial functions.
- Solve linear polynomial functions.

Vocabulary

Study these words to enhance your learning success in this section.

linear function . . . . A polynomial function of the form $y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, where $a_0 \neq 0$, and $n = 1$.

***x*-intercept** . . . . The value of x when $y = 0$; the value of the first coordinate of the point where the line crosses the x-axis.

***y*-intercept** . . . . The value of y when $x = 0$; the value of the second coordinate of the point where the line crosses the y-axis.

zero of a function . . . . A value of x for which the function value is zero; a root or solution of $f(x) = 0$.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

WHAT ARE POLYNOMIAL FUNCTIONS?

The **polynomial functions** are a group of functions in the form

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n,$$

where $a_0, a_1, a_2, \ldots$ are numerical coefficients and n, a positive integer, is the exponent of the variable x. The number n then becomes the degree of the polynomial. For example, a second-degree polynomial would have the form $a_0x^2 + a_1x + a_2$, where a_0, a_1, and a_2 are real-number coefficients.

If $a_0 = 2$, $a_1 = 4$, and $a_2 = -3$, the polynomial would be:

$$2x^2 + 4x - 3$$

SOLVING LINEAR POLYNOMIAL FUNCTIONS

If $a_0 \neq 0$, and $n = 1$, the expression is a linear equation or linear function. Its degree is one, so you can also call it a **first-degree polynomial**. A **linear function** in x is one that can be written in the standard form $y = mx + b$, $m \neq 0$, and where m and b are constants.

Some examples of linear functions are $y = 3x + 2$, $x + y = 6$, and $7x = 3y$. For practice, identify a_0, a_1, and n in each of these examples:

1. $y = 3x + 2$
2. $x + y = 6$
3. $7x = 3y$

You should have found the following:

1. $a_0 = 3$, $a_1 = 2$, $n = 1$
2. $a_0 = -1$, $a_1 = 6$, $n = 1$
3. $a_0 = \frac{7}{3}$, $a_1 = 0$, $n = 1$

It is important to remember that the graph of a linear function is a straight line, but not all lines are linear functions. For example, a vertical line is not a linear function because of how functions are defined: if two or more points of a graph lie on the same vertical line, then the x does not determine a unique y value and the relation is not a function. Also, a linear function is a first-degree polynomial.

Example

Graph $x + y = 2$. Find at least three solutions.

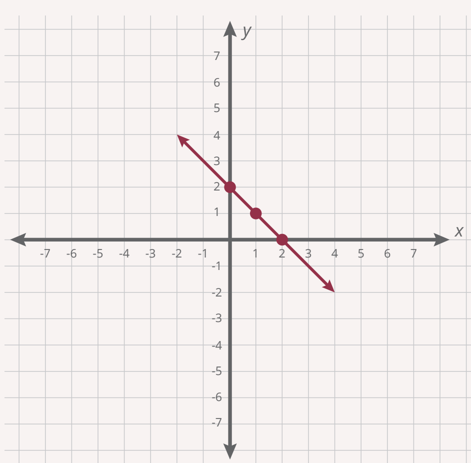
$x + y = 2$. Solve for y.

$y = -x + 2$

Here are three possible solutions:

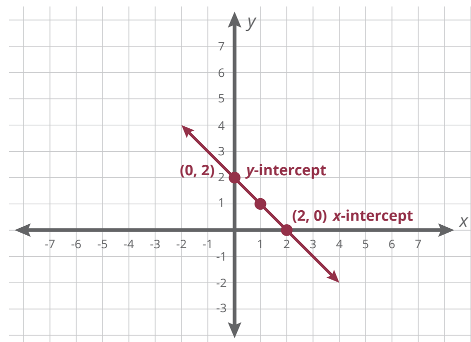
x	y
1	1
0	2
2	0

Locate the solutions as points on the coordinate plane. Do they match the following sample?



When working with functions, there will be certain solutions that will be important for you to identify. With linear equations, it is important that you identify the **zeros of the function**. The zero of a function is a value of x for which the function $f(x) = 0$. Thus, the zeros of a function are **roots** or **solutions** of $f(x) = 0$.

The zero of the function in the preceding example, $x + y = 2$, or, the value of x when $y = 0$, is 2. That is, $f(x) = 0$ when $x = 2$. This value, (2, 0), is also called the ***x*-intercept** and is the point where the line crosses the x-axis. Similarly, we can solve for the point where the line crosses the y-axis. This value, (0, 2), is called the ***y*-intercept**.



"Root," "x-intercept," and "zero of the function" all refer to the same point. You can tell it's important, since we have so many names for it! Usually, we say "intercept" when we are talking about a graph, and "root" or "zero" when we are finding the value using algebra. If you are asked for the intercept, you should provide both coordinates. In our example, the y-intercept is (0, 2).

SOLVING LINEAR EQUATIONS THAT INCLUDE FRACTIONS

A special case of a linear function that we need to consider is in the form of a fraction.

Example

Graph $y = \frac{x^2 - 9}{x + 3}$. Find at least three solutions.

Implied in this rule is that $x \neq -3$; therefore, the domain of this function is the set of all real numbers **except** $x = -3$. If we exclude $x = -3$, then we may simplify $y = \frac{x^2 - 9}{x + 3}$ in the following manner.

Factoring the numerator, we get $y = \frac{(x - 3)(x + 3)}{(x + 3)}$, which can be reduced to $y = x - 3$.

Since $y = x - 3$ is a straight line and $x \neq -3$, we have a line with a hole in it.

Three solutions are:

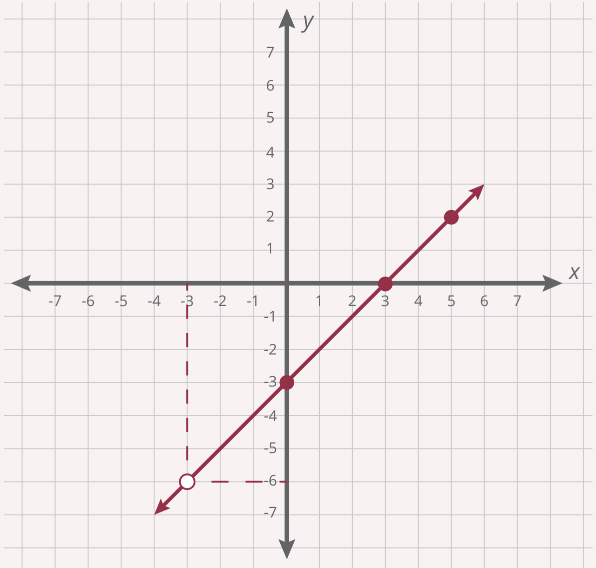
x	*y*
5	2
0	-3
3	0

The zero of this function is 3; that is, $f(x) = 0$ when $x = 3$.

The value of the function at $x = -3$ is undefined.

When you take calculus, you will be able to prove that the closer the value of x gets to -3, the closer the function value, y, will get to -6.

The graph of this function looks like this:



Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

Complete the following activities.

1.1 A function of the form $y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, where $a_0 \neq 0$, and $n = 1$, is called a ________ .

a. second-degree polynomial b. root c. linear equation d. *y*-intercept

1.2 Which of the following is a third-degree polynomial? ________

a. $y = 2x - 5$ b. $y = 3x^2 + 2x - 5$ c. $y = x^3 - 3x^2 + 2x - 5$ d. $y = 3x^4 + x^3 - 3x^2$

1.3 Find three solutions for the following linear function and use them to draw its graph.

$F(x) = x + 3$

x	*y*

1.4 Find three solutions for the following linear function and use them to draw its graph.

$F(x) = 3x - 2$

x	*y*

1.5 Find three solutions for the following linear function and use them to draw its graph.

$F^{-1}(x)$ if $F(x) = x + 3$

x	*y*

1.6 If $F(x) = x + 3$, what is the zero of $F^{-1}(x)$? _______

1.7 What is the *y*-intercept of $G(x)$ if $G(x) = 3x - 2$? _______

1.8 Find three solutions for the following linear function and use them to draw its graph.

$g(x) = \frac{3x - 5}{4}$

x	*y*

1.9 Find three solutions for the following linear function and use them to draw its graph.

$h(x) = \dfrac{x^2 - 16}{x - 4}$

x	***y***

1.10 Find three solutions for the following linear function and use them to draw its graph.

$A(x) = \dfrac{9x^2 - 25}{3x + 5}$

x	***y***

1.11 If $G(x) = \dfrac{3x - 5}{4}$, then $G^{-1}(x) =$ _______ .

a. $\dfrac{4x - 5}{3}$

b. $\dfrac{4x + 5}{3}$

c. $\dfrac{3x + 5}{-4}$

d. $\dfrac{3x - 5}{4}$

1.12 What is the zero of the function $A(x) = \dfrac{9x^2 - 25}{3x + 5}$? _______

1.13 What is the *y*-intercept of $h(x) = \dfrac{x^2 - 16}{x - 4}$? _______

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