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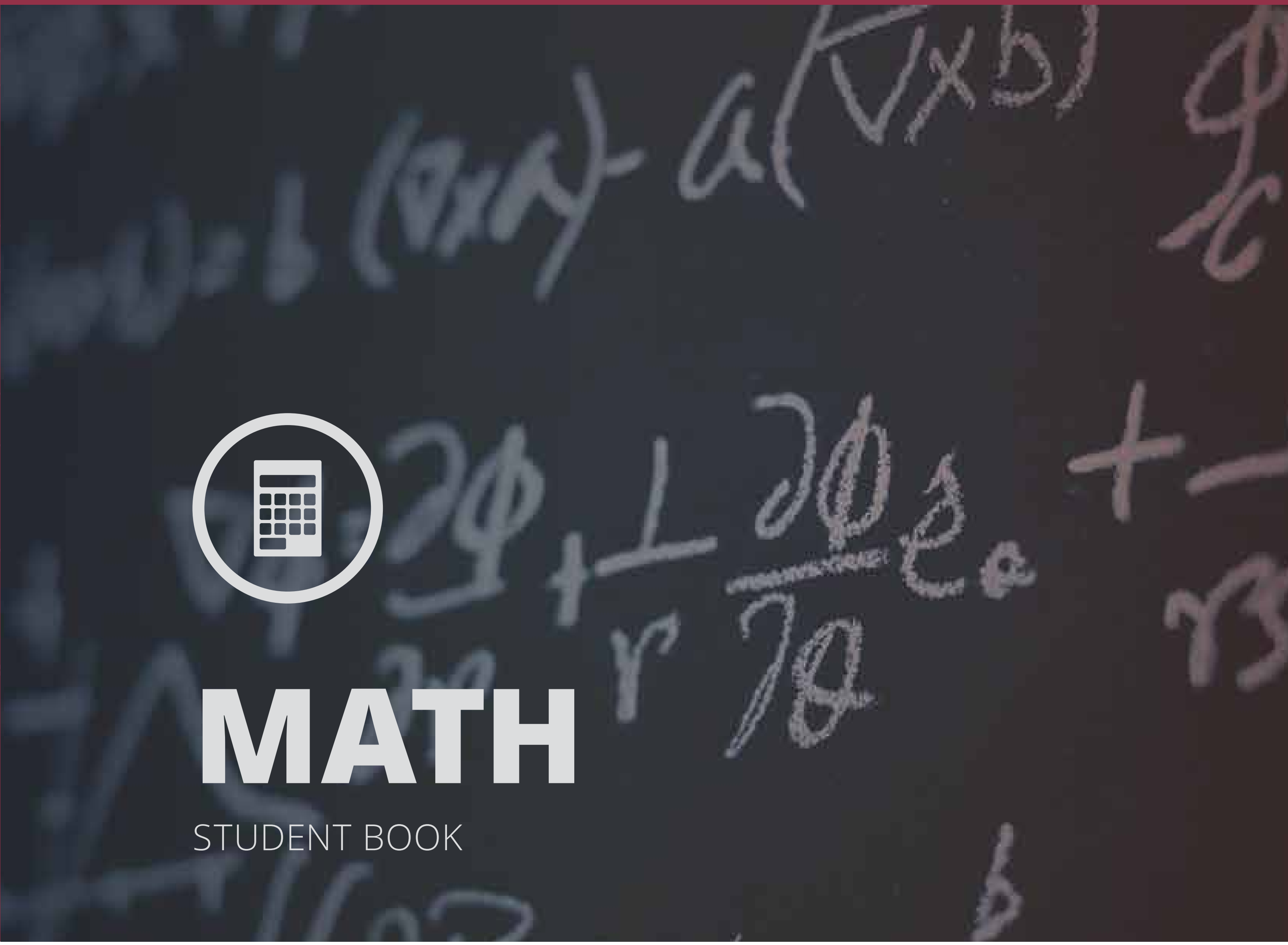

MATH

STUDENT BOOK

12th Grade | Unit 3

Alpha Omega
PUBLICATIONS

MATH 1203

RIGHT TRIANGLE TRIGONOMETRY

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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PUBLICATIONS

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Right Triangle Trigonometry

Introduction

In this unit, the definitions of trigonometric ratios are established and used to determine the measures of angles and sides in right triangles. The unit circle is used on the coordinate plane to evaluate the values of trigonometric ratios. Students are introduced to the Pythagorean trigonometric identities and use them to evaluate values of trigonometric ratios. Lastly, students learn about radian measure and how to apply it to solve problems involving angular velocity.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Use right triangle trigonometry to solve problems.
2. Locate the quadrant in which an angle in standard position terminates.
3. Convert between various forms of angle measure.
4. Express trigonometric functions in terms of acute angles.
5. Determine values of trigonometric functions of an angle from a given point on the terminal side or from one trig function value.
6. Simplify trigonometric expressions.
7. Solve problems involving angular velocity.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. SOLVING A RIGHT TRIANGLE

LENGTHS OF SIDES

What is trigonometry?

Trigonometry is a branch of geometry that deals with triangle measurement. Throughout history, trigonometry ("trig") has been used to, and evolved from the need or desire to, measure that which could not physically be measured. Early astronomers used spherical trig and the chords in a circle to measure distances to stars. Today, plane trig is applied in many fields such as surveying, physics, and engineering.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Express trigonometric functions as ratios in terms of the sides of a right triangle.
- Evaluate trigonometric expressions.
- Use the Pythagorean theorem and trigonometric ratios to calculate side measures in right triangles.

Vocabulary

Study these words to enhance your learning success in this section.

cosine . . . . The trigonometric ratio of adjacent side over hypotenuse for an acute angle in a right triangle.

hypotenuse . . . . The longest side of a right triangle; the side that is opposite the right angle.

sine . . . . The trigonometric ratio of opposite side over hypotenuse for an acute angle in a right triangle.

tangent . . . . The trigonometric ratio of opposite side over adjacent side for an acute angle in a right triangle.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

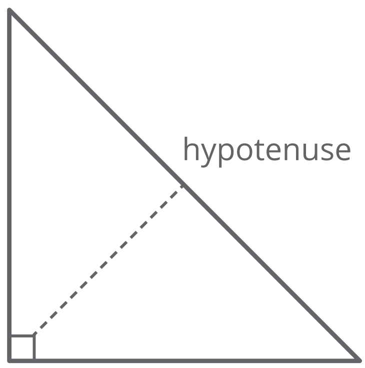
RIGHT TRIANGLES

Let's begin our study of trigonometry with measurements in the right triangle. Let's review some things about the right triangle that you should already be familiar with.

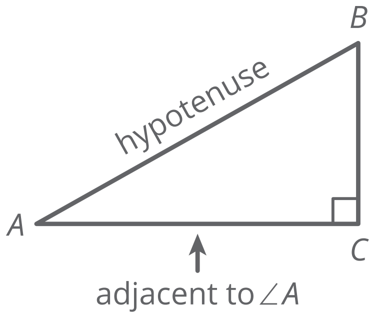
- A right triangle has one right (90-degree) angle and two acute angles.
- The sum of the measures of the two acute angles in a right triangle is 90 degrees. (The sum of the measures of the angles in any triangle is 180 degrees.)
- The longest side of a right triangle is called the **hypotenuse**.
- The two shorter sides of a right triangle are called the legs.
- The Pythagorean theorem ($a^2 + b^2 = c^2$) applies only to right triangles.

Trigonometric ratios express relationships between the sides of a right triangle. Therefore, it is important to be able to identify them.

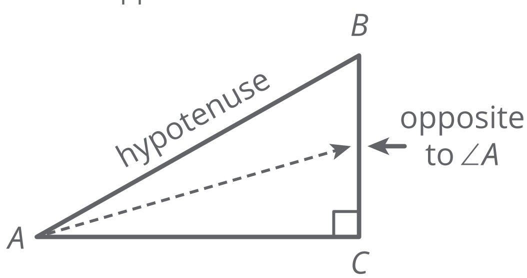
The sides are referred to as the *hypotenuse*, *opposite side*, and *adjacent side*. The hypotenuse is always the longest side of the right triangle and is across from the right angle.



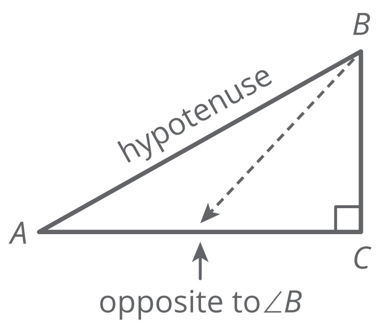
Notice that the hypotenuse always helps to form each of the acute angles. The leg that helps to form an acute angle is called the *adjacent side*.



The remaining leg is called the *opposite side*. Think of going "across" or "through" the triangle from the angle to the opposite side.



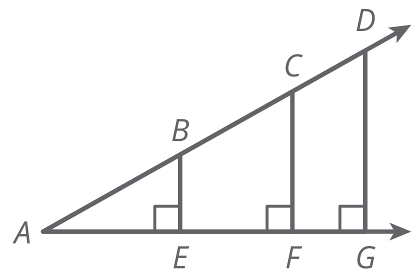
The adjacent and opposite sides switch when using another angle. In the same triangle as above, *AC* is the opposite side in relation to ∠*B*.



TRIGONOMETRIC RATIOS

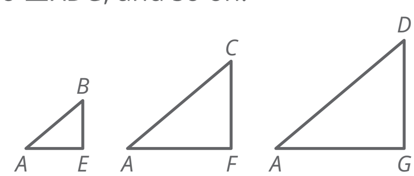
Now that you can identify the opposite and adjacent sides for a given acute angle of a right triangle, let's define the trig ratios.

Given an acute angle, draw parallel lines that are perpendicular to one ray:



From geometry, you should know that if two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

In the diagram below, △*ABE* is similar to △*ACF* is similar to △*ADG*, and so on.



Since the ratios of corresponding sides of similar triangles are equal, $\frac{BE}{AB} = \frac{CF}{AC} = \frac{DG}{AD}$. For ∠*A*, each of these ratios represents the opposite side over the hypotenuse. For all right triangles that can be drawn with the acute ∠*A*, this ratio will always be the same value.

This ratio is called the **sine** of ∠*A* and is written as

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}.$$

By the same reasoning, the ratio of the adjacent side over the hypotenuse is a constant value: $\frac{AE}{AB} = \frac{AF}{AC} = \frac{AG}{AD}$.

This ratio is called the **cosine** of ∠*A* and is written as

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}.$$

The ratio of the opposite side over the adjacent side is also a constant value: $\frac{BE}{AE} = \frac{CF}{AF} = \frac{DG}{AG}$.

This ratio is called the **tangent** of ∠*A* and is written as

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}.$$

Many students use the acronym **SOHCAHTOA** to aid them in remembering the trig ratios.

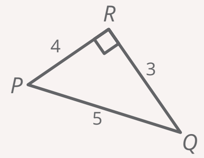
Sine is **O**pposite over **H**ypotenuse;
Cosine is **A**djacent over **H**ypotenuse;
Tangent is **O**pposite over **A**djacent.

This might help!

Some students prefer to use a different memory strategy. A common saying is "Oscar Had A Heap Of Apples Since Christmas Time." In this saying, the ratios are given first, followed by the trig functions. "Oscar Had" is Opp/Hyp, "A Heap" is Adj/Hyp, "Of Apples" is Opp/Adj, "Since" is sine, "Christmas" is cosine, and "Time" is tangent.

Example

Find the value of sin *P*.



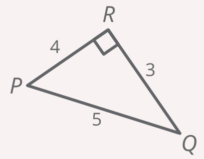
Solution

Begin by labeling the sides of the right triangle using $\angle P$. The hypotenuse of the triangle is *PQ*. For $\angle P$, *PR* is the adjacent side and *QR* is the opposite side.

$$\sin P = \frac{QR}{PQ} = \frac{3}{5}$$

Example

Find the trig values for $\angle Q$.



Solution

Begin by labeling the sides of the right triangle using $\angle Q$. The hypotenuse of the triangle is *PQ*. For $\angle Q$, *QR* is the adjacent side and *PR* is the opposite side.

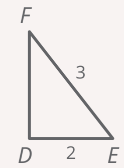
$$\sin Q = \frac{PR}{PQ} = \frac{4}{5}$$

$$\cos Q = \frac{QR}{PQ} = \frac{3}{5}$$

$$\tan Q = \frac{PR}{QR} = \frac{4}{3}$$

Example

Find the value of tan *F*.



Solution

Begin by labeling the sides of the right triangle using $\angle F$. The hypotenuse of the triangle is *EF*. For $\angle F$, *DF* is the adjacent side and *DE* is the opposite side.

Note that you are not given the length of side *DF*, which you need for the tangent ratio. Using the Pythagorean theorem, you can solve for this value:

$$(DF)^2 + 2^2 = 3^2$$

$$(DF)^2 + 4 = 9$$

$$(DF)^2 = 5$$

$$DF = \sqrt{5}$$

Now that you know the adjacent side, you can write the tangent ratio. Don't forget to rationalize the fraction.

$$\tan F = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Example

If $\sin A = \frac{5}{13}$, find cos *A*.

Solution

The sine ratio is opposite over hypotenuse, so the opposite side is 5 and the hypotenuse is 13. The sides of a right triangle must satisfy the Pythagorean theorem.

$$x^2 + 5^2 = 13^2$$

$$x^2 + 25 = 169$$

$$x^2 = 144$$

$$x = 12$$

Therefore, the adjacent side is 12 and $\cos A = \frac{12}{13}$.

EVALUATING

It is important to note that most trigonometric values are irrational numbers. Before the use of scientific calculators, trig values of angles were listed in tables and given to four decimal places. Today, it is common to use the scientific calculator in calculations involving trig functions. This course will require the use of a scientific calculator. If you do not have a scientific calculator, you should be able to find one on your computer or find one online.

Key Point!

A trig function cannot be evaluated without an angle measure!

There are a variety of calculator models to choose from and they vary in their appearance, the layout of the keypad, and use. While you will be provided with tutorials on calculator use in this course, it will not be specific to a brand of calculator. It is important that you follow along with the tutorial to ensure that you are getting the correct results as problems are worked. It will be your responsibility to read the manual on how to operate your individual calculator if you are unable to get the correct answer by following the model provided.

As you will learn later in the course, degrees are not the only means of measuring angles. Your calculator allows you to work with the different measuring systems. Therefore, it is important before beginning any problem to be sure that you are working in the correct system. For the exercises in this lesson, be sure that your calculator is set for degrees. Some calculators have a key labeled as "mode" and you might find the degree setting using the mode key.

To find the cos 25°, the key strokes are:

2 5 cos

Note that is it not necessary to press the equal key. If you do, however, the value does not change. Also, some calculators may require you press the cosine key first, then enter 25, and lastly press "enter."

Closeness applies when working with trig expressions. When a number precedes a trig function, multiplication is implied.

Example

Evaluate 2 sin 30°.

Solution

The expression *2 sin 30°* means "2 times the sine of 30 degrees."

On the calculator, evaluate sin 30° by entering 30 sin; then multiply this value by 2. Try it.

You should have gotten a value of 1.

Compare

You may also enter the problem on your calculator using the following key strokes:

2 × 30 sin =

The answer should still be 1.

Example

Evaluate cos 60° sin 30°.

Solution

The expression *cos 60° sin 30°* means "the cosine of 60 degrees times the sine of 30 degrees."

On the calculator, enter the following:

60 cos × 30 sin =

The cosine of 60° is 0.5 and the sine of 30° is 0.5, so their product is 0.25.

Example

Evaluate tan (20 + 30)°.

Solution

This expression tells you to find the tangent of 20 + 30, or 50 degrees.

tan 50° = 1.1918

Example

Evaluate cos 45° ÷ 5.

Solution

This expression says to divide the cosine of 45 degrees by 5 (not to find the cosine of 45 divided 5 degrees).

On a calculator, enter the following:

45 cos ÷ 5 =

cos 45° ÷ 5 = 0.1414

Try evaluating the expressions below with your calculator. Check your answers to ensure you are using your calculator correctly.

tan 30° = 0.5774

-4 sin 65° = -3.6252

$\frac{15}{\sin 5°} = 172.1057$

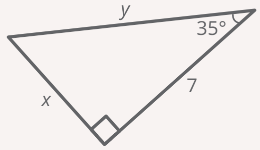
INDIRECT MEASUREMENT

Knowing that the ratios between the sides of a right triangle remain constant allows you to determine measures without actually measuring. You have previously used this concept of indirect measurement in your work with the Pythagorean theorem. If you know the measures of two sides of a right triangle, you can determine the length of the remaining side by using $a^2 + b^2 = c^2$, just as you did in a previous example.

The trig functions allow you to find the lengths of sides in a right triangle given only an acute angle and one side.

Example

For the triangle shown, find the values of x and y to the nearest tenth.



Solution

Label the sides using the 35° angle:

y is the hypotenuse, 7 is the adjacent side, and x is the opposite side.

Using 35°, there are three trig equations that can be written:

$$\sin 35° = \frac{x}{y}$$

$$\cos 35° = \frac{7}{y}$$

$$\tan 35° = \frac{x}{7}$$

The first equation contains two variable and would be impossible to solve.

Solve for x:

$\tan 35° = \frac{x}{7}$ Multiply both sides of this equation by 7:

$$7 \tan 35° = x$$

Evaluate 7 tan 35° on your calculator:

$$4.9014 = x$$

Round the answer: $4.9 = x$

Solve for y:

$$\cos 35° = \frac{7}{y}$$

$$y \cos 35° = 7$$

$$\frac{y \cos 35°}{\cos 35°} = \frac{7}{\cos 35°}$$

$$y = \frac{7}{\cos 35°}$$

Now evaluate 7 divided by the cosine of 35°:

$$y = 8.5454$$

Round the answer: $y = 8.5$

Compare

The Pythagorean theorem could also be used to solve for y since you know that $x = 4.9$.

$$4.9^2 + 7^2 = y^2$$
$$24.01 + 49 = y^2$$
$$73.01 = y^2$$
$$8.5 = y$$

Be careful, though, because if you make a mistake solving for x, this would cause y to be wrong also.

When solving these types of trig equations, it may help to remember:

- If the variable is in the numerator, multiply the number times the trig value.
- If the variable is in the denominator, divide the number by the trig value.

LET'S REVIEW

Before going on to the practice problems, make sure you understand all the main points of this lesson.

- Trigonometric ratios represent the relationships between the sides of a right triangle that contains a given acute angle.
- An acronym for remembering the sine, cosine, and tangent ratios is SOHCAHTOA.
- A trigonometric function cannot be evaluated without an angle measure.
- Indirect measurement can be done using trigonometric ratios.

Match the term with the appropriate definition.

1.1 ________ the trigonometric ratio of opposite side over hypotenuse for an acute angle in a right triangle

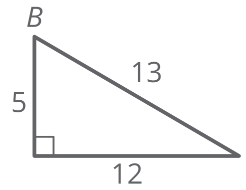
1.2 ________ the trigonometric ratio of opposite side over adjacent side for an acute angle in a right triangle

1.3 ________ the longest side of a right triangle; the side that is opposite the right angle

1.4 ________ the trigonometric ratio of adjacent side over hypotenuse for an acute angle in a right triangle

a. sine
b. cosine
c. tangent
d. hypotenuse

Match the following items. Label the triangle using $\angle B$. The triangles are not always drawn to scale.



1.5 ________ hypotenuse

1.6 ________ opposite side

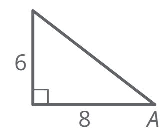
1.7 ________ adjacent side

a. 5
b. 12
c. 13

Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

Complete the following activities. The triangles are not always drawn to scale.

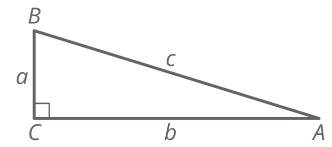
1.8 Evaluate sin *A* for the triangle shown. _______



a. 0.6
b. 0.75
c. 0.8

1.9 Right △*ACB* has side lengths *c* = 17 inches and *a* = 8 inches.

Solve for the exact length of side *b*. _______



a. $b = \sqrt{353}$ inches
b. $b = 9$ inches
c. $b = 15$ inches
d. $b = 10\sqrt{3}$ inches

1.10 Round to four decimal places.

tan75° = ________________

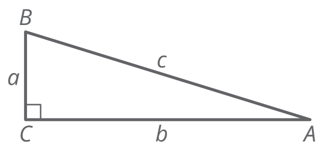
1.11 Evaluate -15 cos 60°. _______

a. -0.5
b. -7.5
c. -14.5

1.12 Evaluate $\frac{\tan 45°}{\cos 45°}$. _______

a. 0.7071
b. 1.0000
c. 1.4142

1.13 Given the following triangle, if *a* = 2 and *b* = 3, find *c*. _______



a. $\sqrt{5}$
b. $\sqrt{10}$
c. $\sqrt{13}$

1.14 In △*MPN*, ∠*P* is a right angle. If $\cos N = \frac{3}{5}$, which of the following statements is true? _______

a. $\sin M = \frac{5}{3}$

b. $\sin M = \frac{5}{4}$

c. $\sin M = \frac{4}{5}$

d. $\sin M = \frac{3}{5}$

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