



# MATH

STUDENT BOOK

▶ **12th Grade | Unit 4**

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# MATH 1204

## GRAPHING AND INVERSE FUNCTIONS

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**LIFEPAC Test is located in the center of the booklet.** Please remove before starting the unit.

**Author:**

Alpha Omega Publications

**Editors:**

Alan Christopherson, M.S.

Lauren McHale, B.A.



**804 N. 2nd Ave. E.  
Rock Rapids, IA 51246-1759**

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# Graphing and Inverse Functions

## Introduction

Drawing and interpreting graphs are important mathematical practices that have many applications: not just in math, but in many other fields as well. Graphs enable us to see what is happening, rather than just relying on words—imagine trying to explain a line to someone who has never seen one!

In this unit, you will learn how to draw and interpret the graphs of the trigonometric functions. You will also learn how to solve trigonometric equations using inverse functions, properties of equality, factoring, and the quadratic formula.

This unit covers graphing the trigonometric functions and their inverses. The graph of each function is connected back to the unit circle, which determines its repeating nature. Transformational geometry is also used for graphing.

The graph of the sine function is used as a mathematical model. The equation can be used to determine attributes of the curve; this information is used in graphing and modeling real-world phenomena. Trig functions are used as functions of real numbers where the independent variable of angle measure is replaced with other measures, such as length or time.

Inverse relations of the trigonometric functions are formed. Restrictions are placed on the domain and range of each inverse relation so that they form a function. The domain and range of the inverse functions are important in the evaluation of inverse trig expressions, solving trig equations, and understanding values that are obtained when using a calculator.

## Objectives

**Read these objectives.** The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Identify, interpret, and draw graphs of sine, cosine, and tangent curves, as well as their reciprocals, inverses, and the inverses of the reciprocals.
2. Identify the domain and range of sine, cosine, and tangent curves, as well as their reciprocals, inverses, and the inverses of the reciprocals.
3. Determine the frequency, amplitude, period, phase shift, and vertical shift from an equation of the form  $y = A \sin [B(x - C)] + D$ .
4. Graph the sine curve corresponding to an equation of the form  $y = A \sin [B(x - C)] + D$ .
5. Write the equation corresponding to a given sine or cosine graph.
6. Solve trigonometric equations using inverse functions, properties of equality, factoring, and the quadratic formula.
7. Graph and interpret periodic data with real-life applications.

Survey the LIFE PAC. Ask yourself some questions about this study and write your questions here.

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# 1. GRAPHING

## GRAPHING AND AMPLITUDE

Have you ever wondered why a sound is soft or loud?

Sound waves can be modeled using the sine function, and attributes of the graph tell us a lot about the sound. In this lesson, you will learn how to graph the trigonometric functions. You will also learn which feature of the sound wave tells us how loud a sound will be.

### Section Objectives

**Review these objectives.** When you have completed this section, you should be able to:

- Identify the graphs of the sine, cosine, and tangent curves.
- State the domain and range of sine, cosine, and tangent curves of the form  $y = A \sin x$ ,  $y = A \cos x$ , and  $y = A \tan x$ .
- Determine the amplitude of a sine or cosine function from a given equation or graph.

### Vocabulary

**Study these words to enhance your learning success in this section.**

<b>amplitude</b> .....	The maximum displacement from the horizontal line of symmetry for the sine and cosine functions.
<b>asymptote</b> .....	A line that a graph approaches but never touches.
<b>domain</b> .....	The set of first coordinates in a relation; the set of independent variable ( $x$ ) values.
<b>periodic function</b> .....	A function that repeats a pattern over its domain.
<b>range</b> .....	The set of second coordinates in a relation; the set of dependent variable ( $y$ ) values.

**Note:** All vocabulary words in this LIFEPAC appear in **boldface** print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.

## THE SINE FUNCTION

From your study of algebra, you should be familiar with function notation and with finding ordered pairs for a given function.

For the function  $f(x) = \sin x$ :

- the **domain**, or set of  $x$  values, represents the angle measures;
- the **range**, or set of  $y$  values, represents the sine values of those angles.

*Note:* It is common to use radian measure for the  $x$ -axis when graphing trig functions.

### Think About It!

What is the advantage of using radians instead of degrees for angle measure?

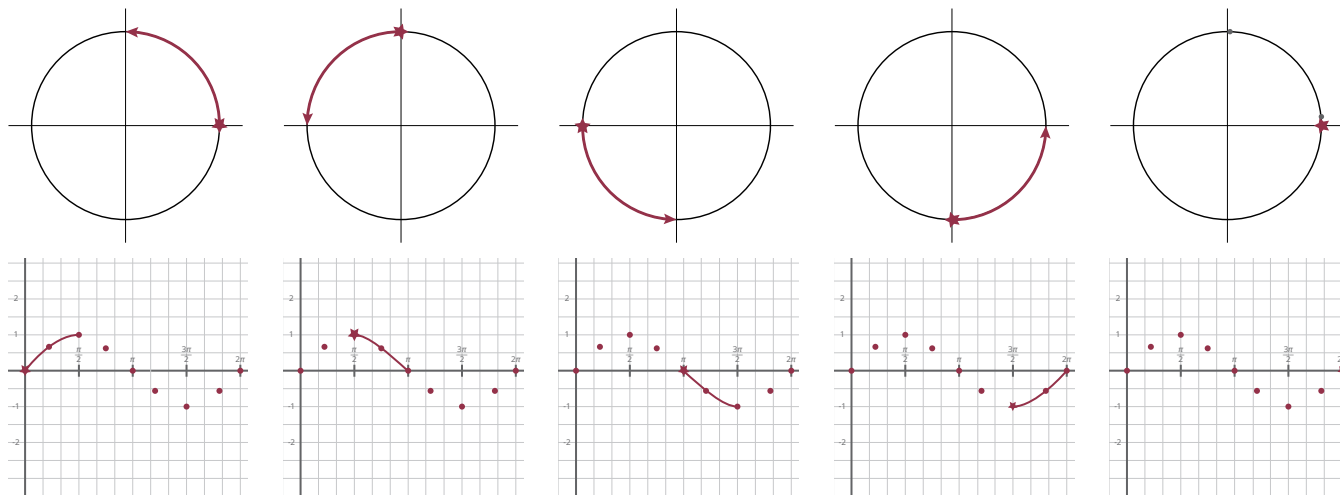
Hint: Think of scaling the  $x$ -axis using real numbers.

Before you begin graphing, let's use the unit circle and your knowledge of special angles to make a table of some ordered pairs on the graph of  $f(x) = \sin x$ .

Irrational values such as  $\frac{\sqrt{2}}{2}$  have been approximated.

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\sin x$	0	.5	.71	.87	1	.87	.71	.5	0

Using the symmetry of the unit circle, you have angles in Quadrants III and IV whose sine function values are the same as their reference angles, but negative.



### Reminder:

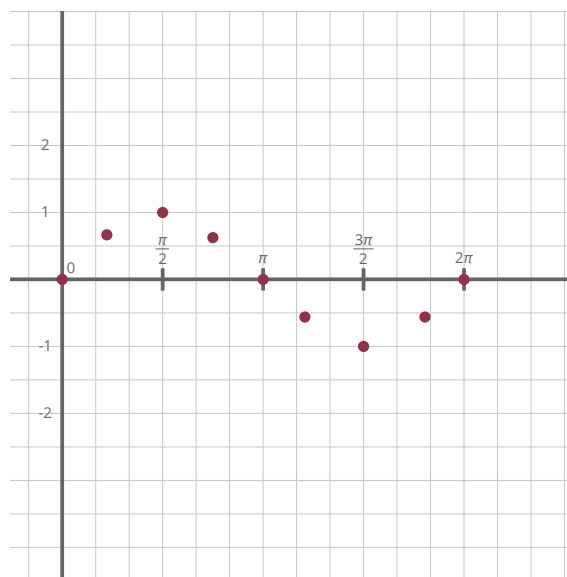
$$\sin \frac{5\pi}{6} = \sin \frac{\pi}{6}$$

$$\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$$

$$\sin \frac{11\pi}{6} = -\sin \frac{\pi}{6}$$

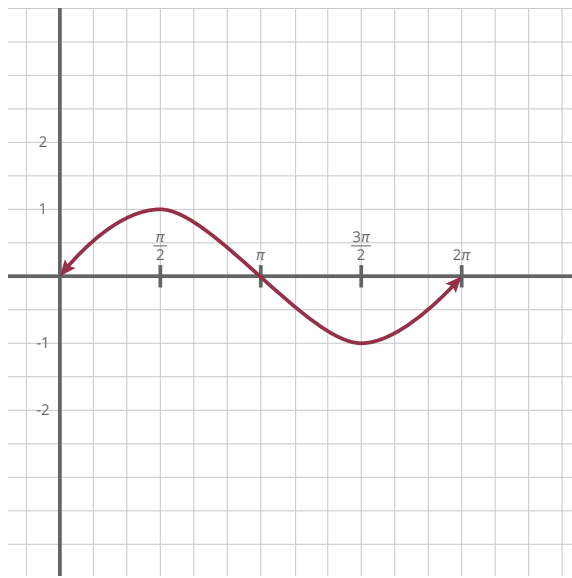
Do you remember why this is true?

Plotting some points yields the following:



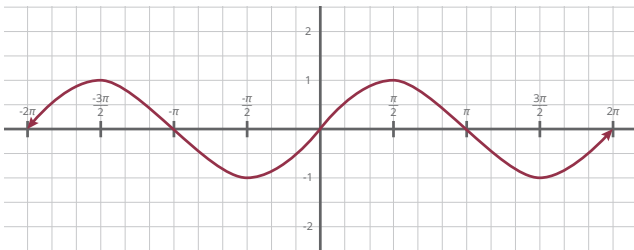
As you move along the unit circle from 0 to  $2\pi$  radians, include all angle measures for  $x$  and all real-number values for  $y$  from -1 to 1 inclusive. Then you can connect these points with a nice smooth curve. Notice that each location on your graph corresponds to a location on the unit circle!

The graph of  $f(x) = \sin x$  for  $0 \leq x \leq 2\pi$  is



Notice the arrows indicating that the graph continues on.

Because coterminal angles have the same trigonometric function values, this pattern will repeat itself again from  $2\pi$  to  $4\pi$ ,  $4\pi$  to  $6\pi$ , and so on. Because  $-2\pi$  is coterminal with  $0$ , you can start there for negative angles and move around the unit circle to  $0$ . The pattern from  $-2\pi$  to  $0$  is the same as from  $0$  to  $2\pi$ .



When a function repeats a pattern over its domain, it is called a **periodic function**. The trigonometric functions are periodic functions because all coterminal angles have the same trig function values.

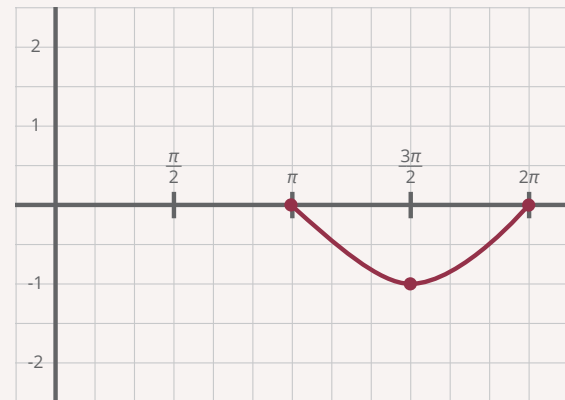
The sine function is defined for all angle measures; hence, the domain of the sine function is all real numbers. Note that the sine values are never greater than  $1$  or less than  $-1$ . The range of the sine function is  $-1 \leq y \leq 1$ .

It is possible to graph over a specified interval of the domain. Look at the following examples to see the sine function graphed over different *restricted domains*.

### Example

Graph  $y = \sin x$  for  $\pi \leq x \leq 2\pi$  and state the range.

### Solution

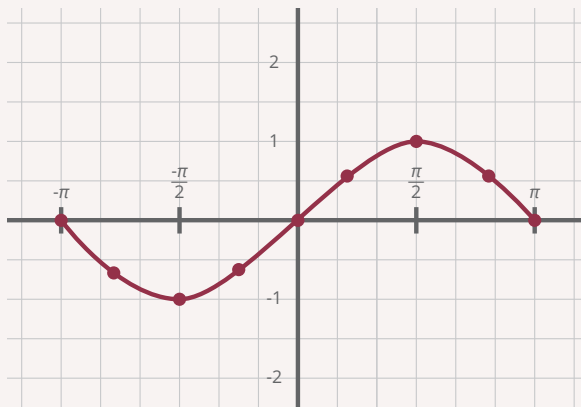


The range is  $-1 \leq y \leq 0$ .

### Example

Graph  $y = \sin x$  for  $-\pi \leq x \leq \pi$ .

### Solution



Notice that the above graphs are simply “sections” of the overall sine function. By restricting the domain, you restrict the graph to a specific section of the sine curve.



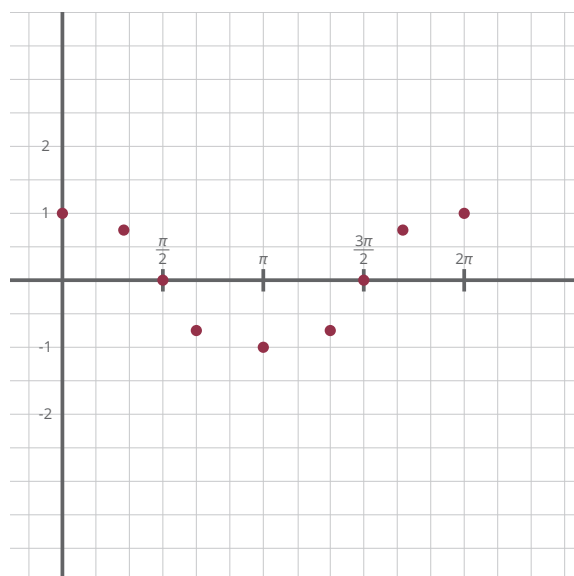
## THE COSINE FUNCTION

You can graph the cosine function  $y = \cos x$ .

From the unit circle and special angles, you can write a table of values:

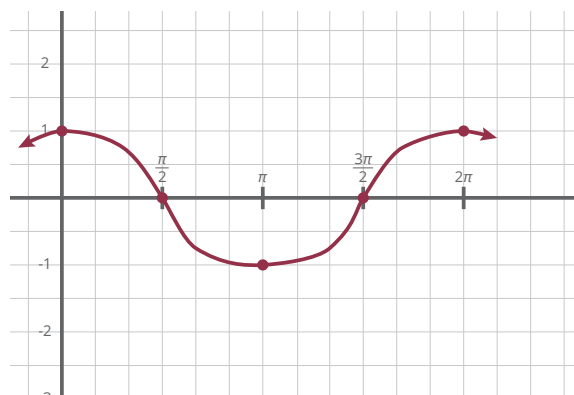
$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\cos x$	1	.87	.71	.5	0	-.5	-.71	-.87	-1

Note that the cosine value starts at 1 and becomes negative in the second and third quadrants. Therefore, the graph of the cosine function is below the  $x$ -axis from  $\frac{\pi}{2}$  to  $\frac{3\pi}{2}$ .



Just as with the sine function:

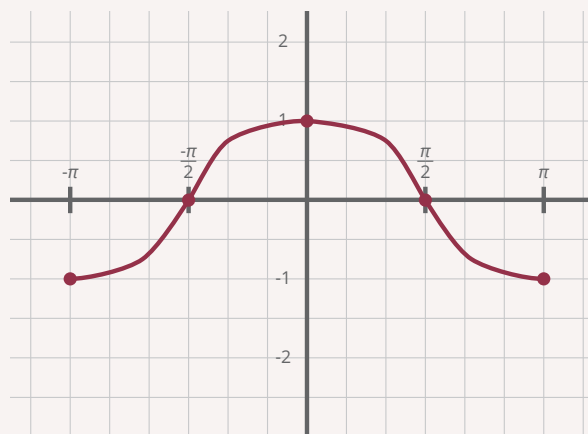
- the domain for the cosine function is all real numbers;
- the range of the cosine function is  $-1 \leq y \leq 1$ ;
- the cosine function is periodic and will repeat this pattern over intervals of  $2\pi$ .



### Example

Graph  $y = \cos x$  for  $-\pi \leq x \leq \pi$  and state the range.

### Solution



The range is  $-1 \leq y \leq 1$ .

## THE TANGENT FUNCTION

Unlike the sine and cosine functions, the tangent function is *not* defined for all angle measures.

Tangent is the ratio of sine over cosine and is undefined when the cosine of an angle is zero. On the interval of 0 to  $\pi$ , cosine is 0 for  $\frac{\pi}{2}$ . What do you think happens to the tangent function as the angle measure gets close to  $\frac{\pi}{2}$ ?

As the angle measure gets closer and closer to  $\frac{\pi}{2}$ , the sine function will get closer and closer to 1. Let's look at what happens when a number close to 1 is divided by a number close to zero:

$\frac{.9}{.1} = 9$	$\frac{.99}{.01} = 99$
$\frac{.999}{.001} = 999$	$\frac{.9999}{.0001} = 9,999$

Since you will always be able to find a smaller number to divide by, the fraction will keep getting bigger and bigger. Therefore, the tangent function continues to go to infinity as the angle measure approaches  $\frac{\pi}{2}$ .

The graph of the tangent function has a line called an **asymptote** at  $\frac{\pi}{2}$ . An asymptote is a line that the graph will approach but not touch. The  $y$  values will approach infinity as the  $x$  value approaches  $\frac{\pi}{2}$ .

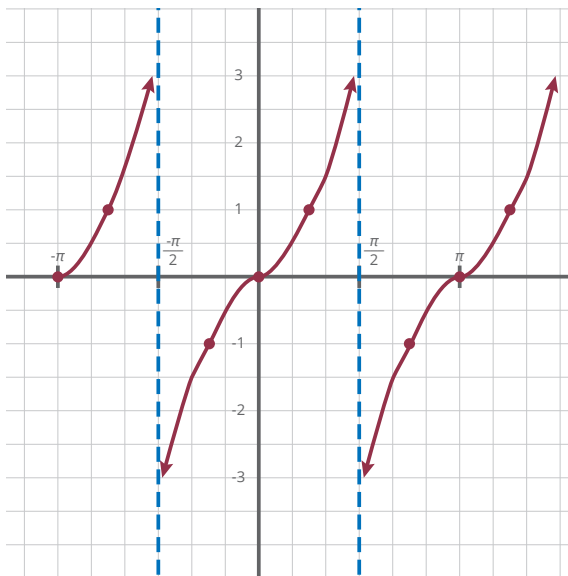
The value of the cosine function is 0 whenever the measure of the angle is  $\frac{\pi}{2} + n\pi$  (where  $n$  is any integer). That is, each time a multiple of  $\pi$  is added to  $\frac{\pi}{2}$ , there is an asymptote in the graph of  $y = \tan x$ .

### Key point!

Asymptotes are typically indicated on a graph by using a dashed line.

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	U*	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1

\*U stands for an undefined value



The domain of  $y = \tan x$  is all real numbers except  $\frac{\pi}{2} + n\pi$  where  $n$  is an integer.

The range of the tangent function is all real numbers.

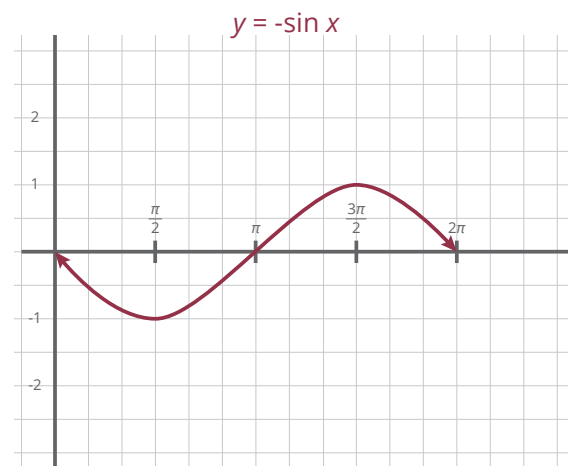
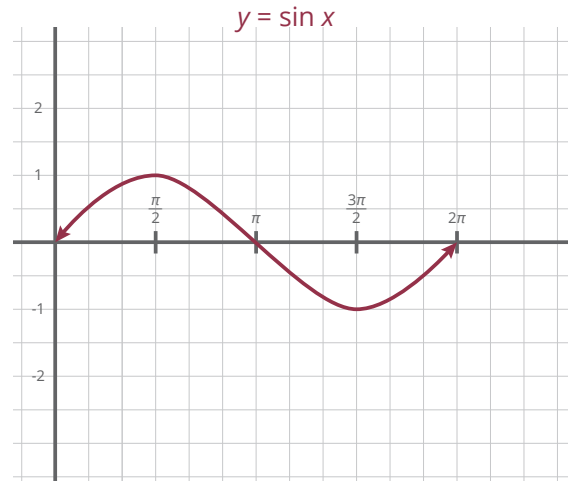
### AMPLITUDE

Now that you can graph the basic trig functions, you will learn how to graph other trig functions by performing transformations on them.

In your study of algebra and functions, you should have explored the connections between transformations of a graph and the related equation. For example,  $y = -x^2$  is a reflection of the graph  $y = x^2$  over the  $x$ -axis, and  $y = 2x^2$  is a stretch of the graph  $y = x^2$ .

These same algebraic principles apply to the graphs of the trigonometric functions.

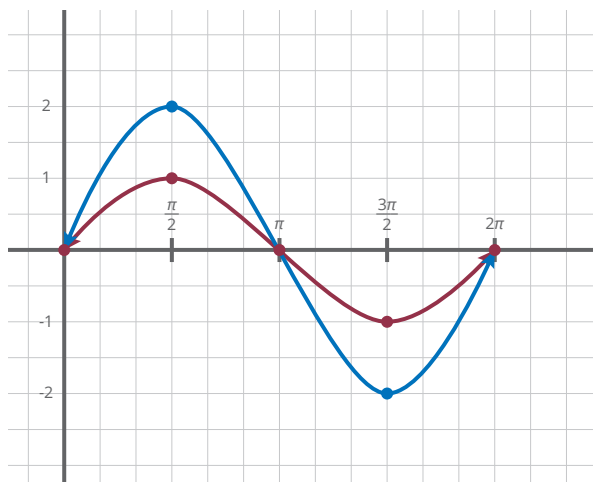
Compare the following graphs of  $y = \sin x$  and  $y = -\sin x$ . Note that the graph of  $y = \sin x$  has been reflected over the  $x$ -axis to obtain the graph of  $y = -\sin x$ .



Now compare the table of values for the functions  $y = \sin x$  and  $y = 2 \sin x$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$2 \sin x$	0	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

To evaluate the function  $y = 2 \sin x$ , we evaluate  $\sin x$  and then multiply by 2. Each  $y$  value on the function  $y = \sin x$  has been multiplied by 2 to obtain the value of  $y$  on the graph  $y = 2 \sin x$ .



Just as with other functions, the graph of  $y = \sin x$  is “stretched” to obtain the graph of  $y = 2 \sin x$ .

The maximum displacement from the horizontal line of symmetry is called the **amplitude**. For  $y = A \sin x$  and  $y = A \cos x$ , the amplitude is  $|A|$ .

### Think About It!

If the amplitude of  $y = \frac{1}{2} \sin x$  is  $\frac{1}{2}$ , what does the graph look like?

For  $|A| < 1$ , how might you describe the change of the graph?

The amplitude of  $y = \sin x$  is 1.

The amplitude of  $y = -\sin x$  is  $|-1| = 1$ .

The amplitude of  $y = 2 \sin x$  is 2.

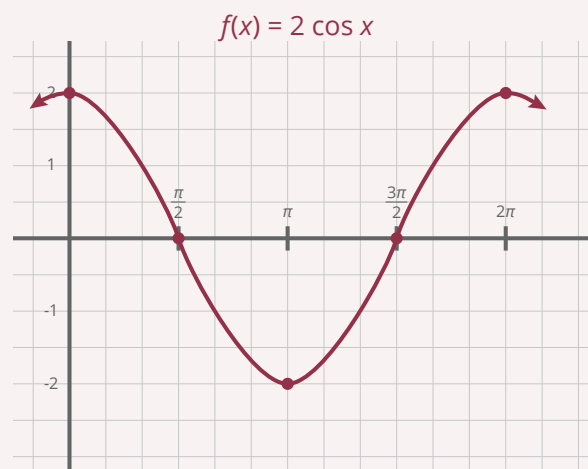
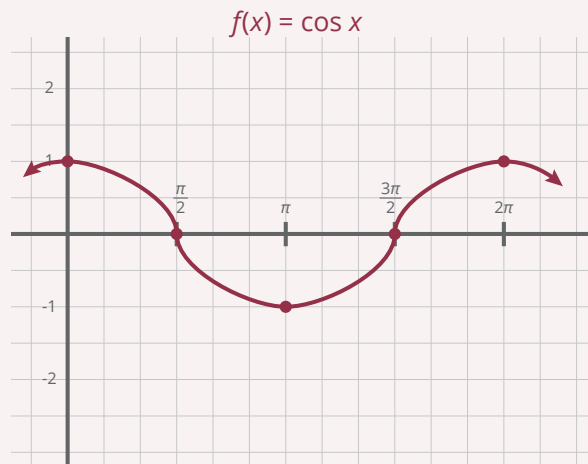
It is the amplitude of a sound wave that determines how loud the sound is. The smaller the amplitude, the softer the sound. As the amplitude of a sound wave increases, the sound gets louder.

### Example

Graph  $f(x) = 2 \cos x$  and state the range.

### Solution

For  $f(x) = 2 \cos x$ , the graph is the same shape as  $y = \cos x$ , but the amplitude is 2.



The range is  $-2 \leq y \leq 2$ .

## LET'S REVIEW

Before going on to the practice problems, make sure you understand all the main points of this lesson.

- Use the unit circle and special angles to help you sketch graphs of the trig functions.
- The trigonometric functions are periodic functions.
- For both the cosine and sine functions, the domain is all real numbers.
- The tangent function is undefined at  $\frac{\pi}{2} + n\pi$  where  $n$  is an integer, so the graph has asymptotes at these values. The domain is the set of real numbers excluding these values. The range is all real numbers.
- For both  $y = \cos x$  and  $y = \sin x$ , the range is  $-1 \leq y \leq 1$ .
- For both  $y = A \cos x$  and  $y = A \sin x$ , the amplitude is  $|A|$ .

Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

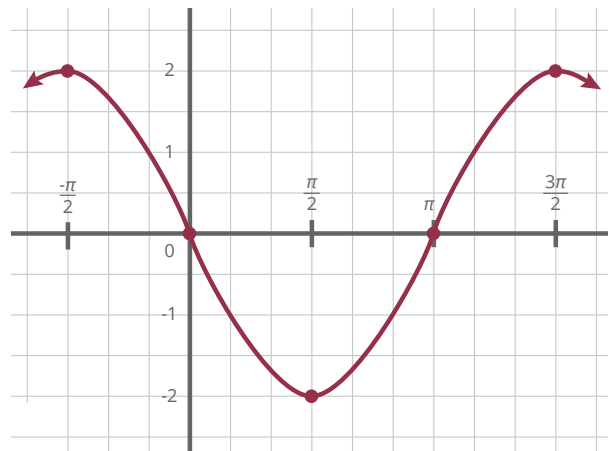
Match each term to its definition.

- |     |       |   |                      |
|-----|-------|---|----------------------|
| 1.1 | _____ | the set of dependent ( $y$ ) values, the set of second coordinates in a relation  | a. amplitude         |
| 1.2 | _____ | a line that a function approaches   | b. asymptote         |
| 1.3 | _____ | the maximum displacement from the horizontal line of symmetry for sine and cosine | c. domain            |
| 1.4 | _____ | a function that repeats a pattern over its domain                                 | d. range             |
| 1.5 | _____ | the set of independent ( $x$ ) values, the set of first coordinates in a relation | e. periodic function |

Complete the following activities.

- 1.6 In two or more complete sentences, describe why the range of  $y = \sin(x)$  is  $-1 \leq y \leq 1$ . Make sure to reference the unit circle in your description.
- 1.7 In two or more complete sentences, describe why the range of  $y = \cos(x)$  is  $-1 \leq y \leq 1$ . Make sure to reference the unit circle in your description.

- 1.8 Graph  $y = \cos(x)$  for  $-\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2}$ . What is the largest value in the range? \_\_\_\_\_
- a. -1                                      b. 0                                      c.  $\frac{1}{2}$                                       d. 1
- 1.9 Which of the following are asymptotes for the function  $y = \tan(x)$ ? Select all that apply.
- \_\_\_\_\_
- a.  $x = \frac{\pi}{4}$                       b.  $x = \frac{3\pi}{4}$                       c.  $x = -\frac{\pi}{2}$                       d.  $x = \frac{5\pi}{6}$                       e.  $x = \frac{\pi}{2}$
- 1.10 What is the range of  $y = \tan(x)$ ? \_\_\_\_\_
- a. all real numbers  
b.  $-1 \leq y \leq 1$   
c.  $y > 0$
- 1.11 Graph  $y = \tan(x)$  for  $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ . What is the range? \_\_\_\_\_
- a.  $-1 \leq y \leq 1$   
b.  $-1 \leq y \leq 0$   
c.  $0 \leq y \leq 1$   
d. all real numbers
- 1.12 Which of the following ordered pairs lies on the graph of  $y = \tan(x)$ ? \_\_\_\_\_
- a.  $(-\frac{5\pi}{2}, -1)$   
b.  $(-\frac{9\pi}{4}, 1)$   
c.  $(\frac{\pi}{6}, \sqrt{3})$   
d.  $(5\pi, 0)$
- 1.13 Choose the equation of this graph. \_\_\_\_\_
- a.  $y = -2 \cos x$   
b.  $y = -2 \sin x$   
c.  $y = 2 \cos x$   
d.  $y = 2 \sin x$





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