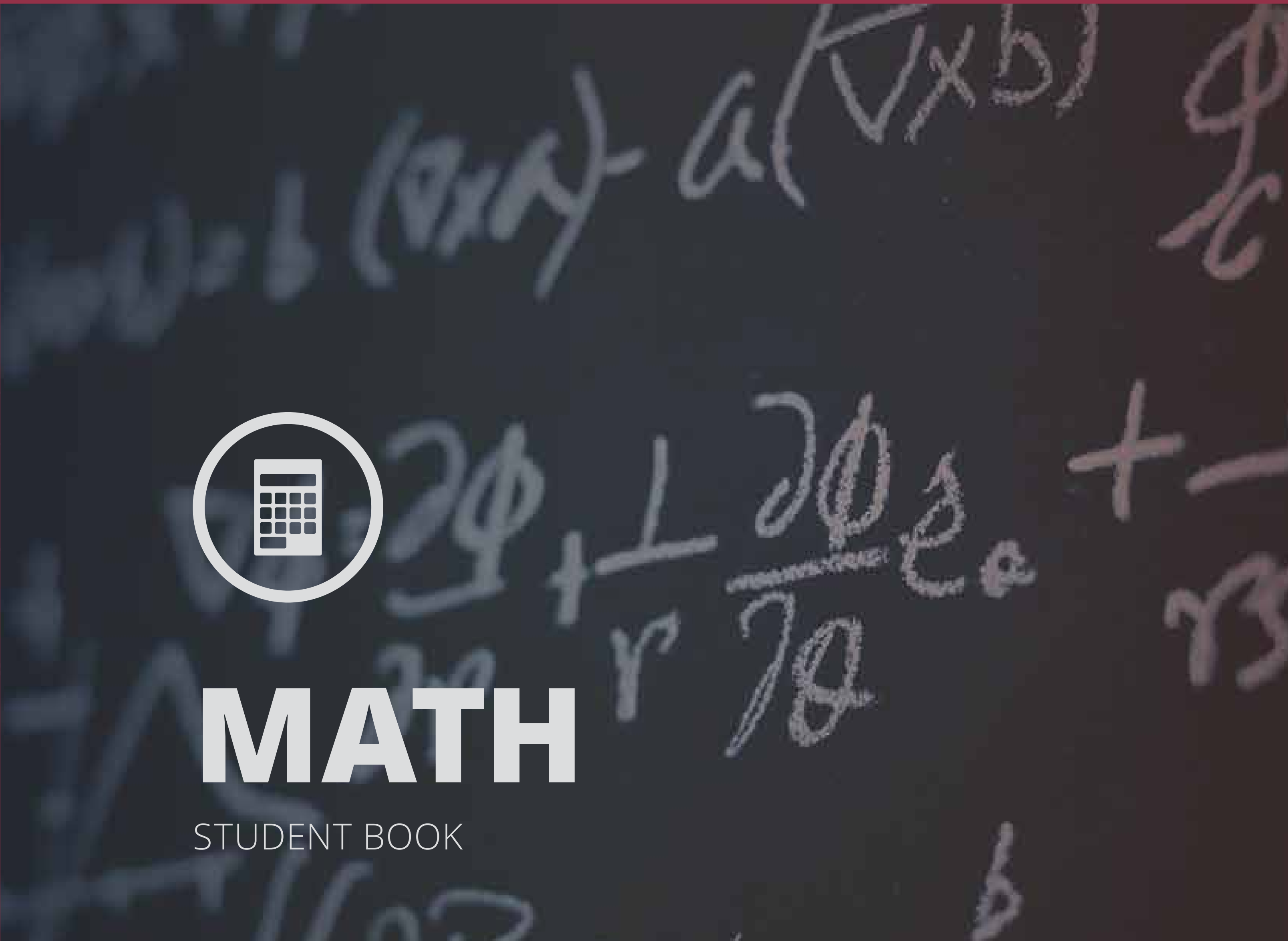
LIFEPAC®



# MATH

STUDENT BOOK

## 12th Grade | Unit 6

Alpha Omega
PUBLICATIONS

# MATH 1206

# TRIGONOMETRIC APPLICATIONS



**LIFEPAC Test is located in the center of the booklet.** Please remove before starting the unit.

**Author:**
Alpha Omega Publications

**Editors:**
Alan Christopherson, M.S.
Lauren McHale, B.A.

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**804 N. 2nd Ave. E.**
**Rock Rapids, IA 51246-1759**

# Trigonometric Applications

## Introduction

This LIFEPAC® reviews trigonometric applications. You will learn to find the trigonometric functions for any given angle and to work with scalar quantities, vectors and resultants. In this LIFEPAC, you will also study the law of sines and the law of cosines and will use them to solve application problems.

## Objectives

**Read these objectives.** The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Find missing angle and side measures of a triangle using the law of sines.
2. Use the law of sines to solve indirect measure problems.
3. Determine the number of triangles that can be formed given two sides and an angle.
4. Find the area of a triangle using two sides and the included angle.
5. Find a side measure of a triangle using the law of cosines.
6. Use the law of cosines to solve problems.
7. Find an angle measure of a triangle using the law of cosines.
8. Define key terms associated with vectors.
9. Add and subtract vectors.
10. Find the magnitude of the resultant of two vectors.
11. Find the direction of the resultant of two vectors.
12. Add two position vectors given their terminal points.
13. Find the horizontal and vertical components of a vector.
14. Find the angles formed between two vectors and their resultant.
15. Solve word problems about navigation.
16. Multiply a vector by a scalar.
17. Subtract vectors.
18. Find the dot product of vectors.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

# 1. TRIGONOMETRY OF OBLIQUE TRIANGLES

## LAW OF SINES

Around 1852, Mt. Everest, known as Peak XV at the time, was declared the highest mountain peak with an estimated height of 8,839 meters. This calculation, done during the Great Trigonometrical Survey of India, was computed using trigonometric formulas; none of the researchers ever scaled the mountain. In 1999, an American team placed global positioning system devices on the mountain and determined the height to be 8,850 meters. Considering the vastness of the mountain and the thickness of snow and ice that are constantly changing, the accuracy of the original trigonometric survey is remarkable. In this lesson, you will learn about one of the formulas that was used to help calculate the mountain's height.

### Section Objectives

**Review these objectives.** When you have completed this section, you should be able to:

- Find missing angle and side measures of a triangle using the law of sines.
- Use the law of sines to solve indirect measure problems.

### Vocabulary

**Study this word to enhance your learning success in this section.**

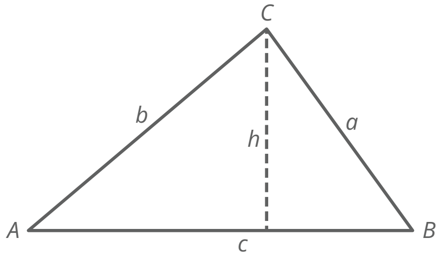
**oblique triangle** .................... A triangle that is not a right triangle.

**Note:** *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

## LAW OF SINES

Let *ABC* be any **oblique triangle**.

Draw the altitude, *h*, from $\angle C$ to *AB*.




Right triangle trigonometry tells us

$\sin A = \frac{h}{b}$

$\sin B = \frac{h}{a}$

Solving each of the equations for *h* results in

$h = b \sin A$
$h = a \sin B$

Therefore, $b \sin A = a \sin B$.

Another way of saying this is $\frac{\sin A}{a} = \frac{\sin B}{b}$.

By the same method, you can draw the altitude, $h_1$, from $\angle B$ to *AC* and show that

$\sin A = \frac{h_1}{c}$

$\sin C = \frac{h_1}{a}$

Therefore, $h_1 = c \sin A$ and $h_1 = a \sin C$, so

$c \sin A = a \sin C$

Therefore, $\frac{\sin A}{a} = \frac{\sin C}{c}$.

Since $\frac{\sin B}{b}$ and $\frac{\sin C}{c}$ are both equal to $\frac{\sin A}{a}$, the ratios of the sine of an angle to the side opposite the angle are all equal.

This is the trigonometric formula known as the *law of sines*.

**FORMULA**

*Law of sines*: In any triangle, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ where $A$, $B$, and $C$ are the angles opposite the sides $a$, $b$, and $c$, respectively.

**Key point!**

The law of sines may also be written as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

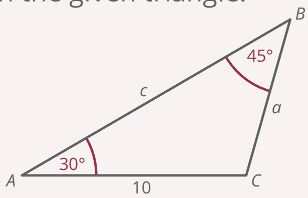
To use the law of sines, you only need to know:

- two sides and one angle; or
- one side and two angles.

Because all three ratios are equal, you can pick out only the two needed ratios and form a proportion.

**Example**

Find the length of $a$ in the given triangle.




**Solution**

Use the law of sines,

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

and substitute the given values:

$$\frac{\sin 30°}{a} = \frac{\sin 45°}{10}$$

Cross-multiply:

$$10 \sin 30° = a \sin 45°$$

Since 30° and 45° are special angles, you know their exact values. Substitute these values and solve for $a$:

$$10\left(\tfrac{1}{2}\right) = a\left(\tfrac{\sqrt{2}}{2}\right)$$

$$5 = a\left(\tfrac{\sqrt{2}}{2}\right)$$

$$5\left(\tfrac{2}{\sqrt{2}}\right) = a\left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{2}{\sqrt{2}}\right)$$

$$5\sqrt{2} = a$$

**Example**

In triangle $ABC$, $a = 18$, $\angle A = 45°$, and $\angle B = 105°$. Find $c$.

**Solution**

Because you wish to find $c$ and you know $a$ and $A$, it makes sense to use the proportion $\frac{\sin A}{a} = \frac{\sin C}{c}$ from the law of sines.

Note that you are not given the measure of $\angle C$. You are, however, given enough information to calculate it.

The angles in a triangle must total 180°, so $\angle C = 180° - (45° + 105°) = 30°$.

Now use the law of sines:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 45°}{18} = \frac{\sin 30°}{c}$$

$$c \sin 45° = 18 \sin 30°$$

$$c\left(\tfrac{\sqrt{2}}{2}\right) = 18\left(\tfrac{1}{2}\right)$$

$$c = 9\sqrt{2}$$

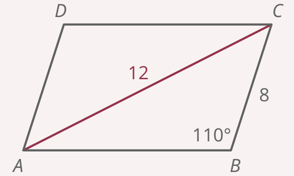
Since polygons with more than three sides can be broken down into triangles, the law of sines can be used to solve problems involving other polygons.

**Example**

In parallelogram $ABCD$, $BC = 8$ inches, $\angle B = 110°$, and diagonal $AC = 12$ inches. Find the measure of $\angle CAB$ to the nearest tenth of a degree.

**Solution**

Sketch and label the figure:




Use the law of sines in triangle $ABC$:

$$\frac{\sin 110°}{12} = \frac{\sin x}{8}$$

$$8 \sin 110° = 12 \sin x$$

$$(8 \sin 110°) \div 12 = \sin x$$

$$x = 38.8°$$

$$\angle CAB = 38.8°$$

**Reminder:**

On your calculator, be sure you are in degree mode and then follow these key strokes:

110 sin × 8 ÷ 12 = INV sin

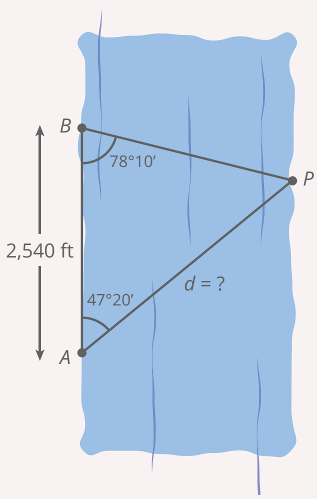
It is not always possible or practical for surveyors to use right triangle trigonometry to calculate measures in the field. For example, there may be a swamp in the location from which a right angle could be measured. With the law of sines, a surveyor need not rely on right triangle trigonometry to calculate necessary measurements.

### Example

To find the distance from an observer's point, *A*, on one side of a river to a point, *P*, on the other side, a line, *AB*, measuring 2,540 feet was laid out on one side and the angles *BAP* and *ABP* were measured and found to be 47°20′ and 78°10′, respectively. Find the distance from *A* to *P* to the nearest foot.

### Solution

Draw a sketch of the problem:




$\angle P = 180° - (47°20' + 78°10') = 54°30'$ or $54.5°$

Use the law of sines:

$$\frac{\sin 78.17°}{d} = \frac{\sin 54.5°}{2{,}540}$$

$$d \sin 54.5° = 2{,}540 \sin 78.17°$$

$$d = \frac{2{,}540 \sin 78.17°}{\sin 54.5°}$$

$$d = \frac{2{,}540(0.9787)}{(0.8141)}$$

$$d = 3{,}053 \text{ ft}$$

**Reminder:**

Degrees can be broken down into smaller units of minutes and seconds.

1 degree = 60 minutes

1 minute = 60 seconds

$10' = \frac{10}{60}°$

## LET'S REVIEW

Before going on to the practice problems, reflect on your work and ensure that you understand all the main points of the lesson.

- The law of sines states that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- The law of sines can be used in the solution of a triangle when you know:
    - two angles and a side opposite one of them; or
    - two sides and an angle opposite one of them.

**Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.**

**Complete the following activities.**

**1.1** An oblique triangle is _______ .

a. a triangle that is not a right triangle
b. a triangle that has an angle that is equal to or greater than 90 degrees
c. a triangle that has an angle that is less than or equal to 90 degrees
d. a triangle that has two angles equal to 45 degrees

**1.2** In triangle *ABC*, if $\angle A = 120°$, $a = 8$, and $b = 3$, then $\angle B =$ ___________ °. (Round your answer to the nearest degree.)

**1.3** In triangle *ABC*, $a = 18.2$, $\angle B = 62°$, and $\angle C = 48°$. Find *b*. _______

a. 15
b. 17
c. 19
d. 22

**1.4** In triangle *ABC*, $c = 8$, $b = 6$, $\angle C = 60°$. $\sin \angle B =$ _______

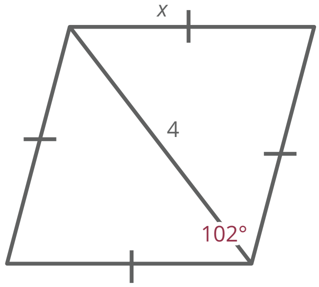
a. $\frac{3}{8}$ b. $\frac{\sqrt{3}}{8}$ c. $\frac{2\sqrt{3}}{3}$ d. $\frac{3\sqrt{3}}{8}$

**1.5** In triangle *ABC*, $b = 600$, $\angle B = 11°$, $\angle C = 75°$. Find *a*. _______

a. 115 b. 1,185 c. 3,037 d. 3,137

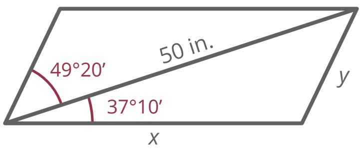
**1.6** One angle of a rhombus measures 102°, and the shorter diagonal is 4 inches long. Approximately how long is the side of the rhombus?

_______ (*Hint:* Diagonals of a rhombus bisect the angles.)




a. 2 in
b. 3 in
c. 4 in
d. 5 in

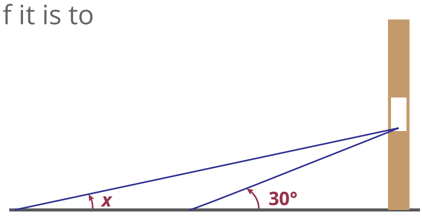
**1.7** A diagonal of a parallelogram is 50 inches long and makes angles of 37°10′ and 49°20′ with the sides.




How long is the longest side? _______

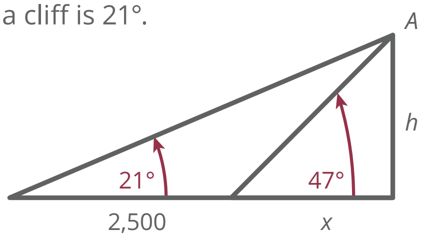
a. 30 in
b. 38 in
c. 40 in
d. 66 in

**1.8** A 10-foot ladder must make an angle of 30° with the ground if it is to reach a certain window. What angle must a 20-foot ladder make with the ground to reach the same window? _______



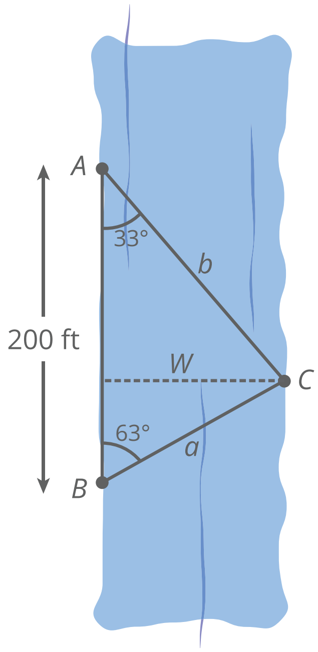

a. 10.5°
b. 12.5°
c. 14.5°
d. 16.5°

**1.9** From a ship, the angle of elevation of a point, *A*, at the top of a cliff is 21°. After the ship has sailed 2,500 feet directly toward the foot of the cliff, the angle of elevation of ∠*A* is 47°. (Assume the cliff is perpendicular to the ground.)



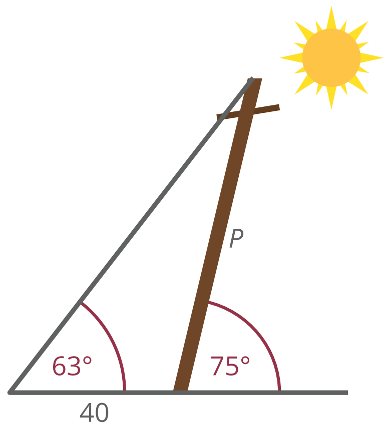

The height of the cliff is _______ .

**1.10** Vertices *A* and *B* of triangle *ABC* are on one bank of a river, and vertex *C* is on the opposite bank. The distance between *A* and *B* is 200 feet. Angle *A* has a measure of 33°, and angle *B* has a measure of 63°. Find *b*. _______




a. 110 ft
b. 168 ft
c. 179 ft
d. 223 ft

**1.11** A powerline pole casts a shadow of 40 feet long when the angle of elevation for the sun is 63°. The pole leans 15° from the vertical, directly toward the sun. Find the length of the pole.




________

a. 37 ft
b. 43 ft
c. 79 ft
d. 171 ft

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804 N. 2nd Ave. E.
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**800-622-3070**
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