LIFEPAC®

# MATH

STUDENT BOOK

## 12th Grade | Unit 7

Alpha Omega PUBLICATIONS

# MATH 1207

# POLAR COORDINATES



**LIFEPAC Test is located in the center of the booklet.** Please remove before starting the unit.

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# Polar Coordinates

## Introduction

This LIFEPAC® reviews the polar plane and its relationship to the Cartesian and complex planes. Students use trigonometry to convert between the different ways of expressing points in each plane. Students also use their knowledge of the trigonometric functions and symmetry to graph polar equations. They also learn about some of the families of polar curves and how the polar form helps to simplify the process of finding products, quotients, powers, and roots of complex numbers.

## Objectives

**Read these objectives.** The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Graph points in the polar plane.
2. Convert between Cartesian and polar coordinates.
3. Convert Cartesian equations to polar equations.
4. Convert polar equations to Cartesian equations.
5. Identify the graphs of polar equations of the forms $r = c$ and $\theta = c$.
6. Determine the slope of a line from its given polar equation.
7. Recognize the equations of the polar curves rose, cardioid, and limaçon.
8. Identify the graphs of polar curves rose, cardioid, and limaçon.
9. Determine the eccentricity of a conic section from its polar equation.
10. Identify and graph a conic section from its equation in polar form.
11. Write the polar equation of a conic section.
12. Represent complex numbers as points in the complex plane.
13. Convert between the rectangular and polar forms of a complex number.
14. Find the absolute value of a complex number.
15. Multiply complex numbers in polar form.
16. Divide complex numbers in polar form.
17. Raise complex numbers in polar form to powers.
18. Find the *n*th roots of a complex number and use this to solve equations.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

# 1. POLAR EQUATIONS

## INTRODUCTION TO POLAR COORDINATES

Radar works by sending signals out from the antenna; these signals then reflect off an object and bounce back. The time that it takes for this reflection to come back to its source is then used to calculate the distance to it. This distance and the angular position of the antenna are used to obtain the exact location of the object. The coordinate system used to locate the object's position is the polar coordinate system.

### Section Objectives

**Review these objectives.** When you have completed this section, you should be able to:

- Graph points in the polar plane.
- Convert between Cartesian and polar coordinates.

### Vocabulary

**Study this word to enhance your learning success in this section.**

**polar coordinate** . . . . . . . . . . . . . . . . . . . . An ordered pair ($r$, θ) where $r$ is the distance from the pole to a point and θ is the angle measured from the initial side to the terminal side.

**Note:** *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

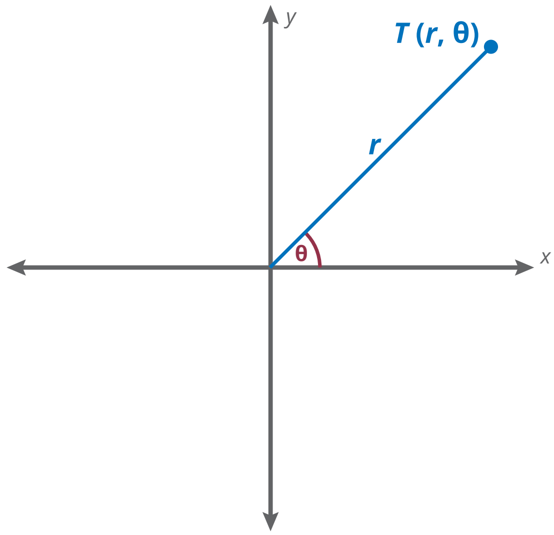
## GRAPHING POLAR COORDINATES

You are familiar with the rectangular coordinate system used for graphing; this system, also called the Cartesian system, consists of a grid of horizontal and vertical lines. Another graphing system uses a grid comprised of concentric circles with rays emanating from the center. This system of graphing is called the **polar coordinate** system.

Just as the rectangular system has the $x$- and $y$-axes intersecting at the origin as a reference point, the polar system has a *pole* and *polar axis* for reference. The pole is the equivalent of the origin, and the polar axis is the equivalent of the positive $x$-axis.

In the polar coordinate system, a point is located using the ordered pair ($r$, θ) where

- $r$ is the distance measured from the pole; and
- θ is the angle measured from the polar axis.



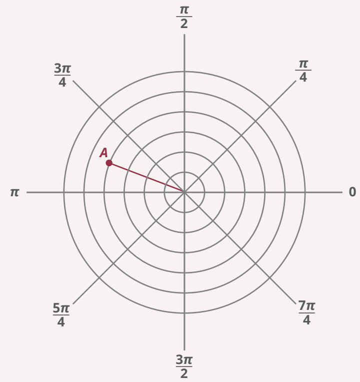

Circles are used in the polar plane so that $r$ can be defined as a specific distance from the pole.

## Example

Graph *A* (4, 150°).

## Solution

Count four spaces from the origin on the polar axis and then rotate 150°.




Recall that there are an infinite number of coterminal angles. Adding or subtracting multiples of 360° from a given angle measure will give you angles that are coterminal with it. For this reason, different coordinates may result in the same point.

### Reminder:

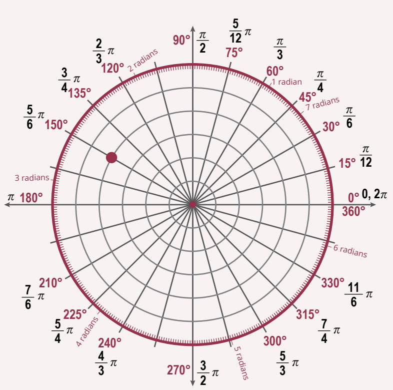
Coterminal angles have the same terminal side.

## Example

Graph *T* (4, -210°).

## Solution

Count four spaces from the origin on the polar axis and then rotate -210°.




Since -210° and 150° terminate at the same place, point *A* (4, 150°) and point *T* (4, -210°) coincide.

Since angles can be measured in either degrees or radians, the angle may be given using either unit.

### Reminder:

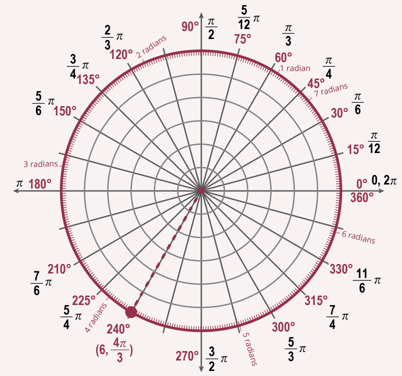
$\pi = 180°$

## Example

Graph *P* $(6, \frac{4\pi}{3})$.

## Solution

Count six spaces out from the pole and rotate $\frac{4\pi}{3}$ (one and one-third *pi*).

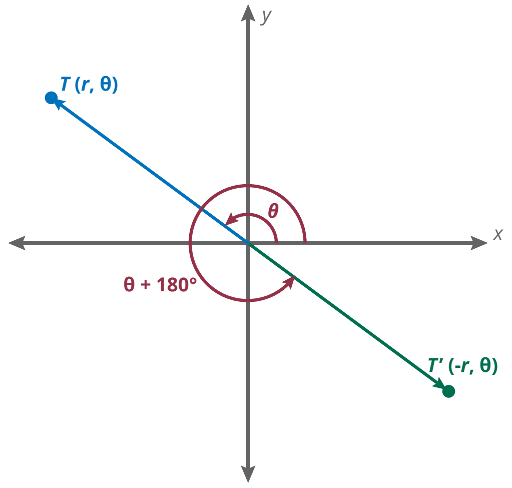



Note that $(6, -\frac{2\pi}{3})$, $(6, \frac{10\pi}{3})$, and $(6, \frac{16\pi}{3})$ are some other equivalent coordinates of point *P*.

The concept of an ordered pair is extremely important in plotting points in the rectangular coordinate system; (3, 4) is different from (4, 3). In other words, you must understand that (3, 4) means you move 3 units in the positive *x* direction and 4 units in the positive *y*. While you would typically follow the order as presented in the coordinates, 3 on *x* and then 4 on *y*, you could arrive at this same location by moving 4 on *y* first and then 3 on *x*.

In the polar system, you can also rotate the angle measure first and then determine the distance on the terminal side of the angle, *r*. When *r* is negative, the distance from the pole to the point is still $|r|$, but it is in the opposite direction.

Note in this diagram that plotting (-*r*, θ) is equivalent to plotting (*r*, θ + 180°).



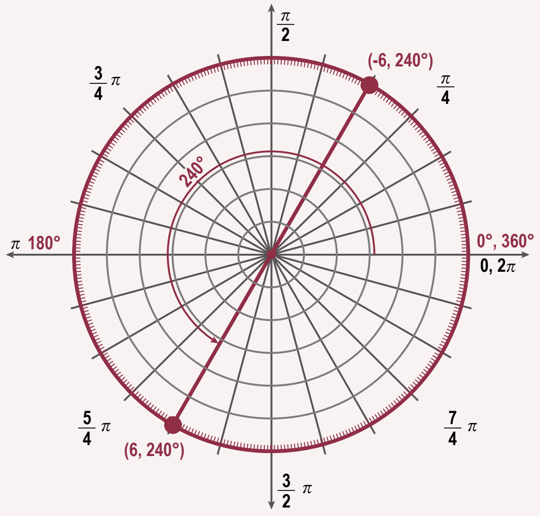

### Example

Graph *P* (-6, 240°).

### Solution

Locate 240° and count six spaces in the opposite direction that you would have if *r* had been positive 6.

The graph shows the points (6, 240°) and (-6, 240°).



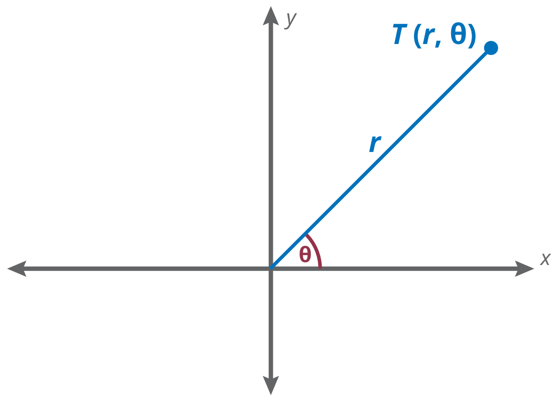

Notice that an equivalent coordinate to (-6, 240°) would be (6, 60°).

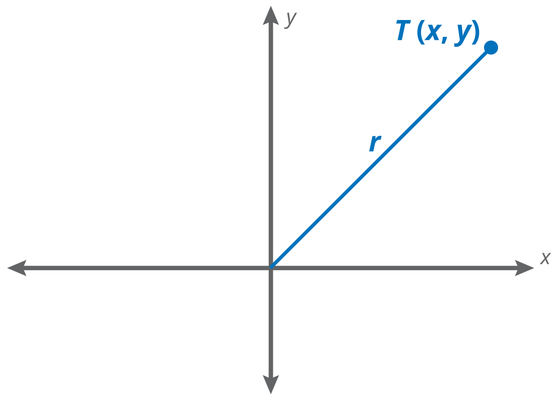
240° + 180° = 420°, and 420° is coterminal with 420° − 360° = 60°.

## CONVERTING COORDINATES

Radar scan converters take data collected by radar and convert it from polar coordinates to rectangular coordinates so that it can be viewed on a computer screen.

When the polar axis coincides with the positive *x*-axis in the Cartesian (rectangular) coordinate system, the trigonometric functions provide the link showing the relationship between the two coordinate systems.




Drop a perpendicular from point *T* to the horizontal axis in each diagram. Using the right triangles results in the following:

| Polar to Cartesian | Cartesian to Polar |
|---|---|
| $\cos\theta = \frac{x}{r}$, so<br>$x = r\cos\theta$ | $x^2 + y^2 = r^2$, so<br>$r = \sqrt{x^2 + y^2}$ |
| $\sin\theta = \frac{y}{r}$, so<br>$y = r\sin\theta$ | $\tan\theta = \frac{y}{x}$, so<br>$\theta = \tan^{-1}\frac{y}{x}$ |

To convert polar coordinates to Cartesian, use the fact that $(r\cos\theta, r\sin\theta) = (x, y)$.

### Example

Find the Cartesian form for the point whose polar form is $(2, \frac{\pi}{6})$.

### Solution

$$x = r\cos\theta = 2\cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

$$y = r\sin\theta = 2\sin\frac{\pi}{6} = 2\left(\frac{1}{2}\right) = 1$$

Therefore, the polar coordinate $(2, \frac{\pi}{6})$ is $(\sqrt{3}, 1)$ in the Cartesian system.

### Example

Convert $(-5, 225°)$ to rectangular coordinates.

### Solution

$$x = r\cos\theta = (-5)\cos 225° = (-5)\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$$

$$y = r\sin\theta = (-5)\sin 225° = (-5)\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$$

**Reminder:**

Sine and cosine are both negative in Quadrant III.

The reference angle is $225° - 180° = 45°$.

$$\cos 225° = -\cos 45°$$

$$\sin 225° = -\sin 45°$$

Therefore, the polar coordinate $(-5, 225°)$ is $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$ in the Cartesian system.

To convert rectangular coordinates to polar, use the following facts:

$$r = \sqrt{x^2 + y^2} \quad (\text{or } r^2 = x^2 + y^2)$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

### Example

Convert $(2, 2)$ to polar form.

### Solution

$$r^2 = x^2 + y^2$$
$$r^2 = (2)^2 + (2)^2$$
$$r^2 = 4 + 4$$
$$r^2 = 8$$
$$r = 2\sqrt{2}$$

Note that the point $(2, 2)$ lies in Quadrant I and then find the angle:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{2}{2}\right)$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45°$$

The polar form of $(2, 2)$ is $(2\sqrt{2}, 45°)$.

### Example

Convert $(-2, 3)$ to polar form.

### Solution

$$r^2 = x^2 + y^2$$
$$r^2 = (-2)^2 + (3)^2$$
$$r^2 = 4 + 9$$
$$r^2 = 13$$
$$r = \sqrt{13}$$

Note that the point $(-2, 3)$ lies in Quadrant II and then find the angle:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{3}{-2}\right)$$

$$\theta = \tan^{-1}(-1.5)$$

Using the calculator tells you that $\theta$ is approximately $-56.3°$, but since the inverse tangent function only gives back angles from $-90°$ to $90°$, you must use this angle as the reference angle.

**Reminder:**

The calculator uses the inverse function which has a restricted domain: $-90°$ to $90°$.

Subtract $56.3°$ from $180°$ to find $\theta$:

$$\theta = 180° - 56.3° = 123.7°$$

The polar form of $(-2, 3)$ is $(2\sqrt{13}, 123.7°)$.

## LET'S REVIEW

Before going on to the practice problems, reflect on your work and ensure that you understand all the main points of the lesson.

- The polar coordinate system for graphing uses concentric circles and rays that emanate from the center.
- The reference points in the polar system are the pole and the polar axis.
- Coordinates in the polar system are in the form ($r$, θ).
- Because there are coterminal angles, an infinite number of coordinates can plot the same point in the polar plane.
- When $r$ is negative, rotate the angle and move $r$ units in the opposite direction from the pole.
- To convert from polar to Cartesian (rectangular) coordinates, use the following information:

$$(x, y) = (r \cos \theta, r \sin \theta)$$

- To convert from Cartesian to polar coordinates, use the following information:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

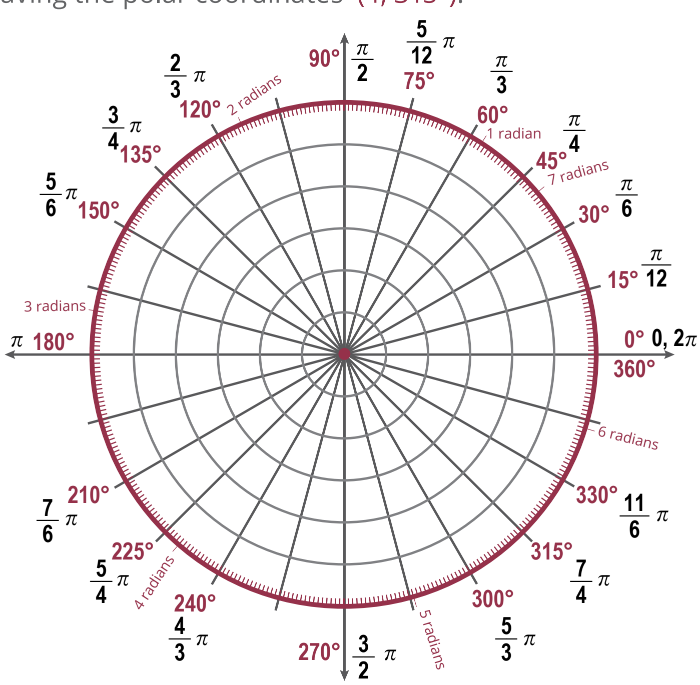
**Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.**

**Complete the following activities.**

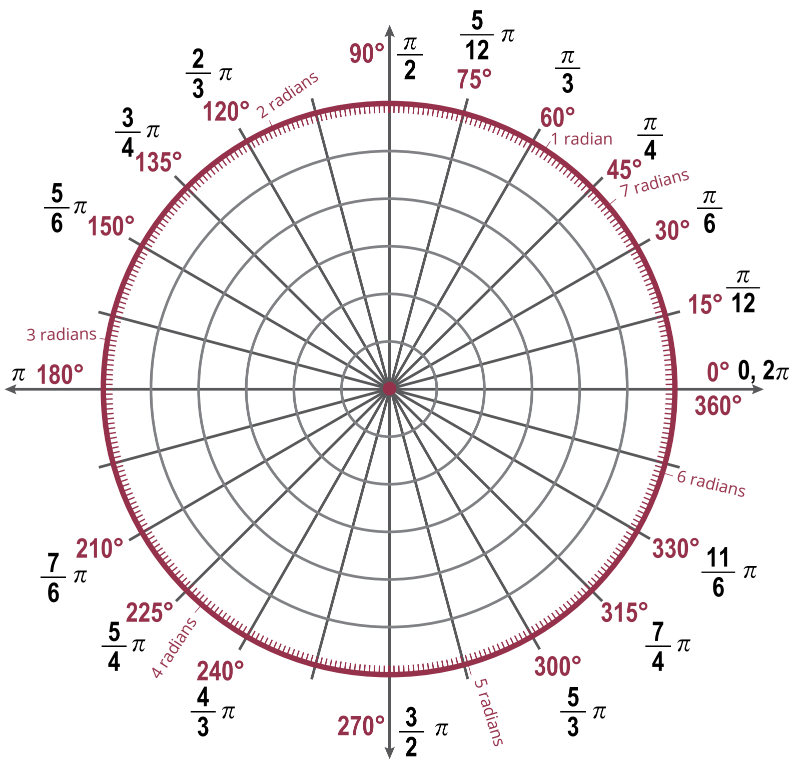
**1.1** Which of the following options is the primary purpose of a polar coordinate system? _______

a. The primary purpose of a polar coordinate system is to create ordered pairs.
b. The primary purpose of a polar coordinate system is to tell $x$ and $y$ locations.
c. The primary purpose of a polar coordinate system is to determine the position of a point.
d. The primary purpose of a polar coordinate system is to calculate lengths and angles between points.

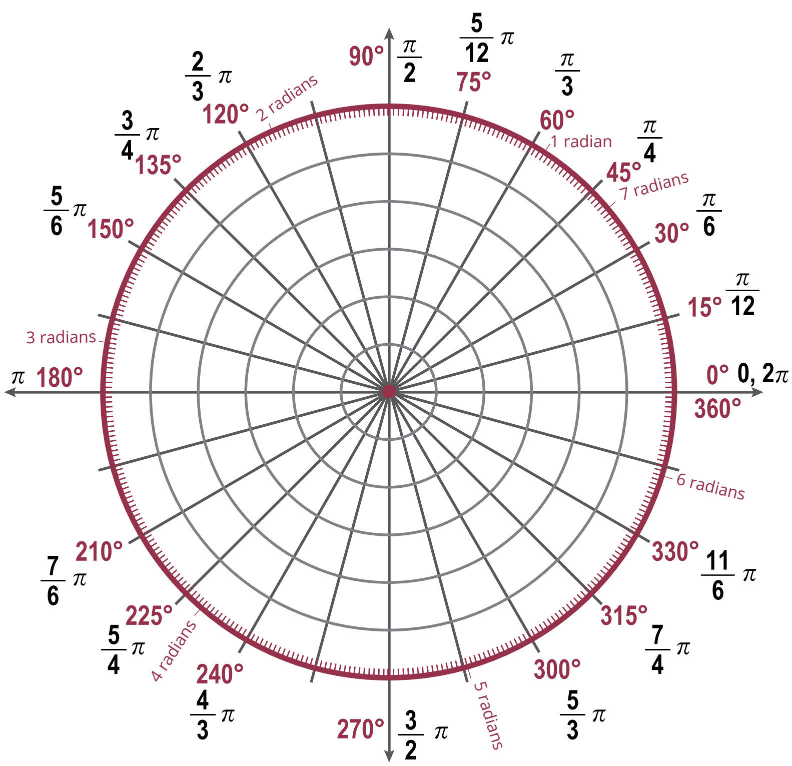
**1.2** Graph the point having the polar coordinates (4, 315°).



**1.3** Graph the points having the polar coordinates $(2, \frac{4\pi}{3})$.




**1.4** Graph the point having the polar coordinates $(2, -800°)$.



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