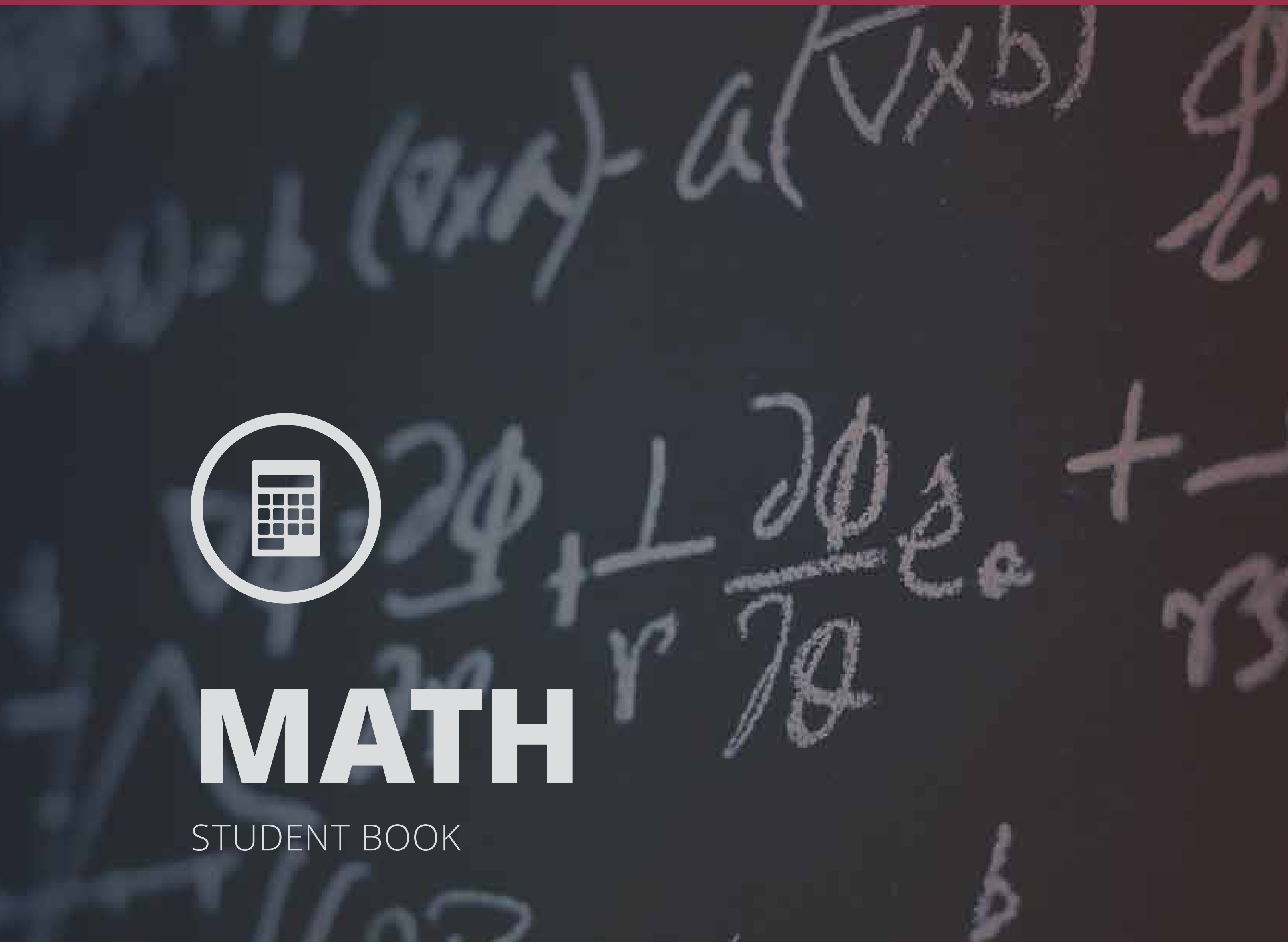
LIFEPAC®



MATH

STUDENT BOOK

12th Grade | Unit 8

Alpha Omega
PUBLICATIONS

MATH 1208

QUADRATIC EQUATIONS

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Alpha Omega
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Quadratic Equations

Introduction

In this unit, you will be studying the conic sections. Conic sections include the circle, ellipse, hyperbola, and parabola. You will study how each of these conic sections differ from each other, what makes them similar, and how to determine what type of conic is being described if you know its equation. As always, it will be helpful to take notes as you go, and refer back to them frequently. Enjoy your study of the conic sections!

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Distinguish between circles, hyperbolas, ellipses, and parabolas.
2. Identify equations of circles and relate them to their corresponding graphs.
3. Compare the standard and general forms of circle equations.
4. Use basic algebra to determine a circle's midpoint, center, and radius.
5. Use basic algebra to determine a circle's proximity to lines.
6. Find circle equations based on given variables of a circle.
7. Identify the properties of an ellipse.
8. Solve ellipse equations.
9. Find the properties of ellipses using general equations.
10. Convert between standard and general elliptical equations.
11. Identify properties of parabolas.
12. Use standard parabolic equations to find properties of parabolas.
13. Write equations in standard parabolic form.
14. Analyze standard parabolic equations.
15. Write general and standard parabolic equations based on a set of givens.
16. Apply parabolic equations to real situations.
17. Identify properties of hyperbolas.
18. Translate points and sections on graphs.
19. Translate equations to pass through a given point.
20. Understand why equations can be translated.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. CIRCLES AND ELLIPSES

THE CIRCLE

In this lesson, you will be introduced to the conic sections and in particular, the circle.

The circle, the ellipse, the parabola, and the hyperbola are called the *conic sections*. When the intersecting plane is perpendicular to the axis of the cone, the section formed is a **circle**. When the intersecting plane is rotated so that it is not perpendicular to the axis of the cone but intersects both edges of the cone, the section formed is an **ellipse**. When the intersecting plane is parallel to one edge of the cone, the section formed is a **parabola**. When the intersecting plane is parallel to the axis of the cone, the section formed is a **hyperbola**.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Distinguish between circles, hyperbolas, ellipses, and parabolas.
- Identify equations of circles and relate them to their corresponding graphs.

Vocabulary

Study these words to enhance your learning success in this section.

circle . . . . A conic section formed when the intersecting plane is perpendicular to the axis of the cone. The locus of points that are at a constant distance from a fixed point.

ellipse . . . . A conic section formed when the intersecting plane is rotated so that it is not perpendicular to the axis of the cone but intersects both edges of the cone. The locus of points the sum of whose distance from two fixed points is constant.

hyperbola . . . . A conic section formed when the intersecting plane is parallel to the axis of the cone. The locus of points such that the difference of the distances from two fixed points is constant.

locus of points . . . . The set of all points, and only those points, that satisfy a given condition.

parabola . . . . A conic section formed when the intersecting plane is parallel to one edge of the cone. The locus of points in a plane such that distances from a line (the directrix) and a point (the focus) are equal.

point circle . . . . A circle that contains only one point.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

DEFINING CONIC SECTIONS

The graphs of the conic sections can assume any position on the coordinate axis. Transformation of the equation simplifies the equation and puts its graph in standard form.

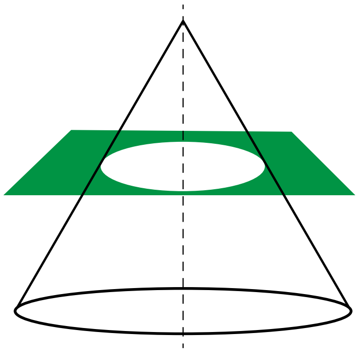
The equation for each of the conic sections will be derived from a geometric definition of **locus of points**.

> **DEFINITION**
>
> The *locus of points* is the set of all the points, and only those points, that satisfy a given condition.

A formal definition of *conic sections*, below, is also simply called *conics*.

> **DEFINITION**
>
> A *conic section* is the locus of a point that moves so that the ratio of its distance from a fixed point to its distance from a fixed line is constant.

The ratio of a curve is called its **eccentricity** and is always denoted by e. When $e = 1$, the conic is a *parabola*. When $e < 1$, the conic is an *ellipse*. When $e > 1$, the conic is a *hyperbola*. As e approaches zero, the corresponding ellipses become more and more circular, approaching the *circle* as a limit. The fixed line is called the **directrix**.

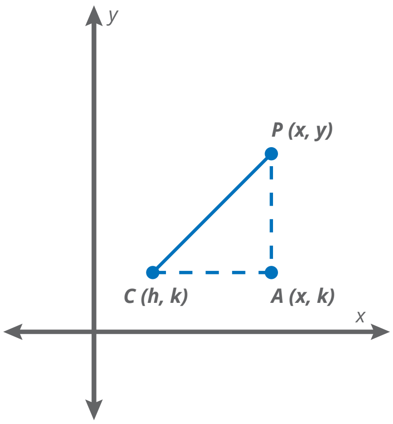


DESCRIPTION: A **circle** is the section of a cone formed when a cutting plane that is perpendicular to the axis of the cone intersects the edges of a cone.

> **DEFINITION**
>
> A *circle* is the locus of points that are at a constant distance from a fixed point.

The fixed point (h, k) is the **center** of the circle and the constant r is the radius of the circle.

The point (h, k) and the constant r are elements in the set of real numbers.



EQUATIONS OF CIRCLES

On the coordinate axis, locate a point C with center (h, k) as the fixed point and point $P\ (x, y)$ as the radius, or the point that moves.

Draw $PA \perp CA$. $\triangle CAP$ is a right triangle and $CP = r$.

Using the distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

$$\overline{CA} = \sqrt{(x - h)^2 + (k - k)^2} = \sqrt{(x - h)^2} = |x - h|$$

$$\text{and } \overline{AP} = \sqrt{(x - x)^2 + (y - k)^2} = \sqrt{(y - k)^2} = |y - k|$$

By the Pythagorean Theorem, $\boldsymbol{(x - h)^2 + (y - k)^2 = r^2}$, which is the standard form of the equation with (h, k) as the center and $|r|$ as the radius. If $(h, k) = (0, 0)$, which means that the center is at the origin, the equation becomes $x^2 + y^2 = r^2$.

> **STANDARD EQUATIONS OF THE CIRCLE**
>
> Center at origin: $x^2 + y^2 = r^2$
>
> Center at (h, k): $(x - h)^2 + (y - k)^2 = r^2$

STUDY THESE EXAMPLES:

Example 1

Find the standard equation of a circle with center at (5, -1) and radius of 6.

Solution

Given: $(h, k) = (5, -1)$ and $r = 6$.

Using the distance formula,

$$(x - h)^2 + (y - k)^2 = r^2,$$

we obtain $(x - 5)^2 + (y + 1)^2 = 36$.

Example 2

Fill in the missing numbers of the standard equation for the circle with center at (3, 5) and radius of 5.

$$(x \_\_\_\_)^2 + (y \_\_\_\_)^2 = \_\_\_\_$$

Solution

Since the center of the circle is given as (3, 5), the first blank will be "-3" and the second blank will be "-5" (don't forget to include the appropriate sign).

To find the third blank, square the radius of 5.

The final answer is:

$$(x - 3)^2 + (y - 5)^2 = 25$$

Example 3

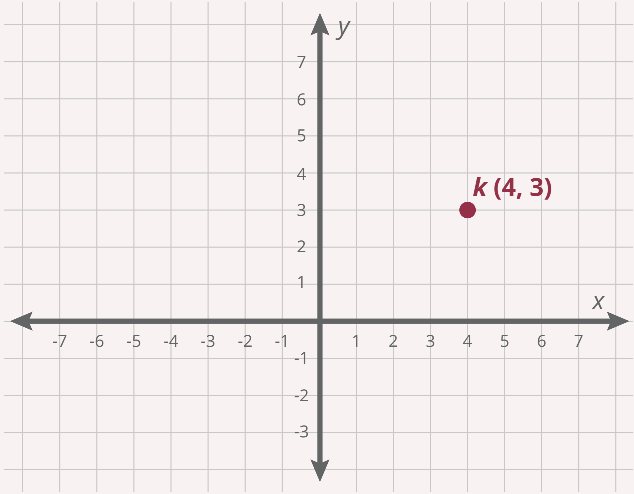
Find the standard equation of a circle with center at the origin and passing through the point (4, 3).

Solution

Using the distance formula,

$$\text{radius} = \sqrt{(4 - 0)^2 + (3 - 0)^2} = 5.$$

Therefore, $x^2 + y^2 = r^2$ and the equation of the circle is $x^2 + y^2 = 25$.



Note: If $r = 0$, so that $x^2 + y^2 = 0$, then the only point belonging to this equation is (0, 0). In this case, we call the circle a **point circle**. If $r < 0$, then the circle does not exist and the radius will be of imaginary numbers.

Match each term to its definition.

1.1 __________ a conic section formed when the intersecting plane is perpendicular to the axis of the cone

1.2 __________ a conic section formed when the intersecting plane is parallel to the axis of the cone

1.3 __________ a conic section formed when the intersecting plane is rotated so that it is not perpendicular to the axis of the cone but intersects both edges of the cone

1.4 __________ a conic section formed when the intersecting plane is parallel to one edge of the cone

1.5 __________ the set of all points that satisfy a given condition

1.6 __________ a circle that contains only one point

a. parabola
b. ellipse
c. hyperbola
d. point circle
e. locus of points
f. circle

Match the following eccentricity values with the conic section they describe.

1.7 ____________ $e < 1$

1.8 ____________ $e = 1$

1.9 ____________ $e > 1$

a. parabola
b. ellipse
c. hyperbola

Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

Complete the following activities.

1.10 Write the equation for the circle with center at (-8, -6) and radius of 10. _______

a. $(x + 8)^2 + (y + 6)^2 = 10$
b. $(x + 8)^2 + (y + 6)^2 = 100$
c. $(x - 8)^2 + (y - 6)^2 = 100$

1.11 Write the equation for the circle with center at (0, 0) and radius of 4. _______

a. $x^2 + y^2 = 4$
b. $x^2 - y^2 = 16$
c. $x^2 + y^2 = 16$

1.12 Write the equation for the circle with center at (6, 0) and radius of 6. _______

a. $(x - 6)^2 + y^2 = 36$
b. $(x + 6)^2 + y^2 = 36$
c. $(x - 6)^2 - y^2 = 36$

1.13 Write the equation for the circle with center at (1, -2) and passing through the origin. _______

a. $(x - 1)^2 + (y + 2)^2 = 0$
b. $(x - 1)^2 + (y + 2)^2 = 5$
c. $(x - 1)^2 + (y + 2)^2 = 25$

1.14 Find the standard equation for the circle with center on the positive x-axis and passing through the origin with radius of 2. _______

a. $x^2 + y^2 = 4$
b. $(x + 2)^2 + y^2 = 4$
c. $(x - 2)^2 + y^2 = 4$

1.15 Find the standard equation for the circle with center on the negative y-axis and passing through the origin with radius of 3. _______

a. $x^2 + y^2 = 9$
b. $(x + 3)^2 + y^2 = 9$
c. $x^2 + (y + 3)^2 = 9$

1.16 Choose the appropriate description for the equation $x^2 + y^2 = 36$. _______

a. circle
b. point circle
c. no circle

1.17 Choose the appropriate description for the equation $(x - 2)^2 + (y + 3)^2 = 49$. _______

a. circle
b. point circle
c. no circle

1.18 Choose the appropriate description for the equation $(x + 3)^2 + (y - 2)^2 = 0$. _______

a. circle
b. point circle
c. no circle

1.19 Choose the appropriate description for the equation $x^2 + y^2 = -4$. _______

a. circle
b. point circle
c. no circle

1.20 Choose the appropriate description for the equation $x^2 + y^2 = 0$. _______

a. circle
b. point circle
c. no circle

1.21 Choose the appropriate description for the equation $(x - 5)^2 + (y + 4)^2 = 18$. _______

a. circle
b. point circle
c. no circle

1.22 Choose the appropriate description for the equation $3x^2 + 3y^2 = -27$. _______

a. circle
b. point circle
c. no circle

1.23 Choose the appropriate description for the equation $10(x - 3)^2 + 10(y + 4)^2 = 100$. _______

a. circle
b. point circle
c. no circle

1.24 Find the center of the circle whose equation is $(x - 5)^2 + (y + 3)^2 = 6$. _______

a. (5, 3)
b. (-5, 3)
c. (5, -3)

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