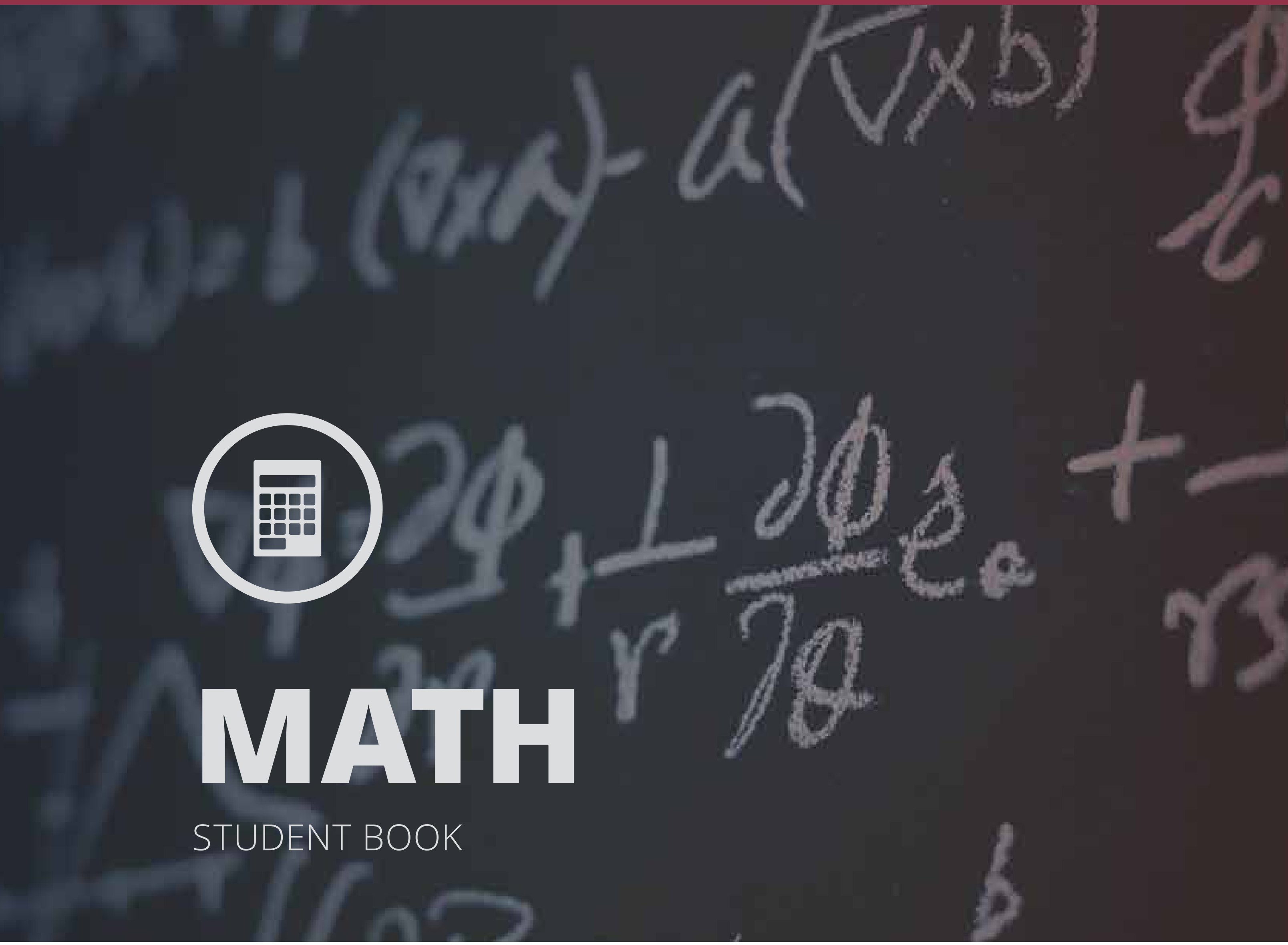
LIFEPAC®



MATH

STUDENT BOOK

12th Grade | Unit 9

Alpha Omega
PUBLICATIONS

MATH 1209
COUNTING PRINCIPLES

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Counting Principles

Introduction

This LIFEPAC® reviews all the basic concepts of probability. Did you know that modern theory of mathematical probability is a little over 300 years old? During the eighteenth century, much attention was given to the studies of popular statistics, the life insurance business, and the use of probability in the analysis of errors in physical and astronomical measurements. These studies included such fields as economics, genetics, physical science, biology, and engineering. Well, these concepts of probability are used in a person's life every day, not only in these fields, but in many other ways.

Permutations, combinations, and theory of probability can easily become difficult, confusing, and frustrating, but this LIFEPAC has done a good job at making the problems clear and precise.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Identify probability, sample space, and equally likely events.
2. Calculate the probability of an event.
3. Calculate the sample space of an event.
4. Combine probabilities by addition.
5. Understand how Venn diagrams relate to probability.
6. Apply probability addition to real situations.
7. Combine probabilities by multiplication.
8. Distinguish between mutually exclusive, independent, and dependent events.
9. Distinguish between combination and permutation.
10. Combine probabilities including multiple conditions.
11. Calculate permutations involving distinct (n) different things.
12. Calculate permutations in which some of the items are the same things.
13. Calculate circular permutations.
14. Distinguish between permutations and combinations.
15. Calculate combinations with one variable.
16. Combine combinations.
17. Use Pascal's triangle to expand binomials.
18. Understand how Pascal's triangle is used to find combinations.
19. Use the explicit formula and the recursive formula to find the nth term as well as the general term of an arithmetic sequence.

20. Use the explicit formula and the recursive formula to find the *n*th term as well as the general term of a geometric sequence.
21. Understand summation notation.
22. Calculate basic and combined summations.
23. Use sigma notation to evaluate finite sums and infinite geometric series.
24. Use sigma notation to represent arithmetic series and geometric series.
25. Identify the logic behind mathematical induction.
26. Apply mathematical induction.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. PROBABILITY

DEFINITIONS, SAMPLE SPACES, AND PROBABILITY

In this lesson, you will work with sets, subsets, sampling, and probability. This lesson explains some key terms, shows how to determine sample spaces, and shows some ways to use probability. As you read through the definitions, think of how they might apply to a practical situation. These types of problems have many real, practical applications. The examples in this lesson are only a few!

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Identify probability, sample space, and equally likely events.
- Calculate the probability of an event.
- Calculate the sample space of an event.

Vocabulary

Study these words to enhance your learning success in this section.

equally likely events . . . . Events with an equal probability of occurring.

event . . . . A subset of a sample space.

sample space . . . . The set of all possible outcomes of a random experiment.

sum of probabilities . . . . The total of all the probabilities in a sample space, equal to 1.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

INTRODUCTION TO PROBABILITY

Imagine you are about to flip a coin. What is the probability that the coin will come up heads?

You probably know that the answer is $\frac{1}{2}$ (assuming the coin is fair and no trickery is involved), but do you know why?

The answer is that there are only two ways the coin can land—heads and tails—and those two **events** are assumed to be equally likely. In general, the probability that a given event A will occur is

$$P(A) = \frac{s}{n}$$

where n is the number of **equally likely events** that are possible, and s is the number of those events that are considered successes (i.e., where A occurs). So, in the example with the coin, $n = 2$ because there are two equally likely events, and $s = 1$, because only one of those events entails the coin coming up heads.

The probability of an event happening will be a positive number from 0 to 1, inclusive.

$$0 \leq P(A) \leq 1$$

If the probability is 0, then there is no chance that the event will occur. If the probability is 1, then the event will definitely occur.

If the probability is not 0 or 1, it will lie between 0 and 1—the event might occur, or it might not.

The sum of probabilities assigned to all the elements of a sample space always equals 1. One of the possible outcomes must occur.

To write this in mathematical language, you can use the symbol A', read "A prime." This is the same as saying "not A", so if $P(A)$ is the probability that A will occur, $P(A')$ is the probability that A will *not* occur. So,

$$P(A) + P(A') = 1$$

For example, if there is a 40% chance of it raining tomorrow, $P(A) = \frac{2}{5}$. Therefore,

$$P(A') = 1 - P(A) = 1 - \frac{2}{5} = \frac{3}{5}$$

So, there is a 60% chance that it *won't* rain tomorrow.

Example

The probability that you will win a card game is $\frac{1}{4}$. What is the probability that you will *not* win the card game?

Solution

$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A) = 1 - \frac{1}{4} = \frac{3}{4}$$

The probability that you will not win is $\frac{3}{4}$.

RECOGNIZING THE SAMPLE SPACE

The sample space for a random experiment is a list of all the equally likely possibilities that can occur. For convenience, you can use abbreviations for a given possibility. For example, if you flip a coin, the sample space would be

$$\{H, T\}$$

where H represents the possibility of the coin coming up heads and T represents the possibility of the coin coming up tails.

Example

A coin is tossed two times in a row. List the sample space of this experiment.

Solution

This experiment has four possible outcomes:

First Toss	Second Toss
H	T
H	H
T	T
T	H

The sample space would be written

$$\{(H, T), (H, H), (T, T), (T, H)\}$$

Example

A coin is tossed three times in a row. List the sample space of this experiment.

Solution

Start by making a table of all the possible outcomes:

First Toss	Second Toss	Third Toss
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

So, there are eight equally likely outcomes. The sample space is

$$\{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)\}$$

Do you notice a pattern with the number of equally likely outcomes for flipping a coin a certain number of times? Look at the information you have:

1 flip	2 outcomes
2 flips	4 outcomes
3 flips	8 outcomes

It turns out that this pattern continues: if a coin is flipped n times, the number of equally likely outcomes will be 2^n. So, if a coin is flipped 4 times in a row, the sample set will contain $2^4 = 16$ possible outcomes. Later in this LIFEPAC, you will explore more deeply why this is true.

USING THE SAMPLE SPACE TO DETERMINE PROBABILITY

Experiments like flipping a coin three times in a row make the process of determining the probability of an event a little bit more interesting.

Example

The sample space for flipping a coin three times in a row is {(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)}.

Determine the probability of the following events:

A: 2 heads and 1 tail
B: 3 heads
C: 1 head and 2 tails
D: 3 tails

Solution

The number of elements in the sample space of the experiment is 8, but you need to determine how many of those 8 count as successes for each of the events you are considering.

There are three distinct ways of accomplishing A: {(H, H, T), (H, T, H), (T, H, H)}, so the probability is

$$P(A) = \frac{3}{8}$$

Similarly, there are three ways of accomplishing C: {(H, T, T), (T, H, T), (T, T, H), so the probability is

$$P(C) = \frac{3}{8}$$

There is only one way, however, that B can occur: {H, H, H}, so the probability is

$$P(B) = \frac{1}{8}$$

There is also only one way of accomplishing D: {T, T, T}, so this probability is also

$$P(D) = \frac{1}{8}$$

Notice that the sum of probabilities is

$$P(A) + P(B) + P(C) + P(D) = \frac{3}{8} + \frac{1}{8} + \frac{3}{8} + \frac{1}{8} = 1$$

This is good news—it means that you haven't missed any possibilities!

Example

Two regular tetrahedrons, one red and one green, are tossed into the air. The faces of each are numbered from 1 to 4. List a sample space of the possible rolls, if the downward face is the one that is counted. Use the sample space to determine the probabilities of the following events:

A: the sum of the down faces is 2
B: the sum of the down faces is 5
C: the sum of the down faces is 6

Solution

The sample space contains 16 possibilities:

Red	Green
1	1
1	2
1	3
1	4
2	1
2	2
2	3
2	4
3	1
3	2
3	3
3	4
4	1
4	2
4	3
4	4

Only one of these possibilities counts as a success for event A: {(1, 1)}. So,

$$P(A) = \frac{1}{16}$$

There are four ways to get a sum of 5: {(1, 4), (2, 3), (3, 2), (4, 1)}. Thus,

$$P(B) = \frac{4}{16} = \frac{1}{4}$$

There are three ways to get a sum of 6: {(2, 4), (3, 3), (4, 2)}. So,

$$P(C) = \frac{3}{16}$$

Notice that the sum of these probabilities is

$$P(A) + P(B) + P(C) = \frac{1}{16} + \frac{4}{16} + \frac{3}{16} = \frac{8}{16} = \frac{1}{2} \neq 1$$

This is because the problem didn't ask you to consider all the possible outcomes—the sum of rolling two tetrahedrons could also be 3, 4, 7, or 8.

Example

Four cards labeled 2, 3, 4, and 5, are placed into a hat. One card is drawn at random, put back into the hat, and then a second card is drawn at random. List the sample space of possible outcomes, then determine the probability of the following events:

- *A*: both cards are even
- *B*: one even and one odd card are drawn
- *C*: the sum of the two cards is ≥ 5
- *D*: the sum of the two cards is between 6 and 10, exclusive

Solution

The sample space is:

First Draw	Second Draw
2	2
2	3
2	4
2	5
3	2
3	3
3	4
3	5
4	2
4	3
4	4
4	5
5	2
5	3
5	4
5	5

There are four ways to draw two even numbers: {(2, 2), (2, 4), (4, 2), (4, 4)}. So,

$$P(A) = \frac{4}{16} = \frac{1}{4}$$

There are eight ways to draw one even and one odd: {(2, 3), (2, 5), (3, 2), (3, 4), (4, 3), (4, 5), (5, 2), (5, 4)}. So,

$$P(B) = \frac{8}{16} = \frac{1}{2}$$

Event *C* provides an opportunity to use a shortcut. There is only one way to get a sum that is *less than* 5: {(2, 2)}. So, $P(C') = \frac{1}{16}$. Since $P(C) + P(C') = 1$, $P(C) = 1 - \frac{1}{16} = \frac{15}{16}$.

There are nine ways to accomplish event *D*: {(2, 5), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5), (5, 2), (5, 3), (5, 4)}. Thus, the probability is $P(D) = \frac{9}{16}$.

In the examples shown in this lesson, all the possibilities are actually written down. However, in more complex problems, there are often so many possibilities that writing down each possible outcome is just not practical. In these cases, formulas like the one we found for the size of the sample space for flipping coins will prove very helpful, so make sure to pay attention to similar formulas during the rest of this LIFEPAC!

Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

Complete the following activities to practice what you've learned.

1.1 The set of all possible outcomes for a random experiment is called the _______ .

a. probability
b. sum of probabilities
c. sample space
d. list of events

1.2 The probability of spinning a one on a certain spinner is $\frac{3}{8}$. What is the probability of *not* spinning a one? _______

a. $\frac{8}{3}$ b. 1 c. $\frac{3}{8}$ d. $\frac{5}{8}$

1.3 There is a 20% chance of rain tomorrow. What are the chances that it will *not* rain? _______

a. 80% b. 20% c. 40% d. 5%

1.4 The senior class at a certain high school contains 300 boys and 200 girls. If a member of the class is chosen at random, what is the probability of choosing a boy? _______

a. $\frac{2}{3}$ b. $\frac{3}{2}$ c. $\frac{5}{3}$ d. $\frac{3}{5}$

1.5 A letter of the English alphabet is chosen at random. What is the probability that a vowel is chosen? (Count *y* as a vowel.) _______

a. $\frac{3}{13}$ b. $\frac{6}{20}$ c. $\frac{1}{2}$ d. $\frac{1}{26}$

1.6 You and five other people are in a drawing to win tickets to a movie. What are the chances that you will win the tickets? _______

a. $\frac{1}{5}$ b. $\frac{1}{6}$ c. $\frac{5}{6}$ d. $\frac{1}{2}$

1.7 If 550,000 sports cars were sold last year, 205,000 with six cylinders and the rest with four cylinders, what is the probability that the next sports car you see from last year will have six cylinders? _______

a. $\frac{110}{151}$ b. $\frac{41}{151}$ c. $\frac{41}{110}$ d. $\frac{1}{2}$

1.8 An integer between 1 and 50 (inclusive) is chosen at random. Find the probability that the chosen integer is divisible by 3. _______

a. $\frac{8}{25}$ b. $\frac{1}{3}$ c. $\frac{9}{50}$ d. $\frac{1}{2}$

1.9 Six cards bearing the letters *a*, *b*, *c*, *d*, *e*, and *f* are put into a hat. Two cards are drawn at random without replacement. What is the probability of drawing *at least one* vowel? _______

a. $\frac{3}{5}$ b. $\frac{2}{5}$ c. $\frac{1}{3}$ d. $\frac{2}{3}$

1.10 A jar contains 10 black paper clips, 8 red paper clips, and 2 white paper clips, all the same size. If one paper clip is drawn from the jar, what is the probability it will be black, red, or white? _______

a. $\frac{1}{2}$ b. $\frac{2}{5}$ c. $\frac{1}{10}$ d. 1

1.11 A small child knows how to write five letters: *a*, *b*, *c*, *i*, and *e*. If the child writes two of these letters at random (possibly the same letter twice), what is the probability that they are both vowels? _______

a. $\frac{2}{3}$ b. $\frac{9}{25}$ c. $\frac{2}{5}$ d. $\frac{1}{3}$

1.12 A small child knows how to write five letters: *a*, *b*, *c*, *i*, and *e*. If the child writes two of these letters at random (possibly the same letter twice), what is the probability that one is a vowel and the other is a consonant? _______

a. 1 b. $\frac{1}{3}$ c. $\frac{8}{25}$ d. $\frac{12}{25}$

1.13 An integer between 1 and 50 (inclusive) is chosen at random. What is the probability that the chosen integer is *not* divisible by 2, 7, or 9? _______

a. $\frac{9}{25}$ b. $\frac{1}{4}$ c. $\frac{1}{5}$ d. $\frac{7}{10}$

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