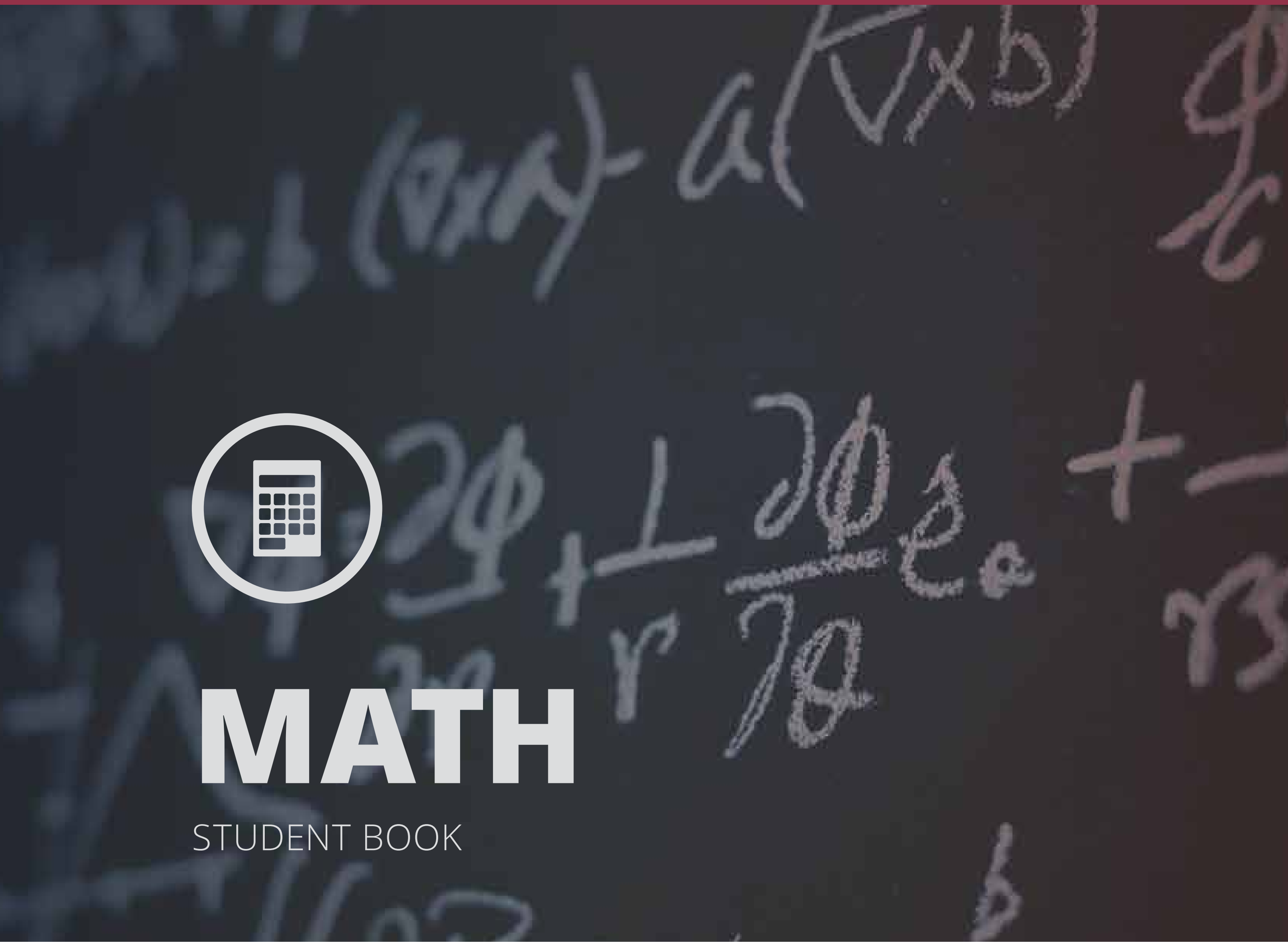
LIFEPAC®



MATH

STUDENT BOOK

12th Grade | Unit 10

Alpha Omega PUBLICATIONS

MATH 1210
CALCULUS

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Author:
Alpha Omega Publications

Editors:
Alan Christopherson, M.S.
Lauren McHale, B.A.

Media Credits:
Page 34: © ChristianChan, iStock, Thinkstock.

Alpha Omega PUBLICATIONS

804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

Calculus

Introduction

In this unit, you will learn some of the introductory concepts of calculus. You will first learn how to find limits, one of the fundamental concepts on which calculus is based. Using that skill, you will move on to finding the slopes of curves at given points. That allows you to calculate the angle where two curves meet. Lastly, you will learn how to find simple derivatives.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Find the value of functions at given points using function notation.
2. Calculate difference quotients.
3. Evaluate limits and use limit notation.
4. Calculate the slope of a function using the definition of limit.
5. Calculate the angle between two curves at the point of intersection.
6. Find the derivative of a function using the constant rule and power rule.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. LIMITS

FUNCTIONAL NOTATION

Functions and **limits** of functions are the two building blocks of calculus. All of calculus is derived by the limit of a function. This lesson introduces these two topics. For convenience, we will assume all functions to be continuous in the domain of definition. The limit theorems will be defined instead of proved. A formal proof of each limit theorem appears in any elementary calculus text.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Recognize and utilize function notation.
- Solve functions involving numbers and conditions.

Vocabulary

Study these words to enhance your learning success in this section.

function . . . . A set of ordered pairs such that for each first element there exists a unique second element.

limit . . . . The value that a function approaches as the input approaches a particular value.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

FUNCTION NOTATION

You probably recall that a function is a set of ordered-pair numbers such that for each first element there exists a unique second element. We use the notation $y = f(x)$, read "y equals a function of x," to represent these numbers. We may use $f(x)$, $f(v)$, $G(x)$, and so on to designate functions. The line $y = 2x$, using function notation, is written:

$$f(x) = 2x$$

Other functions are:

$$f(x) = x^3$$

$$g(x) = (x + 1)^2$$

$$h(x) = \sqrt{x - 1}$$

To find elements of the set of ordered-pair numbers for a function, we choose values of x in the domain of the function and compute the corresponding function values.

Example

Let $f(x) = x^2 + 2x - 1$ and $x \in \{-1, 0, 1, 5\}$. Find $f(x)$ and the set of ordered pairs $(x, f(x))$ for the domain set given.

Solution

$$f(x) = x^2 + 2x - 1$$

$f(-1) = (-1)^2 + 2(-1) - 1 = -2$ $\quad (-1, -2)$

$f(0) = (0)^2 + 2(0) - 1 = -1$ $\quad (0, -1)$

$f(1) = (1)^2 + 2(1) - 1 = 2$ $\quad (1, 2)$

$f(5) = (5)^2 + 2(5) - 1 = 34$ $\quad (5, 34)$

Then $f(x) = \{-2, -1, 2, 34\}$.

Written as ordered pairs, we have

$$(x, f(x)) = \{(-1, -2), (0, -1), (1, 2), (5, 34)\}.$$

FINDING FUNCTIONS FOR A SPECIFIC CONDITION

Example

Given $F(x) = x^2 - x$, find $F(3)$, $F(P)$, and $F(a + h)$.

Solution

$F(x) = x^2 - x$

$F(3) = 3^2 - 3 = 6$

$F(P) = P^2 - P$

$F(a + h) = (a + h)^2 - (a + h)$

$= a^2 + 2ah + h^2 - a - h$

Example

Given the function $H(x) = 3x - 1$, find the value of $H(a + h)$.

Solution

Substitute $(a + h)$ for x in the original function.

$H(a + h) = 3(a + h) - 1$

Now expand and simplify.

$H(a + h) = 3a + 3h - 1$

LET'S REVIEW

In this lesson, you have learned:

- to recognize and utilize function notation; and
- to solve functions involving numbers and conditions.

Multiple-choice questions are presented throughout this unit. To enhance the learning process, students are encouraged to show their work for these problems on a separate sheet of paper. In the case of an incorrect answer, students can compare their work to the answer key to identify the source of error.

Complete the following activities.

1.1 Functions are _______ .

a. sets of ordered pairs for which each first element has a unique second element
b. sets of ordered pairs for which each unique first element has a unique second element
c. sets of ordered pairs for which each unique first element has a second element

1.2 Given the function $F(x) = x^2 - 3x + 1$, find the value of $F(0)$. _______

a. -1
b. 0
c. 1

1.3 Given the function $F(x) = x^2 - 3x + 1$, find the value of $F(1)$. _______

a. -1
b. 0
c. 4

1.4 Given the function $F(x) = x^2 - 3x + 1$, find the value of $F(-3)$. _______

a. 19
b. 16
c. 1

1.5 Given the function $G(x) = \sqrt{2x - 1}$, find the value of $G(0)$. _______

a. -1
b. 0
c. i

1.6 Given the function $G(x) = \sqrt{2x - 1}$, find the value of $G(1)$. _______

a. 0
b. 1
c. i

1.7 Given the function $G(x) = \sqrt{2x - 1}$, find the value of $G(-4)$. _______

a. -3
b. 3
c. $3i$

1.8 Given the function $G(x) = \sqrt{2x - 1}$, find the value of $G(5)$. _______

a. $2\sqrt{2}$
b. 3
c. $3i$

1.9 Given the function $H(x) = 3x - 1$, find the value of $H(2)$. _______

a. 3
b. 5
c. 31

1.10 Given the function $F(x) = x^2 - 3x + 1$, find the value of $F(a)$. _______

a. $-a + 1$
b. $a^2 - 3a + 1$
c. $-3a + 1$

1.11 Given the function $F(x) = x^2 - 3x + 1$, find the value of $F(a - 1)$. _______

a. $a^2 - 3a + 1$
b. $a^2 - 3a + 5$
c. $a^2 - 5a + 5$

1.12 Given the function $H(x) = (3x - 1)^2$, find the value of $H(a)$. _______

a. $3a^2 + 1$
b. $9a^2 + 1$
c. $9a^2 - 6a + 1$

1.13 Given the function $H(x) = 3x - 1$, find the value of $H(a + b)$. _______

a. $a + b + 2$
b. $3a + b - 1$
c. $3a + 3b - 1$

DIFFERENCE QUOTIENT

In this lesson, you will learn about a function that is vital to the study of calculus: the **difference quotient**. The difference quotient is the backbone to techniques in calculus that allow very complicated calculations to be completed precisely.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Identify the difference quotient.
- Calculate difference quotients.

Vocabulary

Study this word to enhance your learning success in this section.

difference quotient . . . . The change in a function $f(x)$ as follows: $\frac{\Delta y}{\Delta x}$; equivalent to the slope of a line.

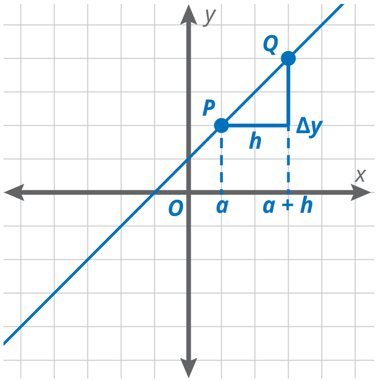
WHAT IS THE DIFFERENCE QUOTIENT?

Consider a linear function such as $f(x) = x + 1$. Select a point in the domain of y, such as $x = a$. From a on the x-axis, move to the right h units. The coordinates of point P are $(a, f(a))$, and the coordinates of point Q are $((a + h), f(a + h))$. The change in the function between point P and point Q, Δy, is represented by the following equation:

$$\Delta y = f(a + h) - f(a)$$

The change in x is h. We shall find the difference quotient of various lines and of various curves. The difference quotient is represented by the following equation:

$$\frac{\Delta y}{\Delta x} = \frac{f(a + h) - f(a)}{h}$$



Notice that the difference quotient is the same as the slope of the function at the point $(a + h, f(a + h))$. This is the beginning of understanding the calculus concept of a derivative.

Example

Given $f(x) = x^2 + 2x + 1$, find $\frac{f(a + h) - f(a)}{h}$.

Solution

$$f(a + h) = (a + h)^2 + 2(a + h) + 1 = a^2 + 2ah + h^2 + 2a + 2h + 1$$

$$f(a) = a^2 + 2a + 1$$

$$\frac{f(a + h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 + 2a + 2h + 1 - (a^2 + 2a + 1)}{h} = \frac{2ah + h^2 + 2h}{h} = 2a + h + 2$$

CALCULATING COMPLEX DIFFERENCE QUOTIENTS

There are times when we need to calculate the difference quotient for more complicated functions. Here are a few examples of how to simplify those difference quotients.

Example

For $F(x) = \frac{2x-2}{3}$, find $\frac{F(a+h)-F(a)}{h}$.

Solution

$$F(a+h) = \frac{2(a+h)-2}{3} = \frac{2a+2h-2}{3}$$

$$F(a) = \frac{2a-2}{3}$$

$$\frac{F(a+h)-F(a)}{h} = \frac{\frac{2a+2h-2}{3} - \frac{2a-2}{3}}{h} = \frac{\frac{2h}{3}}{h}$$

$$= \frac{2}{3}$$

In previous math courses, you learned how to rationalize the denominator of a fraction or expression when there was a radical in the denominator. This practice is rooted in the necessary steps to complete division without the aid of a scientific calculator. However, when working with difference quotients we do the opposite. Instead, we rationalize the numerator. This is a necessary step that will come into play later in this unit when evaluating limits.

Example

For $F(x) = \sqrt{x}$, find $\frac{F(a+h)-F(a)}{h}$.

Solution

For functions that involve radicals, we rationalize the numerator.

$$\frac{F(a+h)-F(a)}{h} = \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$$

$$= \frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

Example

For $f(x) = x^2 - 3$, find $\frac{f(a+h)-f(a)}{h}$.

Solution

$$\frac{f(a+h)-f(a)}{h} = \frac{(a+h)^2 - 3 - (a^2-3)}{h}$$

$$= \frac{a^2 + 2ah + h^2 - 3 - a^2 + 3}{h}$$

$$= \frac{2ah + h^2}{h}$$

$$= 2a + h$$

LET'S REVIEW

In this lesson, you have learned:

- to identify the difference quotient; and
- to calculate difference quotients.

Complete the following activities.

1.14 The difference quotient is closely related to _______ .

a. the curve of a parabola
b. the radius of a circle
c. the slope of a line
d. the translation of a line

1.15 If $f(x) = 2x + 3$, find $f(a + h)$. _______

a. $2a + h + 3$ b. $2a + 2h + 3$ c. $2a + 2h + 5$

1.16 If $f(x) = x - 1$, find $f(a + h) - f(a)$. _______

a. $h - 2$ b. $h - 1$ c. h

1.17 For the function $f(x) = 2x$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. a b. 1 c. 2

1.18 For the function $f(x) = 2x - 1$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. 1 b. 2 c. $\frac{2h - 2}{h}$

1.19 For the function $f(x) = \frac{3x + 2}{5}$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. $\frac{1}{5}$ b. $\frac{3}{5}$ c. $\frac{7}{5}$

1.20 For the function $f(x) = 5 - 7x$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. -7 b. 1 c. $-14x - 7$

1.21 For the function $f(x) = \frac{3 - 7x}{4}$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. $-\frac{16}{5}$ b. $-\frac{8}{5}$ c. $\frac{1}{5}$

1.22 For the function $f(x) = x^2 - x$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. $h - 1$ b. $2a + h$ c. $2a + h - 1$

1.23 For the function $f(x) = 3x^2 + 1$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. $3h$ b. $6a + 3h$ c. $\frac{3a + 2}{h}$

1.24 For the function $f(x) = x^3$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$. _______

a. h^2 b. $a^2 + ah + h^2$ c. $3a^2 + 3ah + h^2$

Alpha Omega
PUBLICATIONS

804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

800-622-3070
www.aop.com

MAT1210 – Jul '18 Printing

ISBN 978-0-7403-3860-1

9 780740 338601