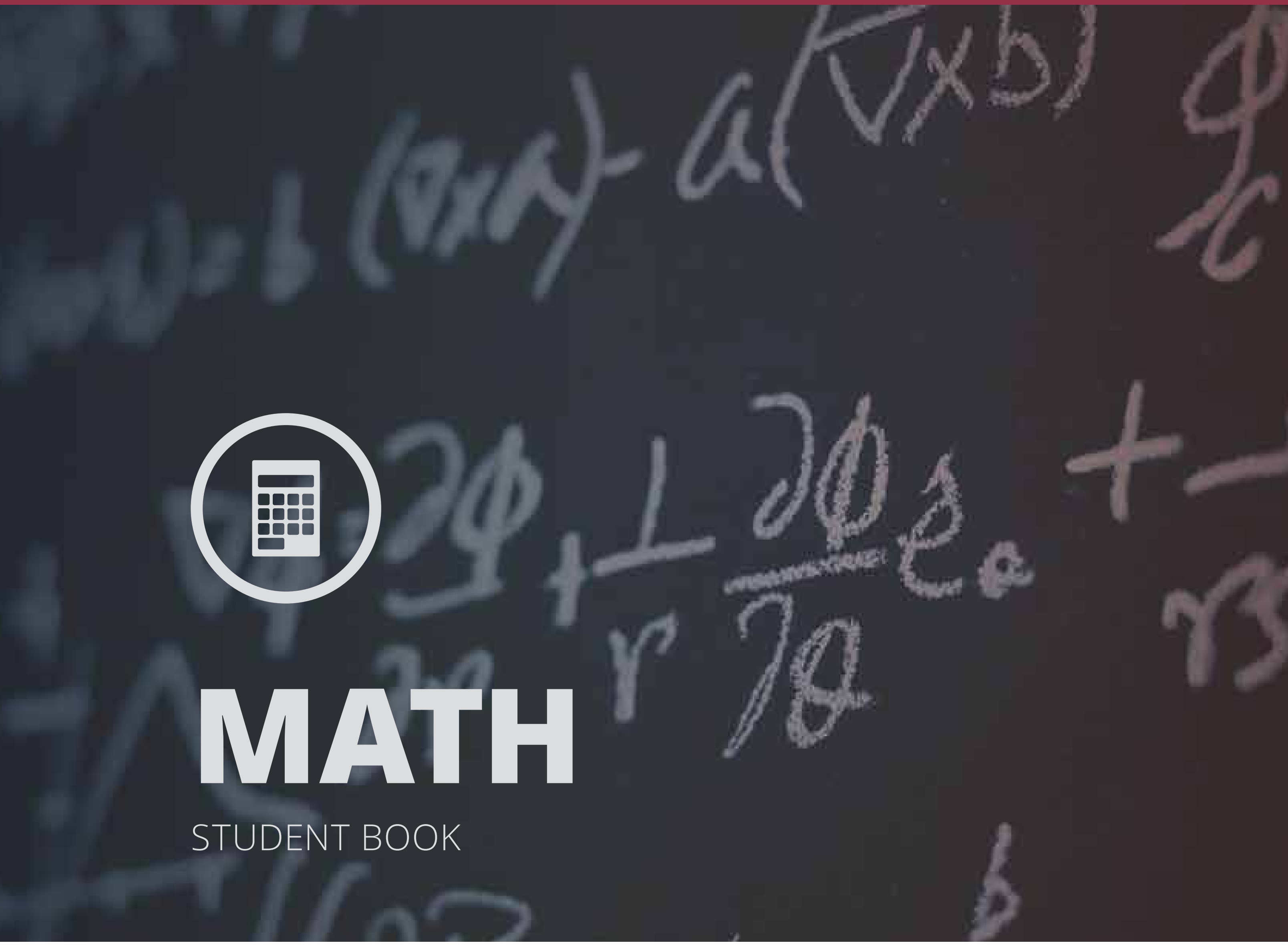
LIFEPAC®



MATH

STUDENT BOOK

10th Grade

Alpha Omega
PUBLICATIONS

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MATH 1001
A Mathematical System

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Proof

Introduction

One of the main categories of items of our geometric system is the properties we call theorems. Theorems are general statements that can be proved. This LIFEPAC® presents methods of proving theorems by using logical thinking and deductive reasoning.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify the various compound sentences.
2. Use truth tables for compound sentences.
3. Explain the difference between inductive and deductive reasoning.
4. Use deductive reasoning in proofs.
5. Describe the six parts of a two-column proof.
6. Write an indirect proof.

1. ANGLE DEFINITIONS AND MEASUREMENT

To continue our study of geometry, we must learn basic angle definitions and measurement methods. Definitions will help in identifying and classifying angles; measurement will allow us to add and subtract angles.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Identify angles as acute, right, or obtuse.
2. Find the measure of angles with a protractor.
3. Add and subtract measures of angles.

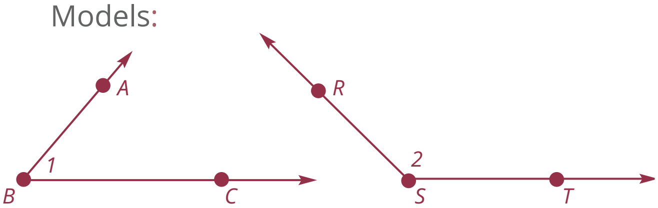
ANGLE DEFINITIONS

These definitions relate to angles. Make sure you know them, because you will be using them many times in this LIFEPAC.

> **DEFINITION**
> **Angle** ($\angle$): the union of two noncollinear rays that have a common end point.

The two rays that form the angle are called its *sides*, and the common end point is called the *vertex* of the angle. The symbol for angle is $\angle$.

Models:

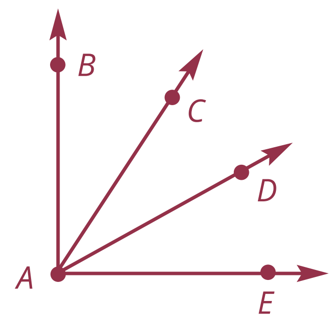


The angle to the left is formed by the union of $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Its sides are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Its vertex is B. We can name the angle $\angle ABC$, $\angle CBA$, $\angle B$, or $\angle 1$. The angle to the right is the union of $\overrightarrow{SR}$ and $\overrightarrow{ST}$. The vertex is point S. This angle can be called $\angle RST$, $\angle TSR$, $\angle S$, or $\angle 2$.

Notice that when three letters are used to name an angle, the vertex letter is always placed between the other two.

If no confusion will result, the vertex letter can be used alone; or a numeral can be used. You should never use a single letter when several angles have the same vertex.

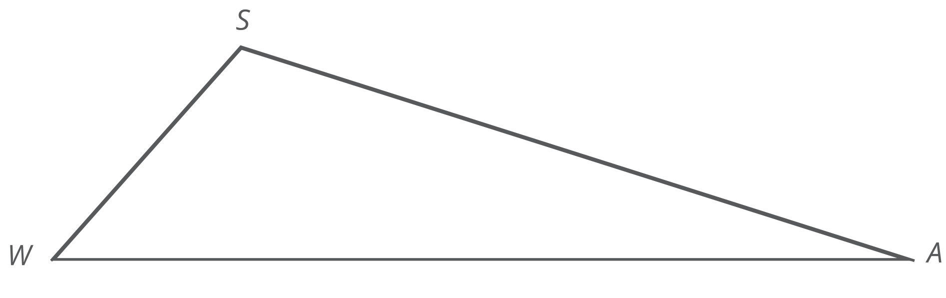
Model:



$\angle A$ could mean:

- $\angle BAC$, the angle formed by $\overrightarrow{AB}$ and $\overrightarrow{AC}$; or
- $\angle CAD$, the angle formed by $\overrightarrow{AC}$ and $\overrightarrow{AD}$; or
- $\angle DAE$, the angle formed by $\overrightarrow{AD}$ and $\overrightarrow{AE}$; or
- $\angle BAD$, the angle formed by $\overrightarrow{AB}$ and $\overrightarrow{AD}$; or
- $\angle CAE$, the angle formed by $\overrightarrow{AC}$ and $\overrightarrow{AE}$; or
- $\angle BAE$, the angle formed by $\overrightarrow{AB}$ and $\overrightarrow{AE}$.

Name the included angle or the included side asked for.



1.28 Included angle between $\overline{WA}$ and $\overline{AS}$: ______________

1.29 Included angle between $\overline{SW}$ and $\overline{WA}$: ______________

1.30 Included side between $\angle S$ and $\angle A$: ______________

1.31 Included side between $\angle S$ and $\angle W$: ______________

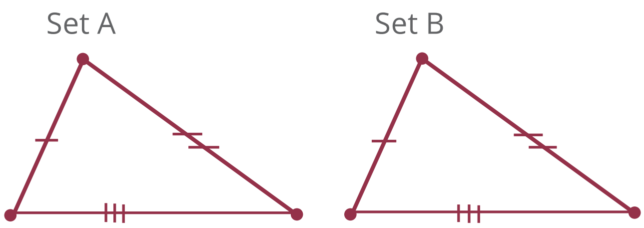
1.32 Included angle between $\overline{SW}$ and $\overline{AS}$: ______________

1.33 Included side between $\angle A$ and $\angle W$: ______________

PROVING TRIANGLES CONGRUENT

Suppose you take two identical sets of three sticks with the sticks in one set the same length as the sticks in the other set.

Connect the three sticks in each set at their end points.



However you put the sticks together, the two Δ's formed will be the same size and shape. The two triangles will be congruent. This result suggests the following postulate.

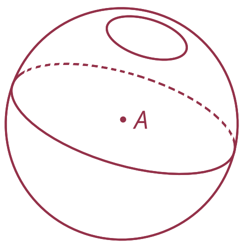
> **POSTULATE 11**
>
> **P11:** If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.
>
> **(SSS Postulate)**

Postulate 11 states that we only need to show that three sides of one triangle are equal to three sides of the other triangle for the triangles to be congruent. We do not need to know anything about the angles to use this postulate. The following two postulates can be used to prove triangles congruent in other ways.

CHARACTERISTICS OF SPHERES

Many properties and parts of a circle suggest similar properties and parts of a sphere. Notice, for example, how close the definition of a circle and a sphere are to each other.

> **DEFINITION**
>
> **Sphere:** the set of all points that are the same distance from a given point.



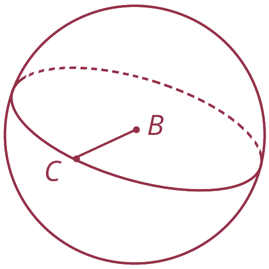
Sphere *A*

Omitting the words **in a plane** from the definition of a circle gives the definition of a sphere. A sphere is named by its center just like a circle is.

Spheres and spherically shaped objects are all about us. Many sports and games are played with balls that are spheres: baseball, tennis, golf, marbles. We eat spherically shaped foods; apples, oranges, tomatoes, cherries. We even live on a sphere.

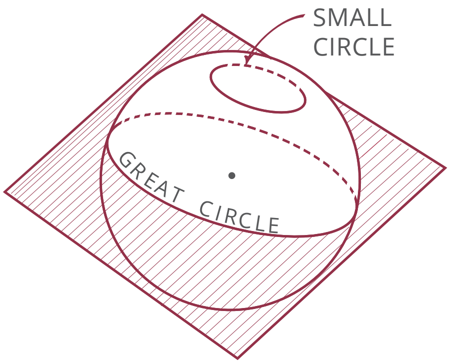
We can often define terms used for spheres by replacing the word **circle** with **sphere** in the definitions for the circle. The center of a sphere is the point from which all the rest are equidistant.

> **DEFINITIONS**
>
> **Radius of Sphere:** a segment with endpoints at the center of the sphere and a point on the sphere
>
> **Diameter of a sphere:** a segment passing through the center with endpoints on the sphere.
>
> **Congruent spheres:** spheres that have equal radii.



A sphere divides space into three sets. Points that are exterior to the sphere have a distance greater than the radius from the sphere. Points that are interior to the sphere have a distance that is less than the radius. Points of the sphere have a distance equal to the radius. Remember, the sphere itself is only the shell of points around its center.

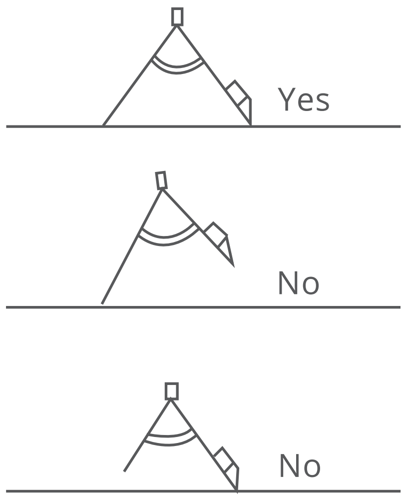
If a plane intersects a sphere at many points, the intersection will be a circle. If the plane does not contain the center of the sphere, the intersection is called a **small circle**. If the plane contains the center, the intersection is called a **great circle**. The equator of our earth is a great circle.



Spheres may be concentric to one another, similar to concentric circles.

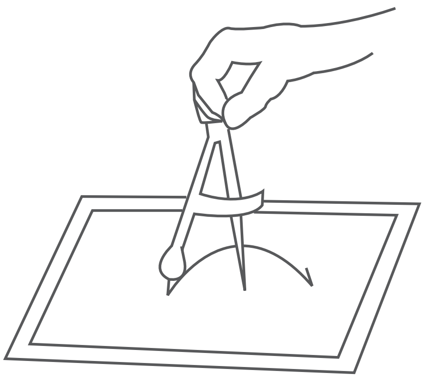
> **DEFINITIONS**
>
> **Great circle:** the intersection of a sphere and a plane containing the center of the sphere.
>
> **Small circle:** the intersection of a sphere and a plane not containing the center of the sphere.
>
> **Concentric spheres:** two or more spheres that have the same center.

other point contains the writing tool, usually a pencil. The pencil point is adjustable up and down. The compass should be adjusted so that the tip of the pencil is the same length as the tip of the center point. The distance between the two points will be the radius of the circle you are drawing.



When using a compass, you may wish to back your paper with some thin cardboard so that the center point of the compass will stick in and not slide off your paper.

When drawing a circle, hold the compass at the top. Do not hold the legs of the compass, or you may change the radius of your circle.



Tip the compass slightly in the direction you are drawing so that it is not perpendicular to the paper and pull the compass through your circle. Never push the compass, because pushing may tear your paper.

Use your straightedge to construct lines through the given points. Use your compass to construct circles.

1.1 $\overline{AB}$

1.2 $\overrightarrow{BC}$

1.3 $\overleftrightarrow{DC}$

1.4 $\overleftrightarrow{BD}$

1.5 $\overline{AD}$

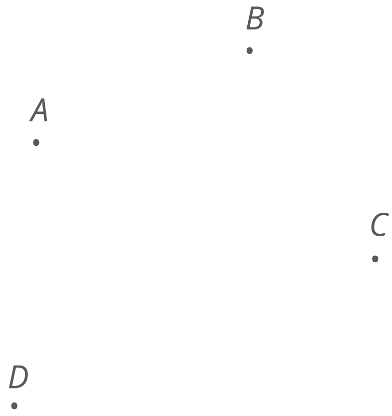
1.6 $\overrightarrow{AC}$

1.7 Circle *A*, any radius

1.8 Circle *B*, any radius

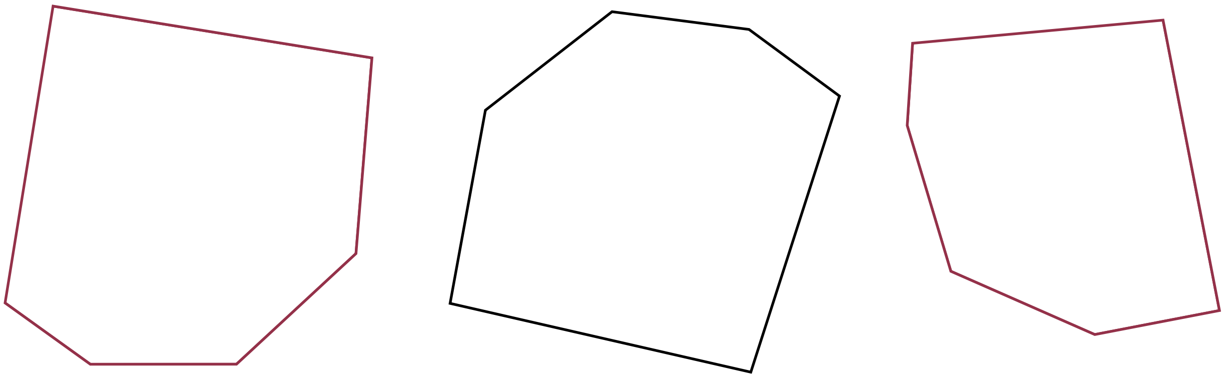
1.9 Circle *C*, same radius as circle *B*

1.10 Circle *D*, same radius as circle *A*

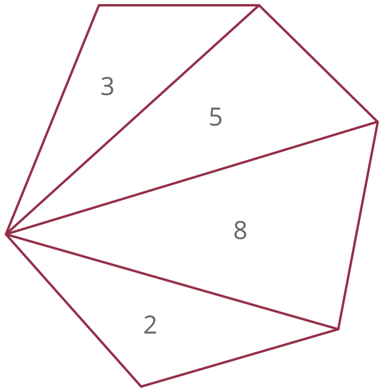


Complete the following activities.

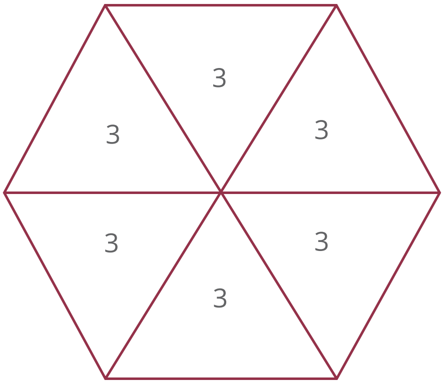
1.1 Divide the three similar polygons into triangular regions in three different ways.



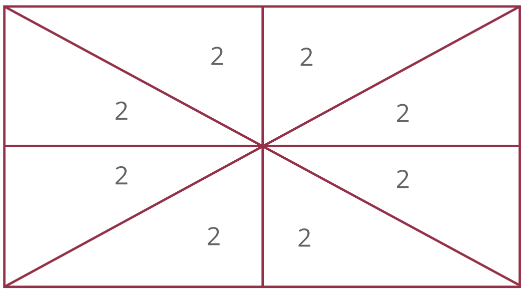
1.2 If the area of each of the triangular regions is as shown, what is the area (A) of each polygon?



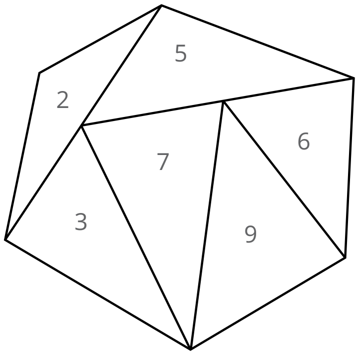
a. A = ____________________



b. A = ____________________



c. A = ____________________



d. A = ____________________

SELF TEST 1

Name the undefined terms for which each is a model (each answer, 3 points).

Figure 1

Figure 2

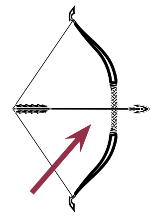


Figure 3

1.01 ________________________________

1.02 ________________________________

1.03 ________________________________

Figure 4

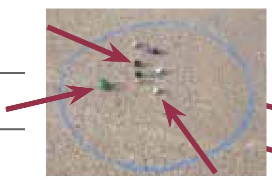


Figure 5

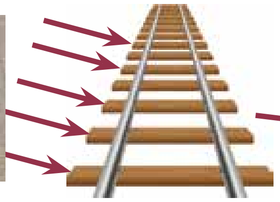


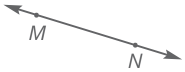
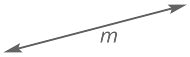
Figure 6

1.04 ________________________________

1.05 ________________________________

1.06 ________________________________

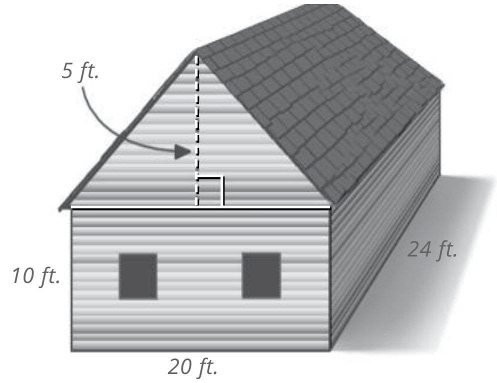
Match the description in Column II with its model in Column I (each answer, 2 points).

Column I			**Column II**
1.07		______________	a. $\overleftrightarrow{MN}$
1.08		______________	b. Point *T*
1.09		______________	c. Plane *R*
1.010		______________	d. Plane *S*
1.011		______________	e. Point *B*
1.012		______________	f. Line *m*

Solve the following problem (5 points).

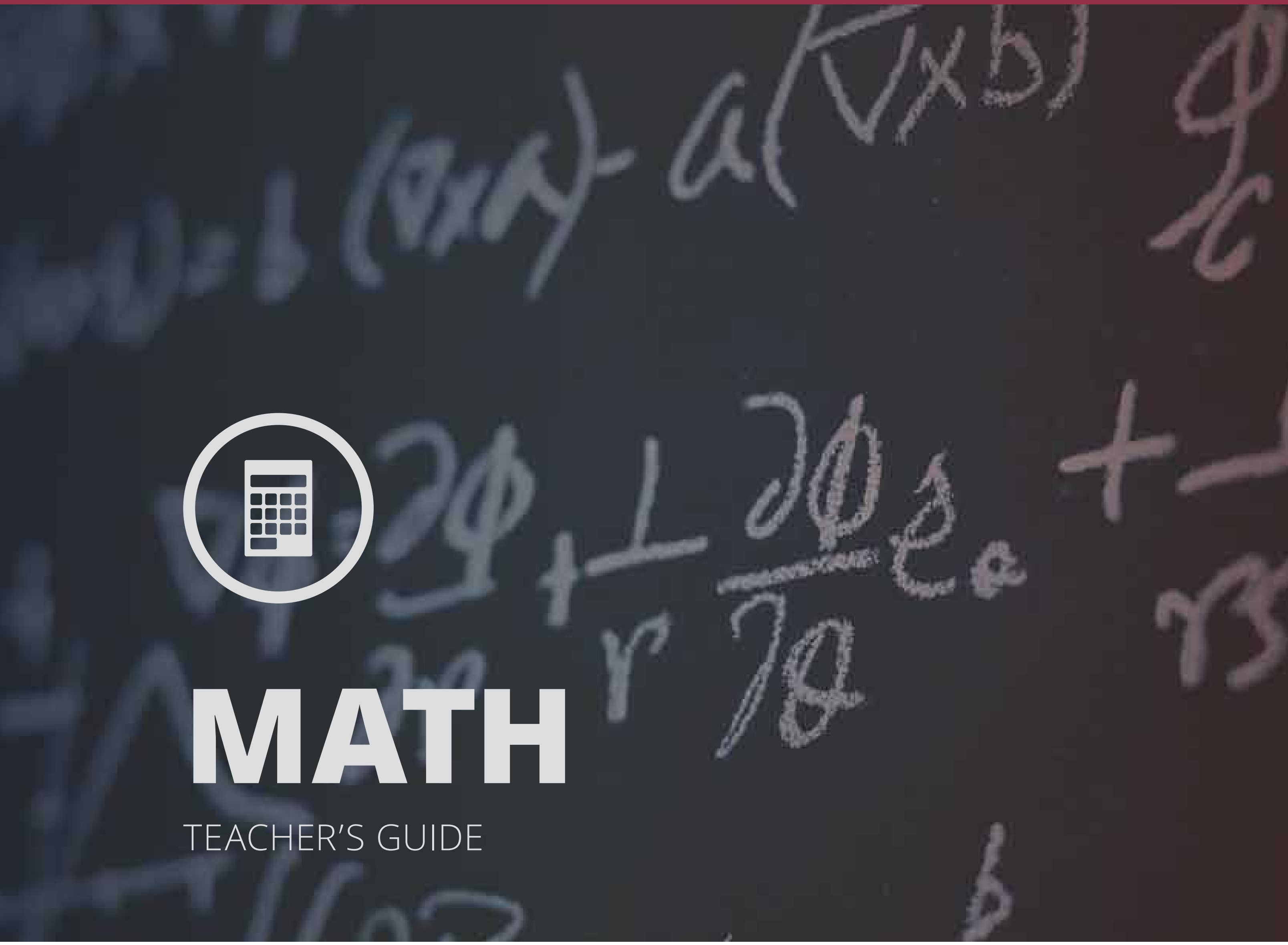
1.022 Find the number of gallons of paint needed to cover the sides of the building shown with two coats of paint if one gallon covers 350 square feet. Disregard windows and doors. Round to the nearest gallon.

____



59/73 **SCORE** ____________ **TEACHER** ____________ ____________
initials date

LIFEPAC®



MATH

TEACHER'S GUIDE

10th Grade

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MATH 1000

Teacher's Guides

INSTRUCTIONS FOR TENTH GRADE MATHEMATICS

The LIFEPAC curriculum from grades two through twelve is structured so that the daily instructional material is written directly into the LIFEPACs. The student is encouraged to read and follow this instructional material in order to develop independent study habits. The teacher should introduce the LIFEPAC to the student, set a required completion schedule, complete teacher checks, be available for questions regarding both content and procedures, administer and grade tests, and develop additional learning activities as desired. Teachers working with several students may schedule their time so that students are assigned to a quiet work activity when it is necessary to spend instructional time with one particular student.

Mathematics is a subject that requires skill mastery. But skill mastery needs to be applied toward active student involvement. Measurements require measuring cups, rulers, empty containers. Boxes and other similar items help the study of solid shapes. Construction paper, beads, buttons, beans are readily available and can be used for counting, base ten, fractions, sets, grouping, and sequencing. Students should be presented with problem situations and be given the opportunity to find their solutions.

Any workbook assignment that can be supported by a real world experience will enhance the student's ability for problem solving. There is an infinite challenge for the teacher to provide a meaningful environment for the study of mathematics. It is a subject that requires constant assessment of student progress. Do not leave the study of mathematics in the classroom.

The Teacher Notes section of the Teacher's Guide lists the required or suggested materials for the LIFEPACs and provides additional learning activities for the students. Additional learning activities provide opportunities for problem solving, encourage the student's interest in learning and may be used as a reward for good study habits.

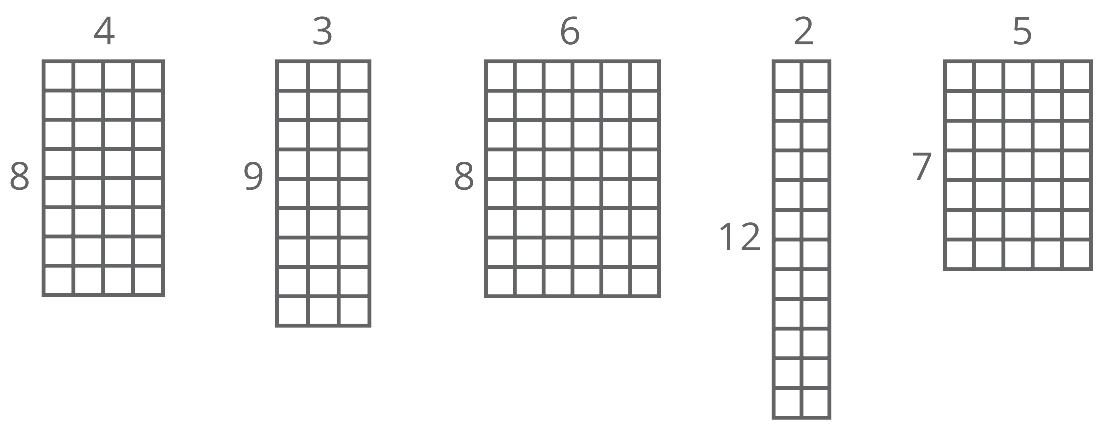
TEACHING NOTES

MATERIALS NEEDED FOR LIFEPAC	
Required	Suggested
(None)	• an instrument to make straight lines such as a ruler or straightedge

ADDITIONAL LEARNING ACTIVITIES

Section 1: Undefined Terms

1. On graph paper, have students make each of the following diagrams that represent tables. Dimensions should be written along the sides.



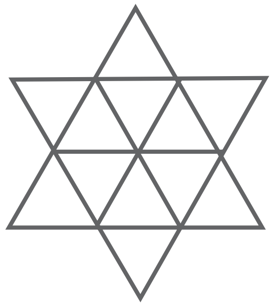
Have students draw the path a small ball would take for each table, starting at the lower left-hand corner, and moving the ball at a 45° angle with each side of the table. The ball always moves one unit up or down for one unit left or right. If the ball stops in a corner, mark the corner with a large dot. At that point the path of the ball terminates; otherwise, it continues rebounding at a 45° angle as it hits each side or end.

Do you think the ball will always end up in a corner?

If the ball starts from the lower left-hand corner, do you think it can stop in any of the four corners?

Section 2: Basic Definitions

1. Discuss these questions with your class.
   a. Can a ray have more than one name?
   b. Can a ray have two end points?
   c. How many line segments are in a line?
2. Have students draw the following figure. Ask them to determine the number of triangles of any size in the figure.



3. During their study of geometry, the students will be learning the definition of many terms. Encourage them to learn each new term as it is presented because later terms will be defined by using earlier terms. New terms will be defined as they need them in their study of geometry. Many of the definitions, theorems, and postulates in this unit will be needed in later units. Lists of these kept and maintained will be very helpful for future reference. The student should start a notebook now! Then as definitions, theorems, and postulates are given in the LIFEPAC they should be added to the notebook and used for reference.

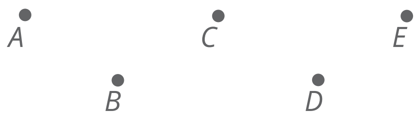
Section 3: Geometric Statements

1. Discuss these questions with your class.
   a. Are any two points always collinear?
   b. Will any two noncollinear lines intersect?
   c. Do any postulates and theorems exist other than the ones used in the LIFEPAC?
2. Research Euclid, a Greek mathematician of 300 B.C., for whom Euclidean geometry is named.
3. Write several general statements such as “all rectangles have four sides.” Then write several specific statements such as “a square has four equal sides.” Devise a postulate or a theorem of your own. Remember that a postulate is a statement accepted without proof and that a theorem is a general statement that can be proved. Prove your theorem(s).

ANSWER KEYS

SECTION 1

1.1 location or position
1.2 a dot
1.3 Example:



1.4 infinite number
1.5 none
1.6 no
1.7 points
1.8 straight
1.9 a. $\overleftrightarrow{AB}$
b. $\overleftrightarrow{CD}$
c. $\overleftrightarrow{EF}$
1.10 infinite number
1.11 A line exceeds indefinitely in both directions.
1.12 a. flat
b. points
1.13 a. plane *R*
b. plane *S*
c. plane *T*
1.14 infinitely long
1.15 no thickness
1.16 no

SELF TEST 1

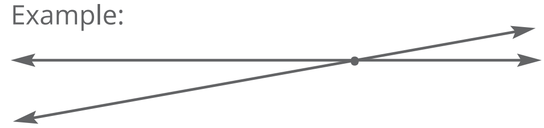
1.01 plane (table top)
1.02 line (arrow)
1.03 planes (cover and pages of book)
1.04 points (marbles)
1.05 lines (parallel railroad tracks)
1.06 points (freckles)
1.07 e
1.08 c
1.09 a
1.010 b
1.011 f
1.012 d
1.013 $\overleftrightarrow{AC}$
1.014 a. intersects
b. *E*
1.015 point *T*
1.016 *B* or *R* (same plane)
1.017 a. $\overleftrightarrow{AC}$
b. line *x*
c. line *w*
1.018 *S*
1.019 *S*, *E*, *A*, *C*, *T*
1.020 *C*, *R*, *A*, *B*

SECTION 3

3.1 Postulate 5: If two planes intersect, then their intersection is a line.
3.2 one
3.3 Postulate 2: Through any two different points, exactly one line exists.
3.4 a. no
b. Postulate 2: Through any two different points, exactly one line exists.
3.5 a. no
b. The three points cannot be on one line.
3.6 Postulate 1: Space contains at least four points not all in one plane.
3.7 Postulate 2: Through any two different points, exactly one line exists.
3.8 Postulate 3: Through any three points that are not on one line, exactly one plane exists.
3.9 Postulate 4: If two points lie in a plane, the line containing them lies in that plane.
3.10 Postulate 1: A plane contains at least three points not all on one line.
3.11 false (undefined terms are used to state some postulates)
3.12 false (a postulate does not require proof)
3.13 false (two planes intersect in exactly one line)
3.14 true
3.15 false (a plane must have at least 3 points)
3.16 false (the intersection of two planes is exactly one line)
3.17 the multiplication by one postulate
3.18 the commutative postulate for addition
3.19 the distributive postulate
3.20 the addition of zero postulate
3.21 the additive inverse postulate
3.22 the multiplication by one postulate
3.23 the addition of zero postulate
3.24 the commutative postulate of multiplication
3.25 the distributive postulate
3.26 the multiplicative inverse postulate
3.27 the addition postulate of inequality
3.28 the multiplication postulate of inequality
3.29 the multiplication postulate of inequality
3.30 the transitive postulate of equality
3.31 the symmetric postulate of equality
3.32 the comparison postulate
3.33 the multiplication postulate of inequality
3.34 the transitive postulate of inequality
3.35 the multiplication postulate of inequality
3.36 the reflexive postulate of equality
3.37 three collinear points
Example:

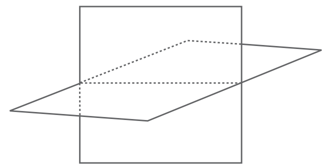
3.38 three noncollinear points
Example:

3.39 two intersecting lines
Example:

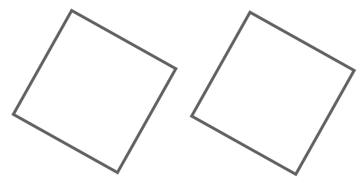


3.40 two nonintersecting lines
Example:

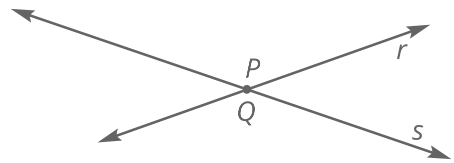
3.41 two intersecting planes
Example:



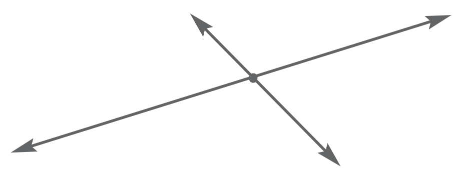
3.42 two nonintersecting planes
Example:



3.43 false (skew lines do not lie in one plane)
3.44 true
3.45 false (two intersecting lines lie in one plane)
3.46 false (three noncollinear points determine a plane)
3.47 true
3.48 They are the same point. Or, they are the point of intersection.

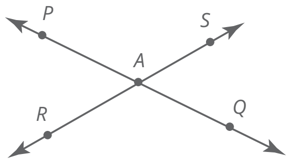


3.49 Theorem 1-1: If two lines intersect, then their intersection is exactly one point.



ALTERNATE LIFEPAC TEST

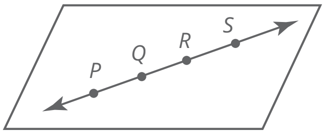
1. e
2. d
3. b
4. a
5. c
6. collinear
7. Either order:
a. *P*
b. *Q*
8. *RS*
9. midpoint
10. postulate
11. theorem
12. line
13. four
14. four
15. c
16.



17.

18.

19.



20.

MATH 1001

ALTERNATE LIFEPAC TEST

NAME ______________________________

DATE ______________________________

SCORE ______________________________

54
68

After each model in Column I, write the matching term from Column II (each answer, 2 points).

Column I

1. 

______

2. 

______

3. 

______

4. 

______

5. 

______

Column II

a. $\overline{AB}$

b. Point Q

c. $AB + BC = AC$

d. $\overleftrightarrow{AB}$

e. $\overrightarrow{AB}$

Complete the following statements (each answer, 3 points).

6. Two or more points all on the same line are called ______________________________ points.

7. The two end points of $\overline{PQ}$ are a. ______________ and b. ______________ .

8. If point P is between R and S, then $RP + PS$ = ______________________ .

9. If $AB = BC$ on AC, point B is called the ______________________ of $\overline{AC}$.

10. A __________________ is a statement we accept without proof.

11. A __________________ is a statement we must prove.

12. A __________________ contains at least two points.

13. Space contains at least _________________ points.

14. How many planes are determined by four noncoplanar points? __________________

15. _________________ + (-c) = 0.

Sketch and label the following conditions (each answer, 5 points).

16. Two lines, $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$, intersecting in a point *A*.

17. A line *n* and a point *B* not on *n* that are both in plane *X*.

18. A segment with midpoint *A* and end points *C* and *D*.

19. Collinear and coplanar points *P*, *Q*, *R*, and *S*.

20. Opposite rays $\overrightarrow{XY}$ and $\overrightarrow{XZ}$.