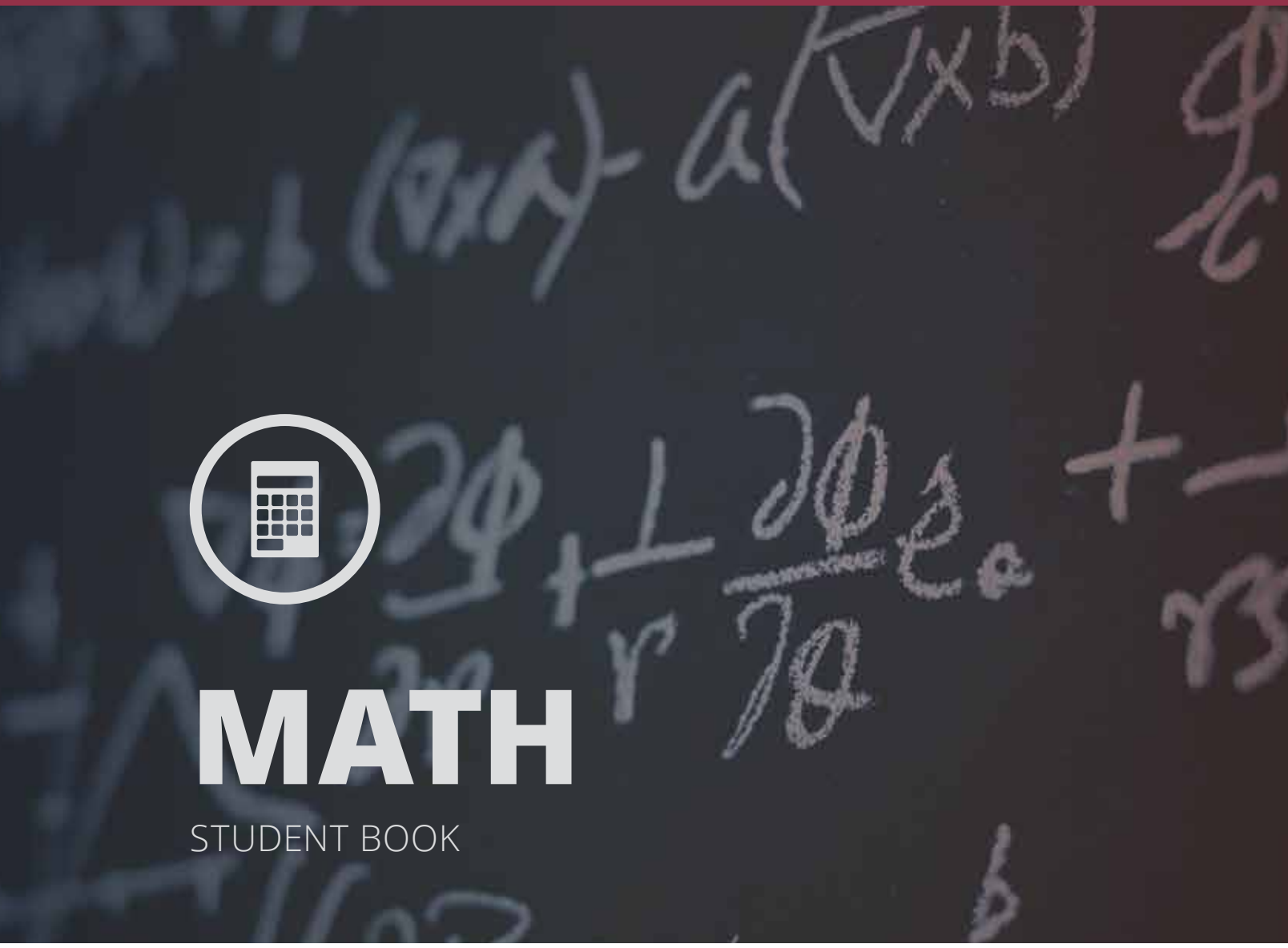
LIFEPAC®



MATH

STUDENT BOOK

11th Grade

Alpha Omega PUBLICATIONS

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MATH 1101

SETS, STRUCTURE, AND FUNCTIONS

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Numbers, Sentences, and Problems

Introduction

Someone has said, "Numbers make expressions, expressions make sentences, and sentences solve problems." Our ultimate goal in the study of math, and particularly of algebra, is to *symbolize* a problem and solve it. Many skills at all levels are needed to accomplish this task. This LIFEPAC® consists of a review of some basic skills relating to numbers, methods of solving equations and inequalities of one variable, and the application of the sentence to problem solving.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Evaluate expressions involving absolute values and integers.
2. Solve linear equations of one variable.
3. Solve and graph linear inequalities.
4. Solve and graph compound sentences.
5. Solve application problems.

1. LINES

Lines are determined by sets of points. Lines can be represented by plotting these points, thus constructing the graph of the line. From the set of points, the equations of the line can also be written. Three equation forms are presented in this section.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Find solutions to the linear functions.
2. Graph the linear function.
3. Identify the slope of a line.
4. Write the equations of lines.

GRAPHS

The graph of a line is the configuration of the set of points that are plotted on the coordinate axes. The points are solutions to a particular equation, the linear function. A linear equation has one or two variables and no variable is raised to a power other than 1 or is used as the denominator of a fraction. The slope of the line is then calculated from the solution set.

FINDING SOLUTIONS

Consider the equation $y = 2x + 3$. To find solutions of this equation, we arbitrarily give the number x a value and find the corresponding y value.

Thus for $y = 2x + 3$,
If $x = 2$, then $y = 2(2) + 3$;
or $y = 7$

One solution is therefore, the ordered pair (2, 7). Another solution would be (3, 9), and so on.

Model 1: Find three solutions to $y = 3x - 8$.

Solution: Let x be any three numbers.

For $x = 1$: $y = 3(1) - 8$; $y = 3 - 8$; $y = -5$ Solution (1, -5)

For $x = 0$: $y = 3(0) - 8$; $y = -8$ Solution (0, -8)

For $x = 3$: $y = 3(3) - 8$; $y = 1$ Solution (3, 1)

MULTIPLYING POLYNOMIALS BY POLYNOMIALS

Multiplying two polynomials together involves the distributive, associative, and commutative properties reviewed in the previous section as well as some memory of products.

Model: $(3x + y)(4x + 2y) = (3x + y)4x + (3x + y)2y$

$= 12x^2 + \underbrace{4xy + 6xy}_{10xy} + 2y^2$

$= 12x^2 + 10xy + 2y^2$

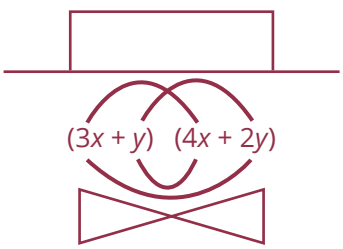
Using either the "foil" method or remembering the unforgettable face of George P. Trinomial can help with finding the product. You should not have to write anything but the answer in finding this product.

The "foil" method involves using the letters in the word to remember that the product consists of the product of the two **f**irst terms $(3x)(4x)$, plus the product of the **o**uter terms $(3x)(2y)$ combined with the product of the **i**nner terms $(y)(4x)$, and finally the product of the two **l**ast terms $(y)(2y)$.

George P. Trinomial is the name of the face you see.

Of course, the crossed eyebrows, nose, and chin help direct us to the products.

In certain polynomials you may group the terms and treat your selected binomials as monomials. With practice these products can be done mentally.



George P. Trinomial

Model: $(a + b - 3)(a + b + 5) = [(a + b) - 3][(a + b) + 5]$

let $x = a + b$

$(x - 3)(x + 5) = x^2 + 2x - 15$

Now replace x with $a + b$: $= (a + b)^2 + 2(a + b) - 15$

$= a^2 + 2ab + b^2 + 2a + 2b - 15$

Find the indicated products mentally when possible.

1.12 $(a + 3)(a - 2)$ _____

1.13 $(3xy - 1)(4xy + 2)$ _____

1.14 $(2x - 3y)(4x - y)$ _____

1.15 $(ab - 9)(ab + 8)$ _____

1.16 $(m^3n + 8)(m^3n - 5)$ _____

1.17 $(3 - c^2d)(4 - 4c^2d)$ _____

1.18 $(1 - 7x)(1 + 9x)$ _____

1.19 $(3m^3 - \frac{1}{2}y)(3m^3 - \frac{1}{2}y)$ _____

1.20 $(a + b)(a^2 - ab + b^2)$ _____

1.21 $(a - b)(a^2 + ab + b^2)$ _____

Find the value when $x = 2$ and $y = 3$.

1.24 y^0 _____ **1.25** $2x^0y^{-2}$ _____

1.26 $x^{-2}y^{-2}$ _____ **1.27** $(\frac{2}{3} + 5y)^0$ _____

1.28 $3x^{-1} + 2y^{-1}$ _____ **1.29** $x^{-3}y^{-3}$ _____

1.30 $x^{-3} + y^{-3}$ _____ **1.31** $(x + y)^{-3}$ _____

REDUCING RATIONAL EXPRESSIONS

A fraction is the quotient of two numbers. In arithmetic, these two numbers were generally integers.

Models: $\frac{2}{3}, \frac{7}{8}, \frac{4}{6}, \frac{9}{12}, \frac{11}{7}, \frac{100}{12}$

You have already learned that any fraction with a common factor for the numerator and denominator can be reduced.

Model 1: $\frac{4}{6} = \frac{2}{3}$ The common factor for 4 and 6 is 2.

Model 2: $\frac{9}{12} = \frac{3}{4}$ The common factor for 9 and 12 is 3.

Model 3: $\frac{100}{12} = \frac{25}{3}$ The common factor for 100 and 12 is 4.

Of course, as you saw in the models, the numerator and denominator are each divided by the largest common factor to give us the reduced or simplified fraction.

You have learned how to factor polynomials. When the numerator or the denominator of a fraction, or both, contain variables, we can simplify these fractions by dividing the numerator and denominator by the largest common factor of each.

Model 1: $\frac{2x + 4}{8} = \frac{\cancel{2}^{1}(x + 2)}{\cancel{8}_{4}}$ or $\frac{x + 2}{4}$

Model 2: $\frac{x^2 - x - 12}{3x + 9} = \frac{\cancel{(x + 3)}^{1}(x - 4)}{3\cancel{(x + 3)}_{1}}$ or $\frac{x - 4}{3}$

Model 3: $\frac{3x^3 - 81y^3}{2x^2 - 18y^2} = \frac{3\cancel{(x - 3y)}^{1}(x^2 + 3xy + 9y^2)}{2(x + 3y)\cancel{(x - 3y)}_{1}} = \frac{3(x^2 + 3xy + 9y^2)}{2(x + 3y)}$

DEFINITIONS

Rational number: A number that can be expressed as a quotient of two other numbers; a terminating or repeating decimal.

Irrational number: A number that cannot be fully expressed as a quotient of two other numbers; a nonterminating, nonrepeating decimal.

The most commonly occurring type of irrational number is a number that contains a radical that cannot be simplified to a rational number.

Models: $\sqrt{16} = 4$, $\frac{4}{6} = \frac{2}{3}$ } rational numbers

$\sqrt{3}$, $\frac{\sqrt{11}}{5}$ } irrational numbers

A **radical** is an indicated root of a number. We know that 9 has two square roots, +3 and -3. Any real number has two square roots. When we use the radical symbol, we are referring only to the principal, or positive, square root.

Radicals occur not only as second-degree radicals (square roots) but as third-degree or higher-degree radicals.

DEFINITIONS

Radical: An expression consisting of a number with a radical sign indicating some root of the number beneath the radical sign.

Index: A small number written over the radical sign indicating which root of the number is being sought.

Radicand: The number inside the radical sign.

Radical sign: A symbol indicating that an expression is a radical.

Model: $\sqrt[5]{79}$

5 is the index.

79 is the radicand.

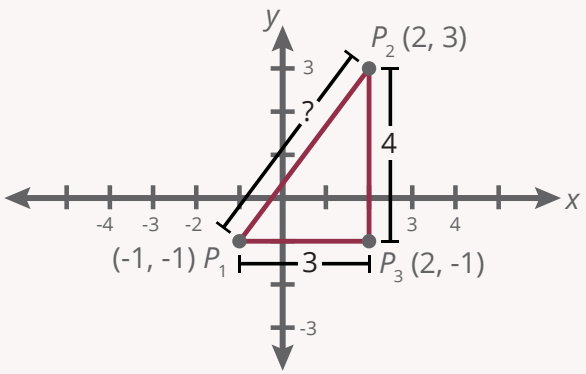
$\sqrt[5]{79}$ is the radical.

$\sqrt{\ }$ is the radical symbol.

Notice that if the radical has no index number, the root indicated is the square root. Therefore, $\sqrt{\ } = \sqrt[2]{\ }$.

See if the results from the distance formula are reasonable by applying the formula to the familiar 3-4-5 right triangle in the model.

Model 1:



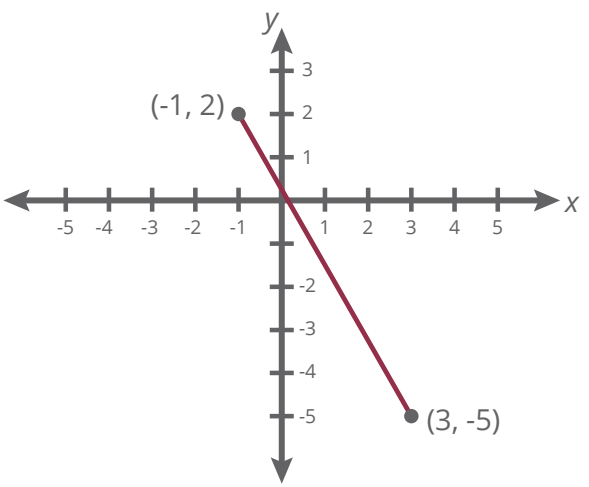
Calculate: $|P_1P_2| = \sqrt{[2-(-1)]^2 + [3-(-1)]^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Should $|P_1P_2|$ be 5? Yes.

Model 2: Graph and find the distance between (-1, 2) and (3, -5).

$d = \sqrt{[3-(-1)]^2 + (-5-2)^2}$

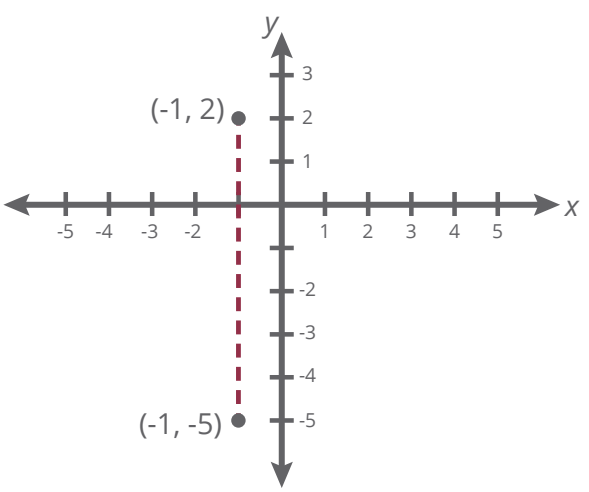
$= \sqrt{16 + 49} = \sqrt{65} \cong 8.1$



Model 3: Graph and find the distance between (-1, 2) and (-1, -5).

We could use the distance formula; however, in cases of horizontal or vertical lines, use the absolute value of the difference between the *x*-coordinates or the *y*-coordinates:

$d = |2-(-5)| = 7$



Express answers with positive exponents.

1.1 $2^0 + 5^0 =$ _____

1.2 $(2 + 5)^0 =$ _____

1.3 $6 \cdot 5^0 =$ _____

1.4 $3^{-2} =$ _____

1.5 $\frac{1}{3^{-2}} =$ _____

1.6 $(2 + 3)^{-2} =$ _____

1.7 $2^{-3} + 4^{-2} =$ _____

1.8 $4^2 \cdot 2^{-3} =$ _____

1.9 $(2 \cdot 3)^{-2} =$ _____

1.10 $\left(\frac{1}{2}\right)^{-1} =$ _____

1.11 $m \cdot n^{-1} =$ _____

1.12 $(m \cdot n)^{-1} =$ _____

1.13 $2x^{-3} =$ _____

1.14 $(2x)^{-3} =$ _____

1.15 $\frac{b^{-2}}{b^{-3}} =$ _____

1.16 $\frac{a^8}{a^6} =$ _____

1.17 $b^5 \cdot b^8 =$ _____

1.18 $(b^4)^2 =$ _____

1.19 $\frac{2ab^{-1}}{3e^{-3}d} =$ _____

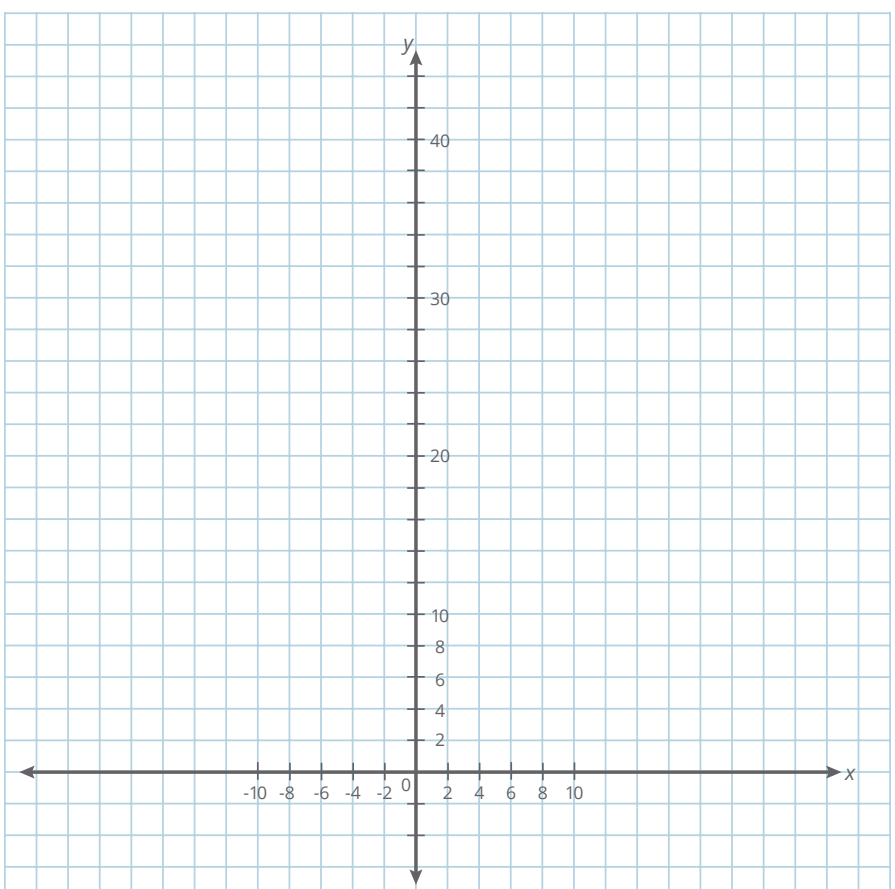
1.20 $\frac{(2x)^{-2}}{(3x)^{-2}} =$ _____

1.21 $\frac{2x^{-2}}{3x^{-2}} =$ _____

Evaluate the exponential function $y = 2^x$ for the values given. Graph.

1.22

x	y
-3	
-2	
-1	
0	
1	
2	
3	
4	



With the Properties of Equality, Properties of Numbers, and Laws of Exponents, many algebraic expressions can be simplified.

Model 1: Simplify $x^2(x^3 - 4x^2 + 3)$

$$x^2(x^3 - 4x^2 + 3) = x^5 - 4x^4 + 3x^2$$

Model 2: Simplify $\frac{x^4}{x^3} \cdot (x^5)^2$

$$\frac{x^4}{x^3} \cdot (x^5)^2 = x \cdot x^{10} = x^{11}$$

Match the best description or example on the right with the term on the left.

1.1 _____ equal sets

1.2 _____ empty set

1.3 _____ intersection

1.4 _____ union

1.5 _____ Reflexive Property of Equality

1.6 _____ Symmetric Property of Equality

1.7 _____ Transitive Property

1.8 _____ Closure Property for Addition

1.9 _____ Commutative Property for Multiplication

1.10 _____ Distributive Property

1.11 _____ ordered pair

1.12 _____ relation, but not a function

1.13 _____ function

1.14 _____ straight line

1.15 _____ abscissa

1.16 _____ ordinate

1.17 _____ coordinates

1.18 _____ $a^m \cdot a^n$

1.19 _____ $a^m \div a^n$

1.20 _____ $(x^a)^b$

a. $a = a$

b. If $a \in R$ and $b \in R$, then $a + b \in R$.

c. (6, 3)

d. x^{ab}

e. {(8, 4), (4, 2), (2, 1)}

f. A ∪ B, the word "or"

g. {(2, 7), (3, 9), (5, 11), (2, 13)}

h. sets with the same elements

i. sets with the same number of elements

j. Ø or { }

k. $a + (b \cdot c) = (a + b)(a + c)$

l. graph of $2x + y = 7$

m. If $a = b$ and $b = c$, then $a = c$.

n. a^{m+n}

o. graph of $2x^2 + 3y = 9$

p. $a + b = b + a$

q. $a(b + c) = ab + ac$

r. 7 in (8, 7)

s. A ∩ B, the word "and"

t. 9 in (9, 4)

u. name for ordered pair in graphing

v. (8, 4, 7)

w. $a \cdot b = b \cdot a$

x. $a^{\frac{m}{n}}$

y. If $a = b$, then $b = a$.

z. a^{m-n}

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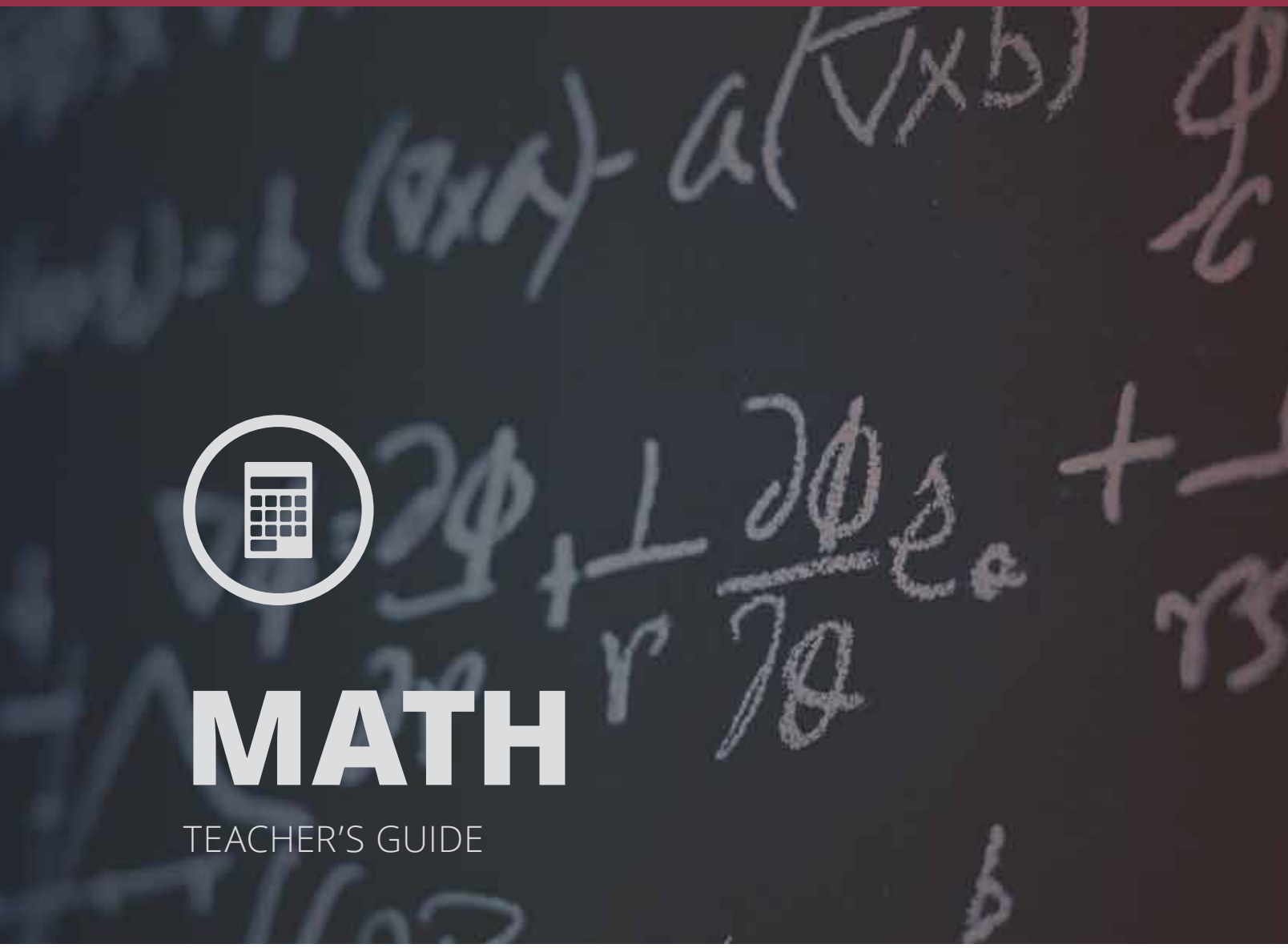

MATH

TEACHER'S GUIDE

11th Grade

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Alpha Omega PUBLICATIONS

MATH 1100

Teacher's Guide

INSTRUCTIONS FOR MATH

The LIFEPAC curriculum from grades two through twelve is structured so that the daily instructional material is written directly into the LIFEPACs. The student is encouraged to read and follow this instructional material in order to develop independent study habits. The teacher should introduce the LIFEPAC to the student, set a required completion schedule, complete teacher checks, be available for questions regarding both content and procedures, administer and grade tests, and develop additional learning activities as desired. Teachers working with several students may schedule their time so that students are assigned to a quiet work activity when it is necessary to spend instructional time with one particular student.

Math is a subject that requires skill mastery. But skill mastery needs to be applied toward active student involvement. Measurements require measuring cups, rulers, and empty containers. Boxes and other similar items help the study of solid shapes. Construction paper, beads, buttons, and beans are readily available and can be used for counting, base ten, fractions, sets, grouping, and sequencing. Students should be presented with problem situations and be given the opportunity to find their solutions.

Any workbook assignment that can be supported by a real world experience will enhance the student's ability for problem solving. There is an infinite challenge for the teacher to provide a meaningful environment for the study of math. It is a subject that requires constant assessment of student progress. Do not leave the study of math in the classroom.

This section of the Math Teacher's Guide includes the following teacher aids: Suggested and Required Material (supplies), Additional Learning Activities, Answer Keys, and Alternate LIFEPAC Tests.

The Teacher Notes section of the Teacher's Guide lists the required or suggested materials for the LIFEPACs and provides additional learning activities for the students. Additional learning activities provide opportunities for problem solving, encourage the student's interest in learning and may be used as a reward for good study habits.

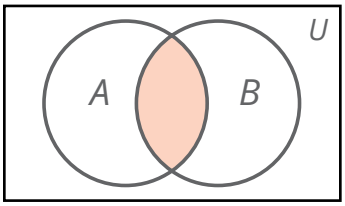
TEACHER NOTES

MATERIALS NEEDED FOR LIFEPAC	
Required	Suggested
(none)	• straightedges

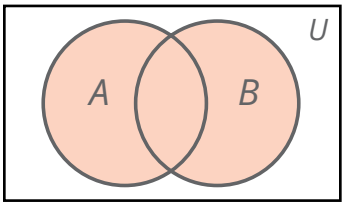
ADDITIONAL LEARNING ACTIVITIES

Section 1: Sets

1. Discuss the use of Venn diagrams as pictorial representations of relationships involving sets. Illustrate with the following examples.



a. $A \cap B$



b. $A \cup B$

2. Divide the class into three groups. Each group writes a description of each of the following sets using set notation.
   a. The set *S* of states having a border on the Pacific Ocean.
   b. The set *C* of consonants in the word Mississippi.
   c. The set *F* of numbers between 3 and 21 that are divisible by 4.

   Each group exchanges its answers with the other groups and critiques one another's solutions.

Section 2: Structure

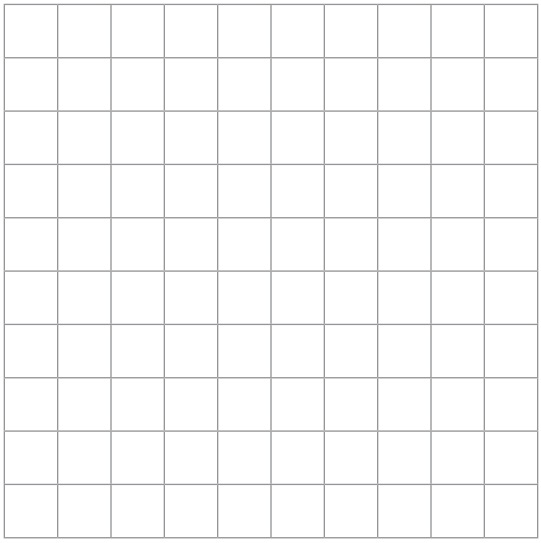
1. Write the following statements on the board and have the students respond with the correct property name. Many other possibilities exist; just change the numbers or variables.

10 = 10	reflexive
5 + 7 = 7 + 5	commutative—addition
2(3 • 9) = (2 • 3)9	associative—multiplication
22 + 0 = 22	identity—addition
If $u = v$ and $v = 6$, then $u = 6$.	transitive
4(2 + 7) = 4 • 2 + 4 • 7	distributive
3 • ⅓ = 1	multiplicative inverse
If $d = e$, then $e = d$.	symmetric
8 • 9 = 9 • 8	commutative—multiplication
6 • 0 = 0	zero—multiplication
4 + (2 + 5) = (4 + 2) + 5	associative—addition
3 + (-3) = 0	additive inverse
5 • 1 = 5	identity—multiplication

» FUNCTIONS

1. Draw a graph for each of these two functions on one pair of axes.

$f(x) = 2x + 1$ $f(x) = 2x + 3$



2. Write the equation for the line midway between the two lines.

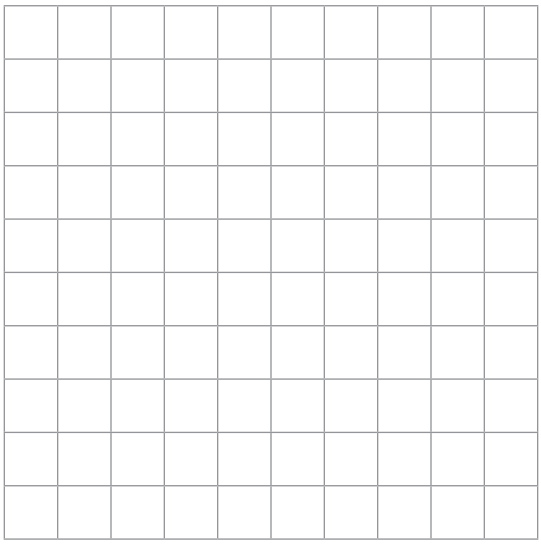
3. Draw the graph for each of these two functions on one pair of axes.

$f(x) = x - 4$ $f(x) = 4 - x$

x	0	1	2	3	4	5
$f(x)$	-4	-3	___	___	___	1

What do you notice about these two lines?

_____



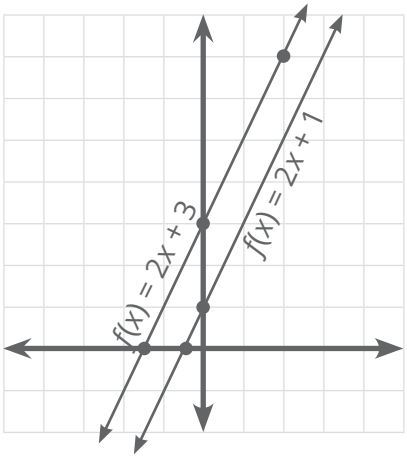
ADDITIONAL ACTIVITY, SOLUTION KEY

1. $f(x) = 2x + 1$

x	0	$-\frac{1}{2}$	-2
$f(x)$	1	0	-3

$f(x) = 2x + 3$

x	0	$-\frac{3}{2}$	2
$f(x)$	3	0	7



2. $f(x) = \frac{1}{2}(2x + 1 + 2x + 3) =$

$\frac{1}{2}(4x + 4) = 2x + 2$

$f(x) = 2x + 2$

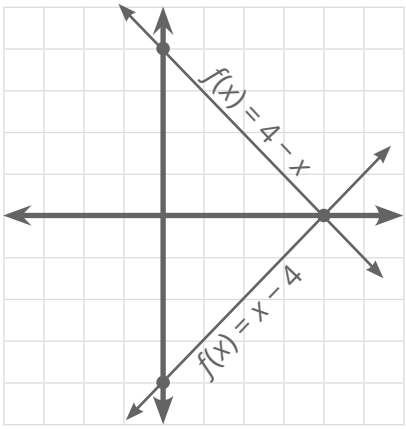
3. $f(x) = x - 4$

x	0	1	2	3	4	5
$f(x)$	-4	-3	-2	-1	0	1

$f(x) = 2 - 4 = -2$
$f(x) = 3 - 4 = -1$
$f(x) = 4 - 4 = 0$

$f(x) = 4 - x$

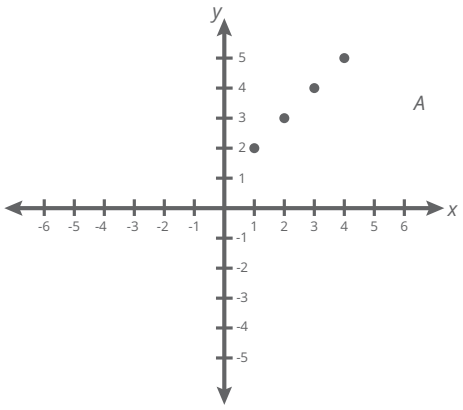
x	0	1	2	3	4	5
$f(x)$	4	3	2	1	0	-1



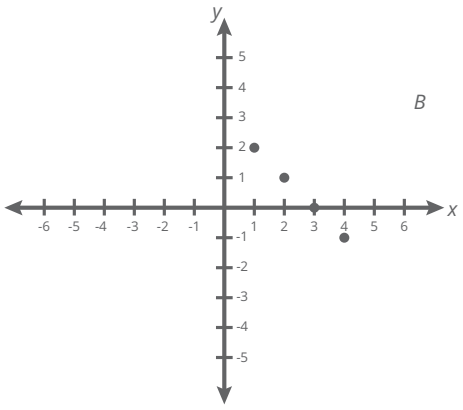
The two lines are perpendicular.

SECTION 3

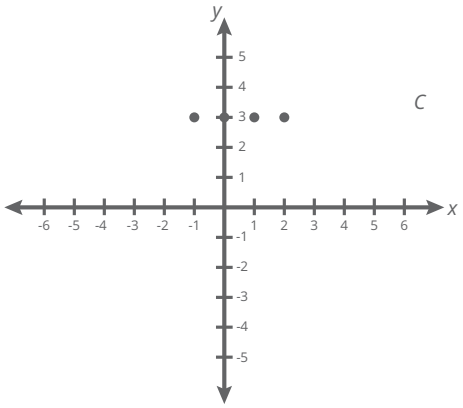
3.1 neither: not a set of paired numbers
3.2 function
3.3 relation
3.4 function
3.5 function
3.6 function
3.7 {2}
3.8 {3, 4, 5, 6}
3.9 domain = range = {set of all real numbers}
3.10 domain = {all real numbers}
range = $\{y : y \geq 0\}$
3.11



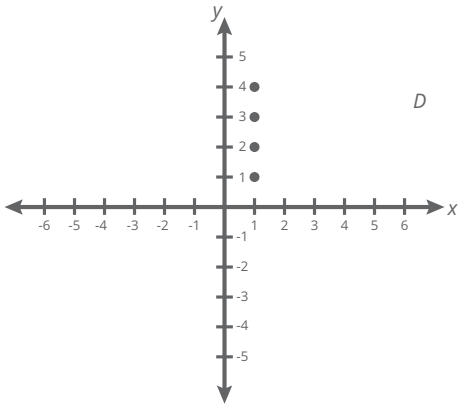
3.12



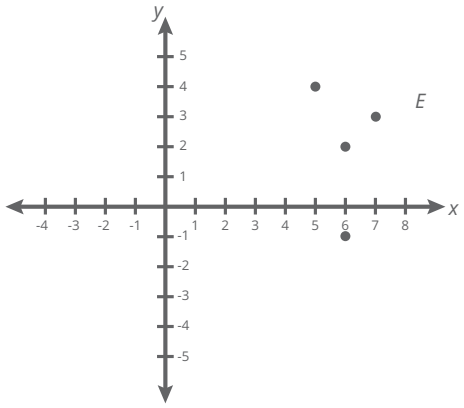
3.13



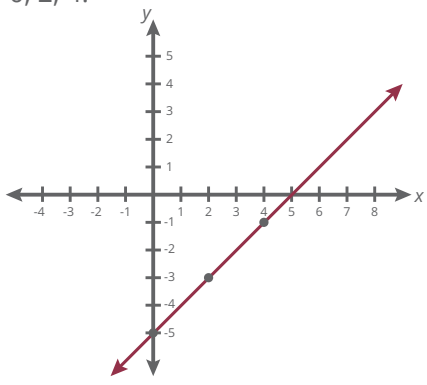
3.14



3.15



3.16 *D* and *E*, Problems 3.14 and 3.15 (vertical-line test)
3.17 Use $x = 0, 2, 4$.



3.18 yes
3.19 yes
3.20 no
3.21 $f(x) = x^2 + 1$
$f(2) = 2^2 + 1 = 4 + 1 = 5$
3.22 $f(x) = x^2 + 1$
$f(6) = 6^2 + 1 = 36 + 1 = 37$
3.23 $f(x) = x^2 + 1$
$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$

LIFEPAC TEST

1. $K \cup G = \{\bigcirc, \square, \triangle, \mathrm{I}, \parallel\!\square\}$

2. $K \cap H = \{\square\}$

3. $K \cap G \cap H = \{\ \} = \varnothing$

4. $\{\square\}, \{\boxtimes\}, \{\otimes\}, \{\square, \boxtimes\}, \{\square, \otimes\}, \{\boxtimes, \otimes\}, \{\square, \boxtimes, \otimes\}, \varnothing$

5. $2^n = 2^6 = 64$

6. 7

7. $12 + 8 \div 2 + 10 = 12 + 4 + 10 = 26$

8. $15 \div 3 + 10 \div 2 = 5 + 5 = 10$

9. associative—addition and commutative—addition

10. multiplicative inverse

11. multiplicative inverse, additive and multiplicative inverse, commutative (addition and multiplication), associative (addition and multiplication)

12. commutative—addition and multiplication

13. range = {1, 2, 3, 7}

14. domain = {5, 6, 7, 8, 9}

15. no: double-valued points, (1, 5) and (1, 8)

16. $f(x) = x^2 + 5x$
$f(-2) = (-2)^2 + 5(-2) = 4 - 10 = -6$

17. $g(x) = 2x + 1$
$g(3) = 2 \cdot 3 + 1 = 6 + 1 = 7$

18. $f(5) + g(6) = 5^2 + 5(5) + 2(6) + 1 = 50 + 13 = 63$

19. $g(3) - f(4) = 2(3) + 1 - (4^2 + 5 \cdot 4) = 7 - 36 = -29$

20. $g(a + h) - g(a)$
$= 2(a + h) + 1 - (2a + 1)$
$= 2a + 2h + 1 - 2a - 1$
$= 2h$

21. a^3

22. $(3b)^4 = 81b^4$

23. $2a^2b^2$

24. $3 \cdot x \cdot x$

25. $3a \cdot 3a \cdot 3a$

26. $abc \cdot abc$

27. $10^2 \cdot 10^3 = 10^{2+3} = 10^5$

28. $x^3x^5 = x^{3+5} = x^8$

29. $a^2 \cdot a^{-3} \cdot a = a^{2-3+1} = a^0 = 1$

30. $\frac{18a^3b^2}{2ab} = 9a^2b$

31. $\frac{12ab^3c^2}{4a^{-2}bc^{-2}} = 3a^{1+2}b^{3-1}c^{2+2} = 3a^3b^2c^4$

32. 8

33. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

34. $7^0 = 1$

35. $5h - 2h = (5 - 2)h = 3h$

36. $2a$

37. $7x^2 + 3x$

38. $5(x - 2) + 3x$
$= 5x - 10 + 3x$
$= 8x - 10$

39. $7 - 2(5 - 2x)$
$= 7 - 10 + 4x$
$= -3 + 4x$ or $4x - 3$

40. $3(x + 2) + 4(x - 5)$
$= 3x + 6 + 4x - 20$
$= 7x - 14$

MATH 1101

ALTERNATE LIFEPAC TEST

NAME _____

DATE _____

SCORE _____

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Complete these activities (each answer, 4 points).

Given A = {a, b, c, d, e}, B = {a, e, i, o, u}, C = {k, m, o}

1. $A \cup C$ _____
2. $A \cap C$ _____
3. $A \cap B \cup C$ _____
4. All subsets of C _____
5. Number of subsets of $A \cup C$ _____
6. Number of elements of $A \cap B \cap C$ _____

Evaluate each expression (each question, 3 points).

7. $16 + 12 \div 3 - 10$ _____
8. $22 \div 11 + 9 \div 3$ _____

Name the axiom(s) that justifies each of the following statements (each answer, 2 points).

9. $6 + (x - 3) = x + 3$ _____
10. $A \cdot \frac{1}{A} = 1$ _____
11. $6 \cdot \frac{1}{6} + 2 + (-1) = 2$ _____
12. $6x + 7x = 13x$ _____

Complete these activities (each answer, 3 points).

Given $F = \{(6, 2), (3, 1), (4, -7), (-2, -1)\}$

13. Write the range set of F. _____

14. Write the domain set of F. _____

15. Is F^{-1} a function? _____

Complete these activities (each answer, 4 points).

Given $F(x) = x^2 - 24$ and $G(x) = 54 - 2x$

16. $F(-2)$ _____

17. $G(3)$ _____

18. $F(5) + G(6)$ _____

19. $G(2) - F(7)$ _____

20. $G(a + h) - G(a)$ _____

Write each expression in exponential notation (each answer, 3 points).

21. $x \cdot x \cdot x \cdot x$ _____

22. $2x \cdot 2x \cdot 2x$ _____

23. $5 \cdot x \cdot y \cdot x \cdot 3$ _____

Write each expression without exponents (each answer, 3 points).

24. $5x^3$ _____

25. $(2x)^3$ _____

26. $(xyz)^2$ _____

Express in simplified exponential notation (each answer, 3 points).

27. $5^2 \cdot 5^4$ _____

28. $a \cdot a^2 \cdot a^4$ _____

29. $x^3 \cdot x^{-3} \cdot x^2$ _____

30. $\frac{12x^2y^3}{4xy}$ _____

31. $\frac{27a^3b^2c}{3a^{-2}bc^{-1}}$ _____