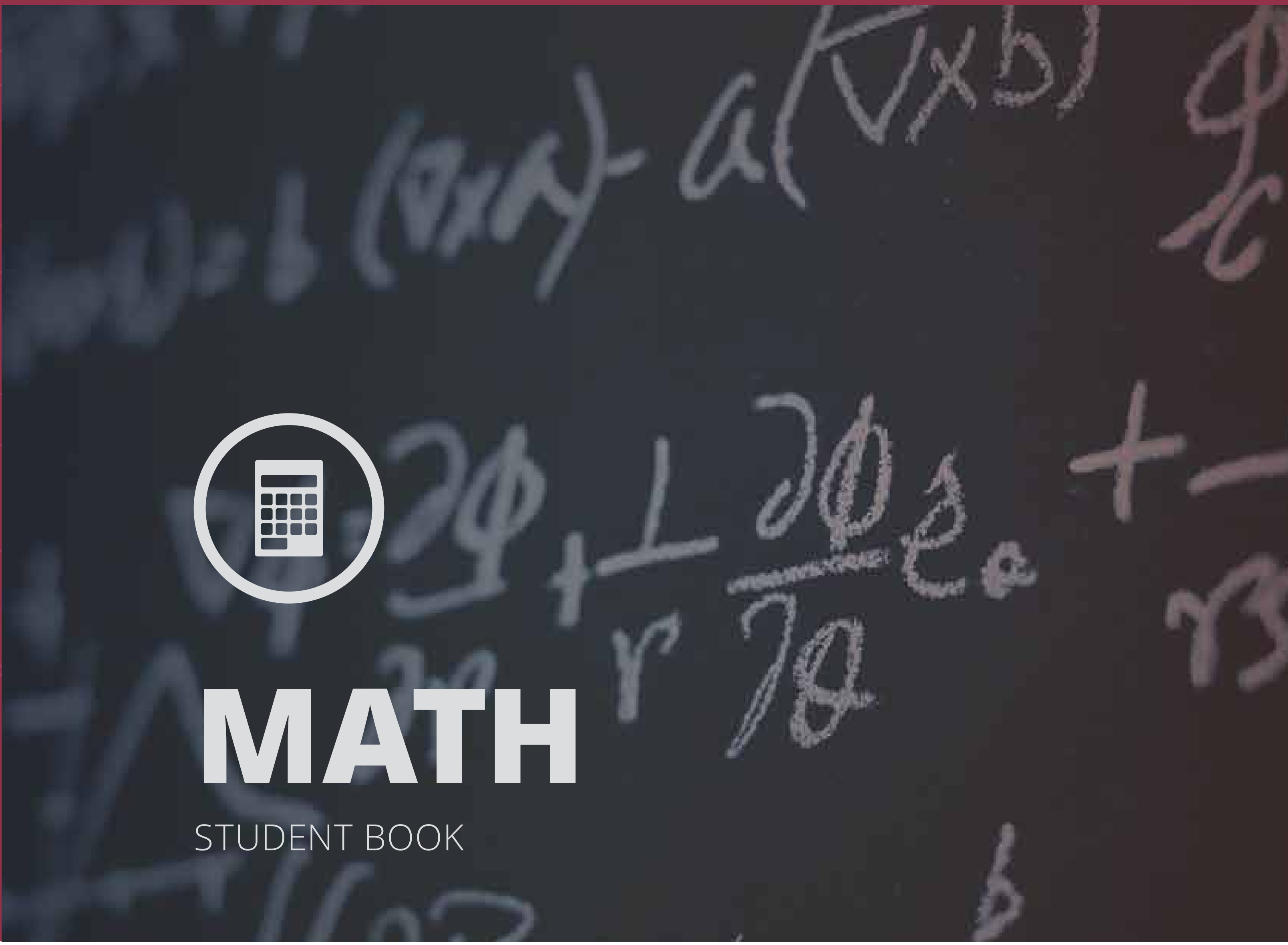
LIFEPAC®



MATH

STUDENT BOOK

12th Grade

Alpha Omega
PUBLICATIONS

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MATH 1201

RELATIONS AND FUNCTIONS

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Functions

Introduction

This LIFEPAC® reviews various types of functions, including linear, quadratic, polynomial, exponential, and logarithmic functions. You will learn how to graph each of these types of functions and solve equations and inequalities in which they appear. You will also learn about complex numbers and their importance in solving polynomial equations.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Identify and solve polynomial, exponential, and logarithmic equations.
2. Graph polynomial, exponential, and logarithmic equations.
3. Recognize various transformations to basic polynomial, exponential, and logarithmic functions and draw appropriate graphs.
4. Use synthetic division to find the roots of a polynomial equation, as well as upper and lower limits on those roots.
5. Geometrically represent basic arithmetic operations of complex numbers on the complex plane.
6. Convert between rectangular and polar forms of complex numbers.
7. Use the complex conjugate to simplify fractions and find the modulus of a complex number.
8. Use interval notation to express the solution to polynomial and rational inequalities in mathematical and real-world contexts.
9. Identify and graph the greatest integer function.
10. Combine functions using basic arithmetic operations.

1. SOLVING A RIGHT TRIANGLE

LENGTHS OF SIDES

What is trigonometry?

Trigonometry is a branch of geometry that deals with triangle measurement. Throughout history, trigonometry ("trig") has been used to, and evolved from the need or desire to, measure that which could not physically be measured. Early astronomers used spherical trig and the chords in a circle to measure distances to stars. Today, plane trig is applied in many fields such as surveying, physics, and engineering.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

- Express trigonometric functions as ratios in terms of the sides of a right triangle.
- Evaluate trigonometric expressions.
- Use the Pythagorean theorem and trigonometric ratios to calculate side measures in right triangles.

Vocabulary

Study these words to enhance your learning success in this section.

cosine . . . . The trigonometric ratio of adjacent side over hypotenuse for an acute angle in a right triangle.

hypotenuse . . . . The longest side of a right triangle; the side that is opposite the right angle.

sine . . . . The trigonometric ratio of opposite side over hypotenuse for an acute angle in a right triangle.

tangent . . . . The trigonometric ratio of opposite side over adjacent side for an acute angle in a right triangle.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

RIGHT TRIANGLES

Let's begin our study of trigonometry with measurements in the right triangle. Let's review some things about the right triangle that you should already be familiar with.

- A right triangle has one right (90-degree) angle and two acute angles.
- The sum of the measures of the two acute angles in a right triangle is 90 degrees. (The sum of the measures of the angles in any triangle is 180 degrees.)
- The longest side of a right triangle is called the **hypotenuse**.
- The two shorter sides of a right triangle are called the legs.
- The Pythagorean theorem ($a^2 + b^2 = c^2$) applies only to right triangles.

Trigonometric ratios express relationships between the sides of a right triangle. Therefore, it is important to be able to identify them.

The sides are referred to as the *hypotenuse*, *opposite side*, and *adjacent side*. The hypotenuse is always the longest side of the right triangle and is across from the right angle.

EVALUATING

It is important to note that most trigonometric values are irrational numbers. Before the use of scientific calculators, trig values of angles were listed in tables and given to four decimal places. Today, it is common to use the scientific calculator in calculations involving trig functions. This course will require the use of a scientific calculator. If you do not have a scientific calculator, you should be able to find one on your computer or find one online.

> **Key Point!**
>
> A trig function cannot be evaluated without an angle measure!

There are a variety of calculator models to choose from and they vary in their appearance, the layout of the keypad, and use. While you will be provided with tutorials on calculator use in this course, it will not be specific to a brand of calculator. It is important that you follow along with the tutorial to ensure that you are getting the correct results as problems are worked. It will be your responsibility to read the manual on how to operate your individual calculator if you are unable to get the correct answer by following the model provided.

As you will learn later in the course, degrees are not the only means of measuring angles. Your calculator allows you to work with the different measuring systems. Therefore, it is important before beginning any problem to be sure that you are working in the correct system. For the exercises in this lesson, be sure that your calculator is set for degrees. Some calculators have a key labeled as "mode" and you might find the degree setting using the mode key.

To find the cos 25°, the key strokes are:

2 **5** **cos**

Note that is it not necessary to press the equal key. If you do, however, the value does not change. Also, some calculators may require you press the cosine key first, then enter 25, and lastly press "enter."

Closeness applies when working with trig expressions. When a number precedes a trig function, multiplication is implied.

Example

Evaluate 2 sin 30°.

Solution

The expression *2 sin 30°* means "2 times the sine of 30 degrees."

On the calculator, evaluate sin 30° by entering 30 sin; then multiply this value by 2. Try it.

You should have gotten a value of 1.

Compare

You may also enter the problem on your calculator using the following key strokes:

2 × 30 sin =

The answer should still be 1.

Example

Evaluate cos 60° sin 30°.

Solution

The expression *cos 60° sin 30°* means "the cosine of 60 degrees times the sine of 30 degrees."

On the calculator, enter the following:

60 cos × 30 sin =

The cosine of 60° is 0.5 and the sine of 30° is 0.5, so their product is 0.25.

THE SINE FUNCTION

From your study of algebra, you should be familiar with function notation and with finding ordered pairs for a given function.

For the function $f(x) = \sin x$:

- the **domain**, or set of *x* values, represents the angle measures;
- the **range**, or set of *y* values, represents the sine values of those angles.

Note: It is common to use radian measure for the *x*-axis when graphing trig functions.

Think About It!

What is the advantage of using radians instead of degrees for angle measure?

Hint: Think of scaling the *x*-axis using real numbers.

Before you begin graphing, let's use the unit circle and your knowledge of special angles to make a table of some ordered pairs on the graph of $f(x) = \sin x$.

Irrational values such as $\frac{\sqrt{2}}{2}$ have been approximated.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin x$	0	.5	.71	.87	1	.87	.71	.5	0

Using the symmetry of the unit circle, you have angles in Quadrants III and IV whose sine function values are the same as their reference angles, but negative.

Reminder:

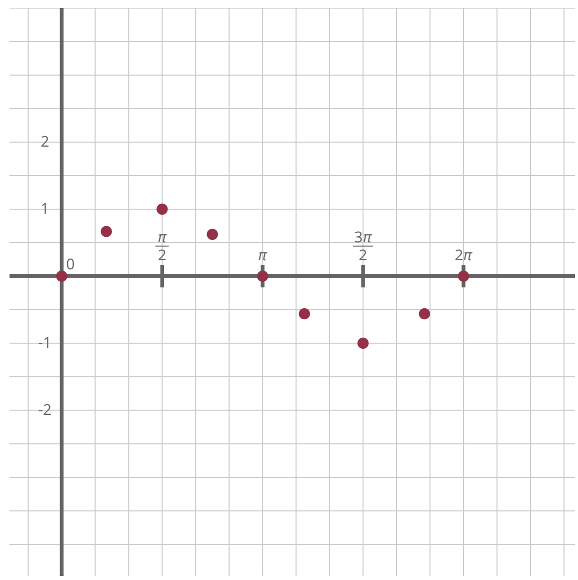
$$\sin \frac{5\pi}{6} = \sin \frac{\pi}{6}$$

$$\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$$

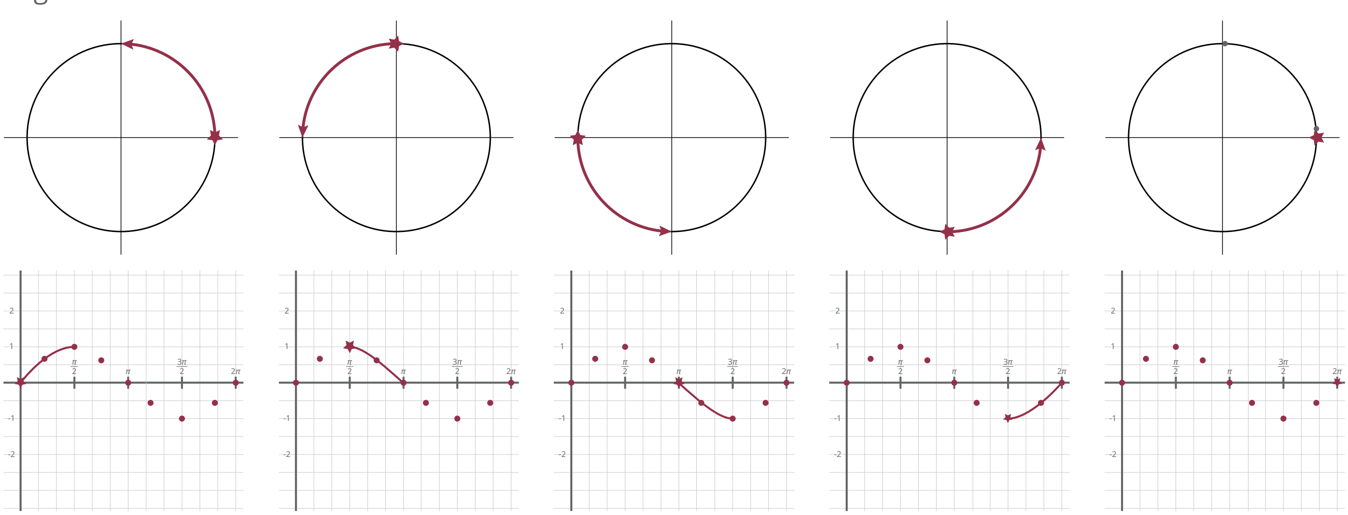
$$\sin \frac{11\pi}{6} = -\sin \frac{\pi}{6}$$

Do you remember why this is true?

Plotting some points yields the following:



As you move along the unit circle from 0 to 2π radians, include all angle measures for *x* and all real-number values for *y* from -1 to 1 inclusive. Then you can connect these points with a nice smooth curve. Notice that each location on your graph corresponds to a location on the unit circle!



1.10 Simplify $\frac{\cot\theta}{\cos\theta\sec\theta}$. _______

a. 1 b. $\cot\theta$ c. $\cot^2\theta$ d. $\tan\theta$

1.11 Simplify $\sin^2\theta - \sec\theta\cos\theta + \cos^2\theta$. ____________________

1.12 Simplify $\cos\theta(\tan\theta + \cot\theta)$. _______

a. 1 b. $\cos^2\theta$ c. $\csc\theta$ d. $\sec\theta$

Match each trig function with its correct value if θ is an acute angle and $\csc\theta = 2\frac{1}{2}$.

1.13 ____________ $\frac{\sqrt{21}}{5}$

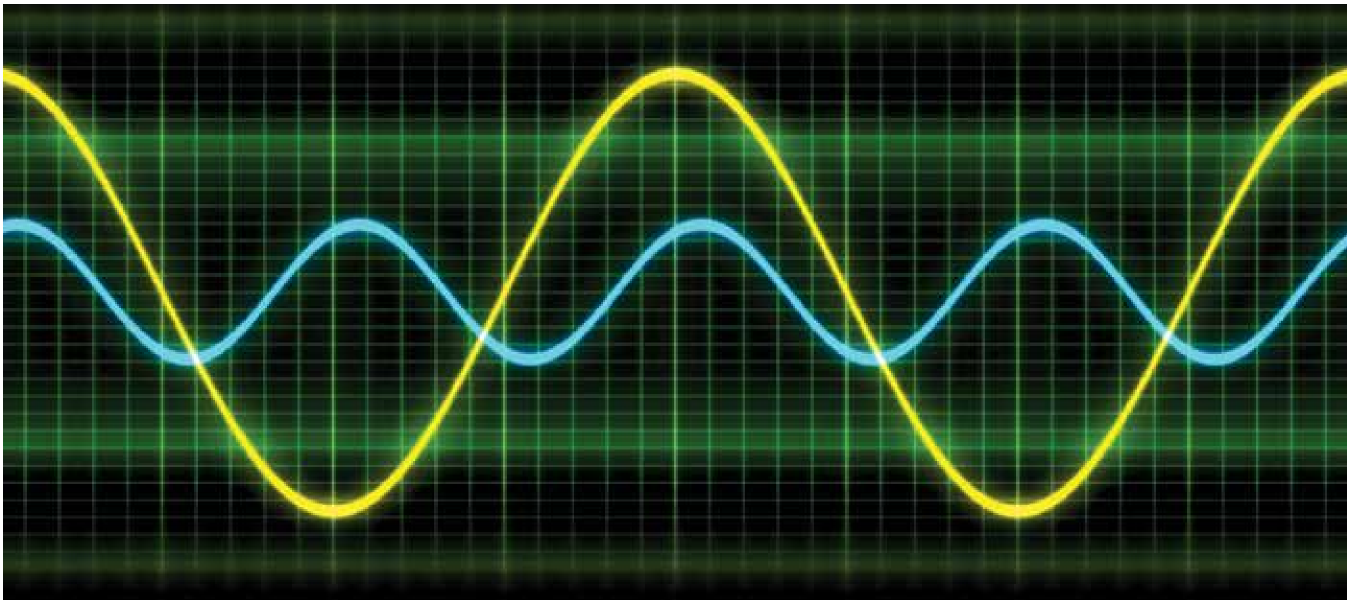
1.14 ____________ $\frac{\sqrt{21}}{2}$

1.15 ____________ $\frac{5\sqrt{21}}{21}$

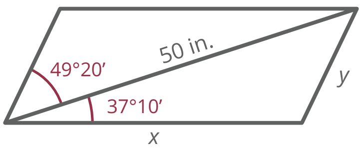
1.16 ____________ $\frac{2}{5}$

1.17 ____________ $\frac{2\sqrt{21}}{21}$

a. $\tan\theta$
b. $\cot\theta$
c. $\sin\theta$
d. $\cos\theta$
e. $\sec\theta$



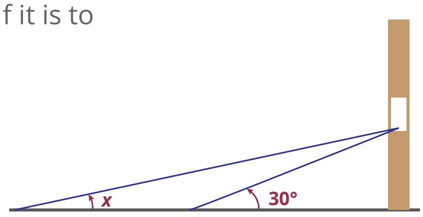
1.7 A diagonal of a parallelogram is 50 inches long and makes angles of 37°10′ and 49°20′ with the sides.



How long is the longest side? _______

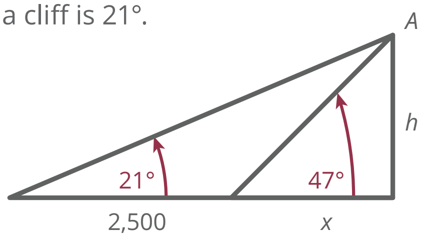
a. 30 in
b. 38 in
c. 40 in
d. 66 in

1.8 A 10-foot ladder must make an angle of 30° with the ground if it is to reach a certain window. What angle must a 20-foot ladder make with the ground to reach the same window? _______



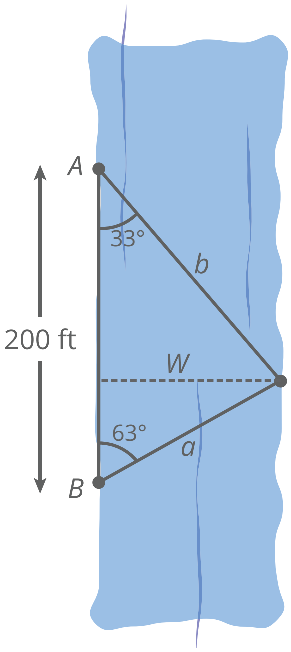
a. 10.5°
b. 12.5°
c. 14.5°
d. 16.5°

1.9 From a ship, the angle of elevation of a point, *A*, at the top of a cliff is 21°. After the ship has sailed 2,500 feet directly toward the foot of the cliff, the angle of elevation of ∠*A* is 47°. (Assume the cliff is perpendicular to the ground.)



The height of the cliff is _______ .

1.10 Vertices *A* and *B* of triangle *ABC* are on one bank of a river, and vertex *C* is on the opposite bank. The distance between *A* and *B* is 200 feet. Angle *A* has a measure of 33°, and angle *B* has a measure of 63°. Find *b*. _______



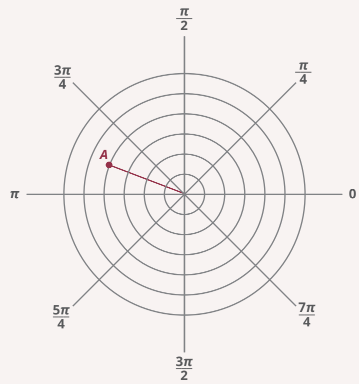
a. 110 ft
b. 168 ft
c. 179 ft
d. 223 ft

Example

Graph *A* (4, 150°).

Solution

Count four spaces from the origin on the polar axis and then rotate 150°.



Recall that there are an infinite number of coterminal angles. Adding or subtracting multiples of 360° from a given angle measure will give you angles that are coterminal with it. For this reason, different coordinates may result in the same point.

Reminder:

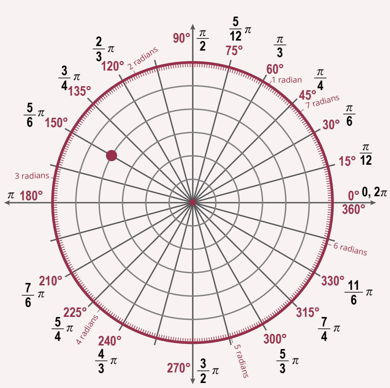
Coterminal angles have the same terminal side.

Example

Graph *T* (4, -210°).

Solution

Count four spaces from the origin on the polar axis and then rotate -210°.



Since -210° and 150° terminate at the same place, point *A* (4, 150°) and point *T* (4, -210°) coincide.

Since angles can be measured in either degrees or radians, the angle may be given using either unit.

Reminder:

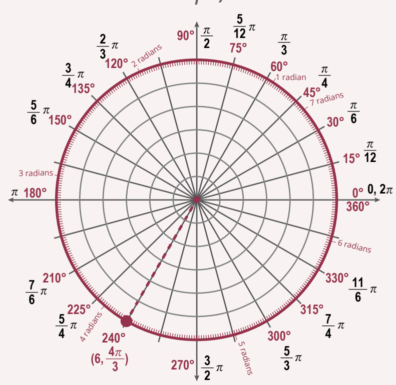
$\pi = 180°$

Example

Graph *P* $(6, \frac{4\pi}{3})$.

Solution

Count six spaces out from the pole and rotate $\frac{4\pi}{3}$ (one and one-third *pi*).



Note that $(6, -\frac{2\pi}{3})$, $(6, \frac{10\pi}{3})$, and $(6, \frac{16\pi}{3})$ are some other equivalent coordinates of point *P*.

The concept of an ordered pair is extremely important in plotting points in the rectangular coordinate system; (3, 4) is different from (4, 3). In other words, you must understand that (3, 4) means you move 3 units in the positive *x* direction and 4 units in the positive *y*. While you would typically follow the order as presented in the coordinates, 3 on *x* and then 4 on *y*, you could arrive at this same location by moving 4 on *y* first and then 3 on *x*.

In the polar system, you can also rotate the angle measure first and then determine the distance on the terminal side of the angle, *r*. When *r* is negative, the distance from the pole to the point is still $|r|$, but it is in the opposite direction.

1.16 Choose the appropriate description for the equation $x^2 + y^2 = 36$. _______

a. circle
b. point circle
c. no circle

1.17 Choose the appropriate description for the equation $(x - 2)^2 + (y + 3)^2 = 49$. _______

a. circle
b. point circle
c. no circle

1.18 Choose the appropriate description for the equation $(x + 3)^2 + (y - 2)^2 = 0$. _______

a. circle
b. point circle
c. no circle

1.19 Choose the appropriate description for the equation $x^2 + y^2 = -4$. _______

a. circle
b. point circle
c. no circle

1.20 Choose the appropriate description for the equation $x^2 + y^2 = 0$. _______

a. circle
b. point circle
c. no circle

1.21 Choose the appropriate description for the equation $(x - 5)^2 + (y + 4)^2 = 18$. _______

a. circle
b. point circle
c. no circle

1.22 Choose the appropriate description for the equation $3x^2 + 3y^2 = -27$. _______

a. circle
b. point circle
c. no circle

1.23 Choose the appropriate description for the equation $10(x - 3)^2 + 10(y + 4)^2 = 100$. _______

a. circle
b. point circle
c. no circle

1.24 Find the center of the circle whose equation is $(x - 5)^2 + (y + 3)^2 = 6$. _______

a. (5, 3)
b. (-5, 3)
c. (5, -3)

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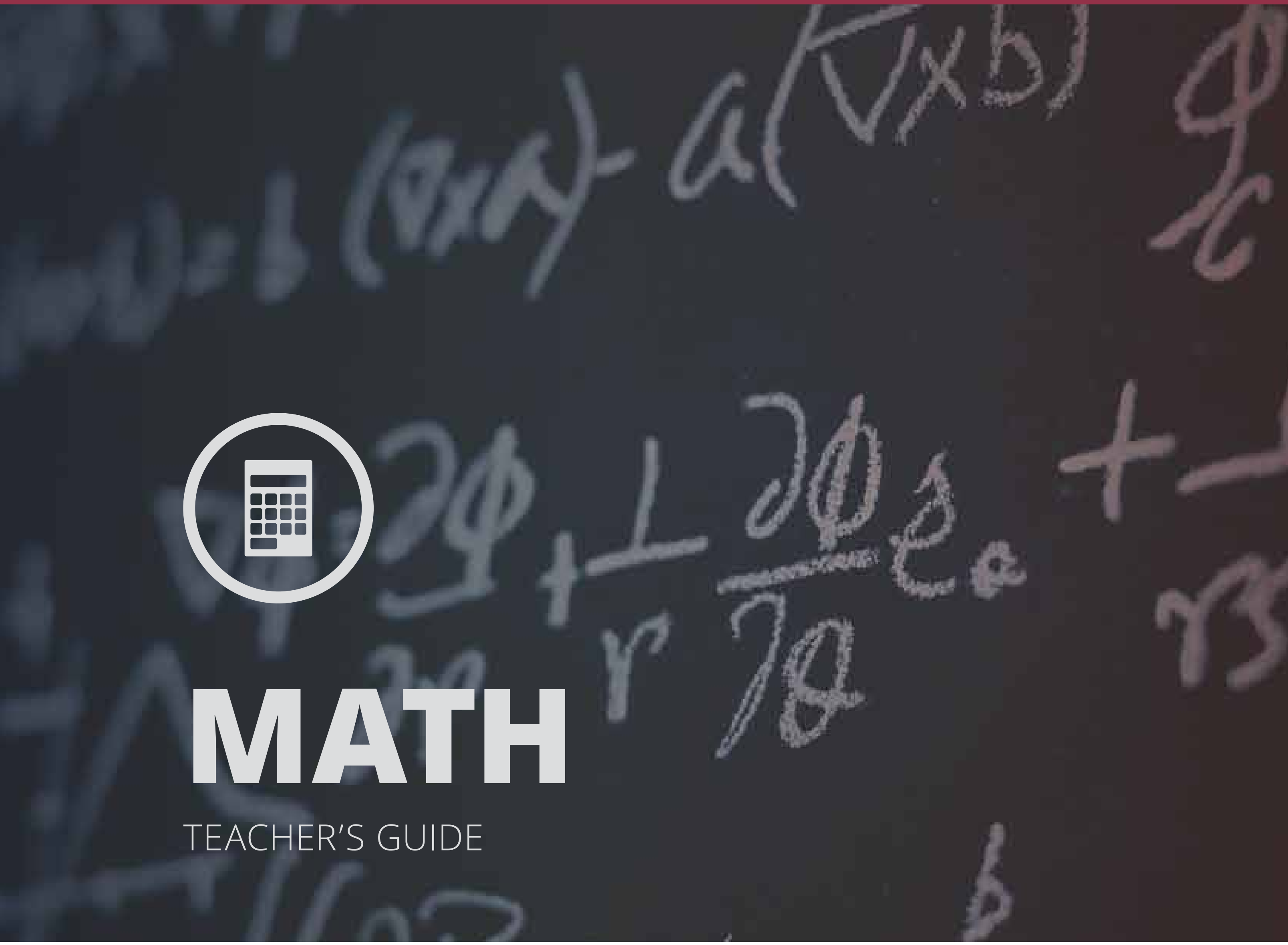

MATH

TEACHER'S GUIDE

12th Grade

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Alpha Omega PUBLICATIONS

MATH 1200

Teacher's Guide

INSTRUCTIONS FOR MATH

The LIFEPAC curriculum from grades 2 through 12 is structured so that the daily instructional material is written directly into the LIFEPACs. The student is encouraged to read and follow this instructional material in order to develop independent study habits. The teacher should introduce the LIFEPAC to the student, set a required completion schedule, complete teacher checks, be available for questions regarding both content and procedures, administer and grade tests, and develop additional learning activities as desired. Teachers working with several students may schedule their time so that students are assigned to a quiet work activity when it is necessary to spend instructional time with one particular student.

Math is a subject that requires skill mastery. But skill mastery needs to be applied toward active student involvement. Measurements require measuring cups, rulers, and empty containers. Boxes and other similar items help the study of solid shapes. Construction paper, beads, buttons, and beans are readily available and can be used for counting, base ten, fractions, sets, grouping, and sequencing. Students should be presented with problem situations and be given the opportunity to find their solutions.

Any workbook assignment that can be supported by a real world experience will enhance the student's ability for problem solving. There is an infinite challenge for the teacher to provide a meaningful environment for the study of math. It is a subject that requires constant assessment of student progress. Do not leave the study of math in the classroom.

This section of the Math Teacher's Guide includes the following teacher aids: Answer Keys and reproducible Alternate LIFEPAC Tests.

ANSWER KEY

SECTION 1: Ordered-Pair Numbers

Ordered-Pair Numbers: Relations

1.1 h. domain
1.2 f. element
1.3 e. function
1.4 d. ordered pair
1.5 a. range
1.6 c. relation
1.7 b. $R \times R$
1.8 g. set
1.9 i. subset

1.10 b. {5, 6, 7}
The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {5, 6, 7}.

1.11 a. {0, 1, 2}
The range is the set of all second numbers of each ordered pair in a relation. Therefore, the range of this relation is {0, 1, 2}.

1.12 a. {6, 7, 8, 9}
The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {6, 7, 8, 9}.

1.13 b. $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$
The range is the set of all second numbers of each ordered pair in a relation. Therefore, the range of this relation is $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$.

1.14 b. 1
The domain is the set of all first numbers of each ordered pair in a relation. Since the first number of each ordered pair in this relation is the same, the number is only listed once. The domain of this relation is $\{\frac{1}{2}\}$.

1.15 c. 4
The range is the set of all second numbers of each ordered pair in a relation. The range of this relation is $\{\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$.

1.16 c. {6.2, 7.3, 8.4, 9.5}
The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {6.2, 7.3, 8.4, 9.5}.

1.17 a. {0.3}
The range is the set of all second numbers of each ordered pair in a relation. Since the second number of each ordered pair in this relation is the same, the number is only listed once. The range of this relation is {0.3}.

1.18 a. {(10, 2), (15, 3), (20, 4), (30, 6), (60, 8), (90, 10)}
The domain is the first number of the ordered pair and the range is the second. Therefore, combining the elements of the domain, *D*, with the elements of the range, *R*, will give the set of the ordered pairs, *Q*.
Q = {(10, 2), (15, 3), (20, 4), (30, 6), (60, 8), (90, 10)}.

1.19 b. {(1, 16), (2, 64), (3, 144), (4, 256), (5, 400)}
The domain is the first number of the ordered pair and the range is the second. In this case, *D* = {1, 2, 3, 4, 5}, and *R* = {16, 64, 144, 256, 400}. Therefore, combining the elements of the domain, *D*, with the elements of the range, *R*, will give the set of the ordered pairs, *F*. *F* = {(1, 16), (2, 64), (3, 144), (4, 256), (5, 400)}.

1.20 b. Domain $\{x: x \in R, x \geq 0\}$
Since the relation contains a square root radical, domain values must be excluded that would make the radicand negative. Otherwise, you have imaginary numbers instead of real numbers. Therefore, $y \geq 0$ and Domain $\{x: x \in R, x \geq 0\}$.

1.21 d. Domain $\{r: r \in R, r \neq 0\}$
Since the relation contains a fraction, domain values must be excluded that would make the denominator zero. Therefore, $13r \neq 0$ or $r \neq 0$. Domain $\{r: r \in R, r \neq 0\}$.

1.22 b. Domain $\{a: a \in R, a \neq 0\}$
The relation is $ab = 12$, or $b = \frac{12}{a}$. Therefore, $a \neq 0$. Domain $\{a: a \in R, a \neq 0\}$.

1.23 b. odd integers
If x = an even integer, then $x + 1$ must be an odd integer = y. Therefore, Domain $\{x: x$ is an even integer$\}$ and Range $\{y: y$ is an odd integer$\}$.

1.24 c. Domain $\{x: x \in R\}$, Range $\{y: y \in R\}$

ordered pairs, (2, 15), to determine the equation.
$y - y_1 = m(x - x_1)$
$y - 15 = 5(x - 2)$
$y - 15 = 5x - 10$
$y = 5x + 5$

1.43 $y = x^2$

Check for a linear function. First, determine the ratios of the difference in the y-coordinates compared to the x-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 9}{5 - 3} = \frac{16}{2} = 8$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{4 - 9}{-2 - 3} = \frac{-5}{-5} = 1$

Since the ratios are not the same, it is not a linear function. Check the ordered pairs to see if it is a quadratic function. Note that the second term in the ordered pair, y, is the square of the first term, x. Therefore, it is a quadratic function and can be written as $y = x^2$.

1.44 b. $50t$

Note that D is 50 times t. Hence, this is a linear function and $D = 50t$.

1.45 c. $\frac{n(n - 3)}{2}$

1.46 c. $S = 16t^2$

1.47 a. 0

$D = \{(n, S): (0, 0), (1, 1), (2, 3), (3, 6), (100, 5{,}050)\}$

$S = \frac{n}{2}(n + 1)$

For $n = 0$, $S = \frac{0}{2}(0 + 1) = 0$

For $n = 1$, $S = \frac{1}{2}(1 + 1) = \frac{1}{2}(2) = 1$

For $n = 2$, $S = \frac{2}{2}(2 + 1) = 1(3) = 3$

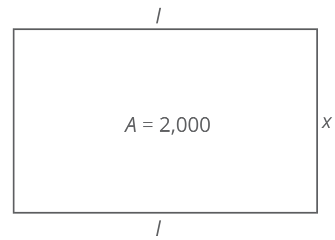
For $n = 100$, $S = \frac{100}{2}(100 + 1) = 50(101) = 5{,}050$

For $S = 6$,

$$6 = \frac{n}{2}(n + 1)$$
$$12 = n(n + 1)$$
$$12 = n^2 + n$$
$$n^2 + n - 12 = 0$$
$$(n + 4)(n - 3) = 0$$
$$n + 4 = 0 \qquad n - 3 = 0$$
$$n = -4 \qquad n = 3$$

Therefore,
$D = \{(n, S): (0, 0), (1, 1), (2, 3), (3, 6), (100, 5{,}050)\}$

1.48 b. $x > 0$



$xl = 2{,}000$

$l = \frac{2{,}000}{x}$

$C(x) = 0.50x + 0.30(x + 2l)$

Substitute for l:

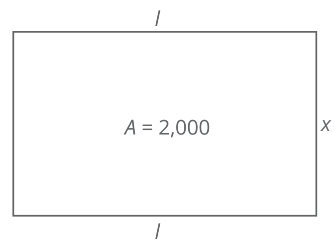
$C(x) = 0.50x + 0.30(x + 2[\frac{2{,}000}{x}])$

$C(x) = 0.50x + 0.30x + \frac{0.30(2)(2{,}000)}{x}$

$C(x) = 0.80x + \frac{1{,}200}{x}$

The domain is $x > 0$.

1.49 b. $0.8x + \frac{1{,}200}{x}$



$xl = 2{,}000$

$l = \frac{2{,}000}{x}$

$C(x) = 0.50x + 0.30(x + 2l)$

Substitute for l:

$C(x) = 0.50x + 0.30(x + 2[\frac{2{,}000}{x}])$

$C(x) = 0.50x + 0.30x + \frac{0.30(2)(2{,}000)}{x}$

$C(x) = 0.80x + \frac{1{,}200}{x}$

The domain is $x > 0$.

2.21 a. $\frac{6}{7}$

First, find the common domain of *f* and *g*. Then, divide the range elements of the common domain. The answer is

$\frac{f(x)}{g(x)} = \{(2, \frac{4}{5}),(4, \frac{6}{7}),(6, \frac{8}{9})\}$

2.22 $3x^2 + 11x + 10$

$(F \bullet G)(x) = (x + 2)(3x + 5)$
$= 3x^2 + 5x + 6x + 10$
$= 3x^2 + 11x + 10$

2.23 b. $\frac{f(x)}{g(x)} = \frac{x + 2}{3x + 5}$

2.24 $x^2 + 4x + 4$

$f(x) = x + 2$
$[f(x)]^2 = (x + 2)^2$
$= x^2 + 4x + 4$

2.25 a. $\frac{1}{4}$

$f(x) = x + 2$

$h(x) = \frac{1}{x - 1}$

$\frac{[h(x)]^2}{f(x)} = \frac{[\frac{1}{x-1}]^2}{(x + 2)} = \frac{1}{(x - 1)^2(x + 2)}$

2.26 $x^2 + 2xh + h^2 + 2x + 2h + 1$

$f(x) = x^2 + 2x + 1$
$f(x + h) = (x + h)^2 + 2(x + h) + 1$
$= x^2 + 2xh + h^2 + 2x + 2h + 1$

2.27 c. $2x + h$

$F(x) = x^2$

$\frac{F(x + h) - F(x)}{h} = \frac{(x + h)^2 - x^2}{h}$

$= \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h}$

$= 2x + h$

2.28 b. 2

$F(x) = 2x + 3$

$\frac{F(x + h) - F(x)}{h} = \frac{[2(x + h) + 3] - (2x + 3)}{h}$

$= \frac{2x + 2h + 3 - 2x - 3}{h} = \frac{2h}{h} = 2$

Algebra of Functions: Composition

2.29 d. composition of functions
2.30 b. constant function
2.31 a. identity function
2.32 c. zero function
2.33 d. $F[G(x)]$
2.34 a. $3x + 7$

$g[f(x)] = (3x + 2) + 5$
$= 3x + 7$

2.35 b. $2x^2 + 11$

$g[f(x)] = 2(x^2 + 6) - 1$
$= 2x^2 + 12 - 1$
$= 2x^2 + 11$

2.36 a. $x^2 + 8$

$g[h(x)] = (x^2 + 1) + 7$
$= x^2 + 8$

2.37 c. $\frac{1}{3x - 2}$

$h[k(x)] = \frac{1}{(3x - 4) + 2} = \frac{1}{3x - 2}$

2.38 a. + $16x + 16$

$f[g(x)] = \frac{1}{(\frac{1}{2x + 4})^2} = \frac{1}{\frac{(1)^2}{(2x + 4)^2}} = \frac{1}{\frac{1}{4x^2 + 16x + 16}}$

$= \frac{1}{1} \bullet \frac{4x^2 + 16x + 16}{1} = 4x^2 + 16x + 16$

2.39 c. 169

$f(x) = x^2$
$g(x) = x + 6$
$h(x) = 7$

$f\{g[h(x)]\} = \{[(7) + 6]\}^2$
$= \{13\}^2$
$= 169$

2.40 b. 55

$f(x) = x^2$
$g(x) = x + 6$
$h(x) = 7$

$g\{f[h(x)]\} = [(7)]^2 + 6$
$= 49 + 6$
$= 55$

2.41 a. 7

$h(x)$ is constant and equal to 7 for any value of x.

2.42 c. $4x^2 + 4x + 1$

$f[g(x)] = f(2x + 1)$
$= (2x + 1)^2$
$= 4x^2 + 4x + 1$

2.43 c. 5

$g[f(-2)] = g[(x^2)]$
$= (-2)^2 + 1$
$= 5$

2.44 c. zero
2.45 b. constant
2.46 a. identity

Algebra of Functions: Inverse

2.47 c. the range and the domain are interchanged
2.48 b. Yes, each element in the domain has only one range value.

2.017 b. 7

$f(x) = 2x^2 - 3x + 1$
$f(3) = 2(3)^2 - 3(3) + 1$
$= 2(9) - 9 + 1$
$= 18 - 8 = 10$

$f(2) = 2(2)^2 - 3(2) + 1$
$= 2(4) - 6 + 1$
$= 8 - 5 = 3$

$f(3) - f(2) = 10 - 3$
$= 7$

2.018 a. Yes, no two ordered pairs in this list have a repeat of the domain element.

2.019 b. $y^2 = x$

Note that the first number of each ordered pair is the square of the second number. Therefore, $y^2 = x$.

2.020 c. No, two ordered pairs in this list have a repeat of the domain element.

2.021 a. $\frac{n(n+1)}{2}$

2.022 b. $\frac{x+2}{5}$

To find the inverse, let $y = G(x)$ and interchange x and y:

$G(x) = 5x - 2 \quad \rightarrow \quad y = 5x - 2$
$x = 5y - 2$
$x + 2 = 5y$
$\frac{x+2}{5} = y \quad \rightarrow \quad G^{-1}(x) = \frac{x+2}{5}$

2.023 If $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then $f(x)$ and $g(x)$ are inverses of each other.

$$(f \circ g)(x) = \frac{2}{(\frac{6x+2}{x}) - 6} = \frac{2}{6x + 2 - 6x} = \frac{2x}{2} = x$$

and

$$(g \circ f)(x) = \frac{6(\frac{2}{x-6}) + 2}{\frac{2}{x-6}} = \frac{\frac{12}{x-6} + \frac{2(x-6)}{x-6}}{\frac{2}{x-6}}$$

$$= \frac{\frac{12 + 2x - 12}{x-6}}{\frac{2}{x-6}} = \frac{\frac{2x}{x-6}}{\frac{2}{x-6}} = \frac{2x}{x-6} \cdot \frac{x-6}{2} = x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, $f(x)$ and $g(x)$ are inverses of each other.

2.024 If $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then $f(x)$ and $g(x)$ are inverses of each other.

$$(f \circ g)(x) = \tfrac{4}{5}(\frac{5x-5}{4}) + 1$$
$$= \frac{20x - 20}{20} + 1$$
$$= \frac{20x}{20} - \frac{20}{20} + 1$$
$$= x - 1 + 1$$
$$= x$$

and

$$(g \circ f)(x) = \frac{5(\frac{4}{5}x + 1) - 5}{4}$$
$$= \frac{\frac{20}{5}x + 5 - 5}{4}$$
$$= \frac{4x + 5 - 5}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, $f(x)$ and $g(x)$ are inverses of each other.

LIFEPAC TEST

1. c. $\{x: x \in R, x \neq -4, x \neq 7\}$

$\{(x, y): y = \frac{x(x-3)}{(x+4)(x-7)}\}$

A rational expression has a domain of all real numbers with the exception of values that make the denominator zero:

$x + 4 \neq 0$ $\quad x - 7 \neq 0$

$x \neq -4$ $\quad x \neq 7$

2. b. $(\sqrt{3}, -4)$ and c. $(-\sqrt{3}, 4)$

3. c. 2

4. c. $\{x: x \geq 3\}$

5. b. all real numbers

6. b. $x^2 + 2$

7. b. 13

$F(-2) = 3(-2)^2 + 1$
$= 3(4) + 1$
$= 12 + 1$
$= 13$

8. a. $3x^2 + 2x - 2$

$F(x) + G(x) = (3x^2 + 1) + (2x - 3)$
$= 3x^2 + 2x - 2$

9. c. $12x^2 - 36x + 28$

$F(x) = 3x^2 + 1$
$G(x) = 2x - 3$
$H(x) = x$
$F \circ G = F[G(x)]$
$= 3(2x - 3)^2 + 1$
$= 3(4x^2 - 12x + 9) + 1$
$= 12x^2 - 36x + 27 + 1$
$= 12x^2 - 36x + 28$

10. b. $\frac{x+3}{2}$

$F(x) = 3x^2 + 1$
$G(x) = 2x - 3$
$H(x) = x$
$G(x) = y = 2x - 3$

Interchange x and y:
$x = 2y - 3$
$x + 3 = 2y$
$y = \frac{x+3}{2}$
$G^{-1}(x) = \frac{x+3}{2}$

11. a. x

$H(x) = y = x$

Interchange x and y:
$x = y$
$H^{-1}(x) = x$

12. b. 23

$F(3) + G(4) - 2H(5) = 3[(3)^2 + 1 + [2(4) - 3] - 2(5)$
$= [3(9) + 1] + 8 - 3 - 10$
$= 27 + 1 - 5$
$= 23$

13. c. $\frac{x+26}{5}$

$F(x) = 5x - 6$
$G(x) = x - 4$

First find the composition $F \circ G$ and then find the inverse of that composition.

$F \circ G = F[G(x)]$
$= 5(x - 4) - 6$
$= 5x - 20 - 6$
$= 5x - 26$

Find $(F \circ G)^{-1}$:
$y = 5x - 26$

Interchange x and y:
$x = 5y - 26$
$x + 26 = 5y$
$y = \frac{x+26}{5}$
$(F \circ G)^{-1} = \frac{x+26}{5}$

14. a. $\frac{x+26}{5}$

$F(x) = 5x - 6$
$G(x) = x - 4$
$F^{-1}(x) = y = 5x - 6$

Interchange x and y:
$x = 5y - 6$
$x + 6 = 5y$
$y = \frac{x+6}{5}$
$F^{-1}(x) = \frac{x+6}{5}$

$G(x) = y = x - 4$

Interchange x and y:
$x = y - 4$
$y = x + 4$
$G^{-1}(x) = x + 4$
$G^{-1} \circ F^{-1} = G^{-1}[F^{-1}(x)]$
$= (\frac{x+6}{5}) + 4$
$= \frac{x+6+20}{5}$
$= \frac{x+26}{5}$

MATH 1201

ALTERNATE LIFEPAC TEST

43/54

NAME ______________________

DATE ______________________

SCORE ______________________

Work the following problems (each answer, 3 points).

1. What is the domain of the relation $\{(x, y): y = \frac{x(x-3)}{(x+4)(x-7)}\}$? ______

a. $\{x: x \approx R, x \leq -4, x \neq 7\}$
b. $\{x: x \notin R, x = -4, x \neq 7\}$
c. $\{x: x = R, x \neq -4, x \geq 7\}$
d. $\{x: x \in R, x \neq -4, x \neq 7\}$

2. Choose the ordered pairs that belong to the given relation. Select all that apply.

$\{(x, y): x^2 < |y|^3 - 5\}$ ____________

a. (0, 1)
b. (2, 3)
c. (-2, 3)
d. (0, 0)

3. Complete the ordered pair for the relation $\{(x, y): y = 3|x + 2|$ and $x \in \{-2, -1, 0, 1, 2\}\}$.

(-2, ______)

a. 12
b. 0
c. 3

4. $\{(x, y): y = \sqrt{x - 4}\}$
The domain of the set is represented by ______ .

a. $\{x: x \in R\}$
b. $\{x: x \geq 4\}$
c. $\{x: x \leq 4\}$
d. $\{x: x \geq 0\}$

13. Given: $F(x) = 5x - 6$ and $G(x) = x - 4$ F^{-1} = _______

a. $\frac{x + 6}{5}$

b. $-6x + 10$

c. $\frac{x + 10}{5}$

14. Given: $F(x) = 5x - 6$ and $G(x) = x - 4$ $G^{-1} \circ F^{-1}$ = _______

a. $\frac{x + 26}{5}$

b. $-6x + 10$

c. $\frac{x + 10}{5}$

15. Given: $F(x) = 3x$ and $G(x) = x^2 + 1$ Find $(F + G)(x)$. _______

a. $3x^3 + 1$

b. $x^2 + 3x + 1$

c. $3x^2 + 1$

16. Given: $F(x) = 3x$ and $G(x) = x^2 + 1$ Find $(G - F)(x)$. _______

a. $x^2 - 3x - 1$

b. $-2x + 1$

c. $x^2 - 3x + 1$

17. Given: $F(x) = 3x$ and $G(x) = x^2 + 1$ Find $\frac{F}{G}(-1)$. _______

a. $-\frac{2}{3}$

b. 1

c. $-\frac{3}{2}$

18. Verify if $f(x) = \frac{x - 3}{5}$ and $g(x) = 5x - 3$ are inverses.

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