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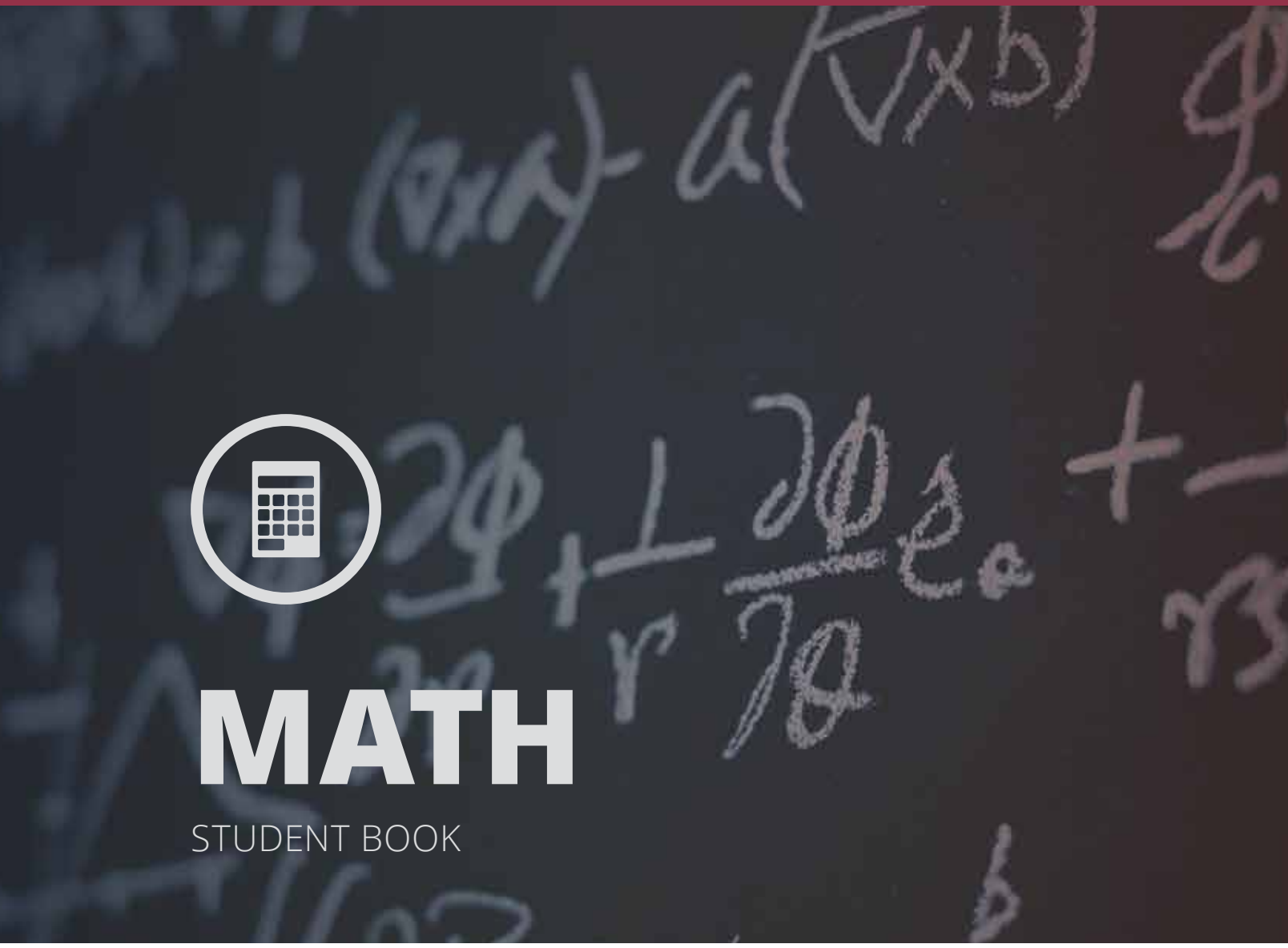

MATH

STUDENT BOOK

6th Grade

Alpha Omega PUBLICATIONS

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MATH 601

Whole Numbers and Algebra

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Data Analysis

Introduction

In this unit, you will be introduced to the topic of Data Analysis. You will learn about ways to collect and organize data, and many ways to display data. You'll learn what type of graph is best to display different data sets. You'll also discover different ways to describe data, called measures of central tendency. You will learn many new terms and have several concepts to build on in your future studies of statistics.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Determine whether a sample is biased or random.
- Find the measures of central tendency.
- Organize and display data in frequency tables, histograms, line plots, stem-and-leaf plots, bar graphs, and line graphs.
- Use Venn diagrams to solve problems, including counting problems
- Solve route problems with vertex-edge graphs.

1. FACTORS AND FRACTIONS

DIVISIBILITY AND PRIME FACTORIZATION

Do you remember what a prime number is? A prime number is a number that has only two **factors**, or numbers, that evenly go into it: 1 and itself. For example, the smallest prime number is 2, because only 1 and 2 evenly go into it. And, although it may seem that the number 1 itself may be a prime number, it is not.

The number of prime numbers is actually infinite. The list goes on forever! However, an ancient Greek mathematician, Eratosthenes, developed a way to make a list of at least the small prime numbers, called the Sieve of Eratosthenes.

Objectives

Review these objectives. When you have completed this section, you should be able to:

- Determine whether a number is prime or composite.
- Use divisibility rules to find the prime factorization of a number.
- Express a number as a product of prime numbers.
- List all the factors of a number.
- Find the GCF of two numbers.
- Use divisibility rules to find factors of a number.
- Use a fraction to show part of a whole.
- Represent a fraction on the number line.
- Identify and find equivalent fractions.
- Reduce fractions to lowest terms.

Vocabulary

composite number. A number that has more factors than just 1 and itself.

denominator. The number under the fraction line; tells how many equal parts the whole was broken into.

equivalent fractions. Fractions with the same numerical value; fractions that are equal to each other.

factor. A number that divides evenly into another number.

factor tree. An organized way of finding the prime factorization of a number.

fraction. A number that shows part of a whole.

fraction bar. The line between the numerator and the denominator of a fraction.

So, to represent amounts that are part of a whole, we use fractions. The top number, or the numerator, in a fraction tells how many parts we have, and the bottom number, or the denominator, tells how many total parts are in the whole. For example, the fraction $\frac{4}{10}$ tells us that we have four of ten parts. Fractions that have a denominator of ten or a power of ten (like 10, 100, or 1,000) are called decimal fractions. That's because they can be written shorthand as decimal numbers, using a decimal point. The digits to the left of the decimal point represent the whole number part of the number. The digits to the right of the decimal point represent the fraction part of a number.

This might help!

Decimal fractions can be written shorthand because they have a denominator that is a power of ten and our decimal system is based on the number ten. In fact, the prefix "deci-" means ten.

Notice in the chart that the places to the right of the decimal point all end in *-ths*. For example, the hundreds place is to the left of the decimal point. But, the hundred*ths* place is to the right of the point. The tens place is to the left of the decimal point. And, the ten*ths* place is to the right. Also, notice that there is *no* "oneths" place. The first place value to the right of the decimal point is the tenths.

Whole numbers ← → Fractions (decimals)

Thousands			Units			Fractions (decimals)		
hundreds	tens	ones	hundreds	tens	ones	tenths	hundredths	thousandths
100,000	10,000	1,000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

decimal point

Example:

Which digit is in the hundredths place? **0.861**

Solution:

The hundredths place is the second place to the right of the decimal. So, the 6 is in the hundredths place. There are six-hundredths.

Key point!

In this decimal number, there is no whole number part. So, we write a zero to the left of the decimal point. This decimal number is between 0 and 1 on the number line.

Example:

What place is the 0 in? **7.08**

Solution:

The zero is in the first place to the right of the decimal point, or the tenths place. That means that there no tenths.

Place a check mark next to each correct answer (you may select more than one answer).

1.14 Which of the following statements are true of the greatest common factor of two numbers?

- ☐ Each number must share the factor.
- ☐ The number must be large.
- ☐ The greatest common factor could be 1.
- ☐ The number must be the largest factor both numbers share.

1.15 Use the divisibility rules to determine which of the following numbers 528 is divisible by.

☐ 2	☐ 4	☐ 6	☐ 9	☐ 11
☐ 3	☐ 5	☐ 7	☐ 10	

1.16 Choose all the factors of 5.

☐ 1	☐ 2	☐ 3	☐ 4	☐ 5

1.17 Choose all the factors of 12.

☐ 1	☐ 4	☐ 7	☐ 10
☐ 2	☐ 5	☐ 8	☐ 11
☐ 3	☐ 6	☐ 9	☐ 12

1.18 Choose all the factors of 8.

☐ 1	☐ 3	☐ 5	☐ 7
☐ 2	☐ 4	☐ 6	☐ 8

Answer true or false.

1.19 _____ The list of all factors for 21 is 1, 3, 7, and 21.

1.20 _____ The list of all factors for 32 is 1, 2, 4, 8, and 32.

Circle the letter of each correct answer.

1.21 Find the GCF of 14 and 22.

a. 1 b. 2 c. 4 d. 7

1.22 Find the GCF of 20 and 30.

a. 2 b. 5 c. 10 d. 15

1.23 Find the GCF of 16 and 27.

a. 1 b. 2 c. 3 d. 8

Write the correct answer on each blank

1.24 The GCF of 18 and 36 is _____ .

1.25 The GCF of 44 and 66 is _____ .

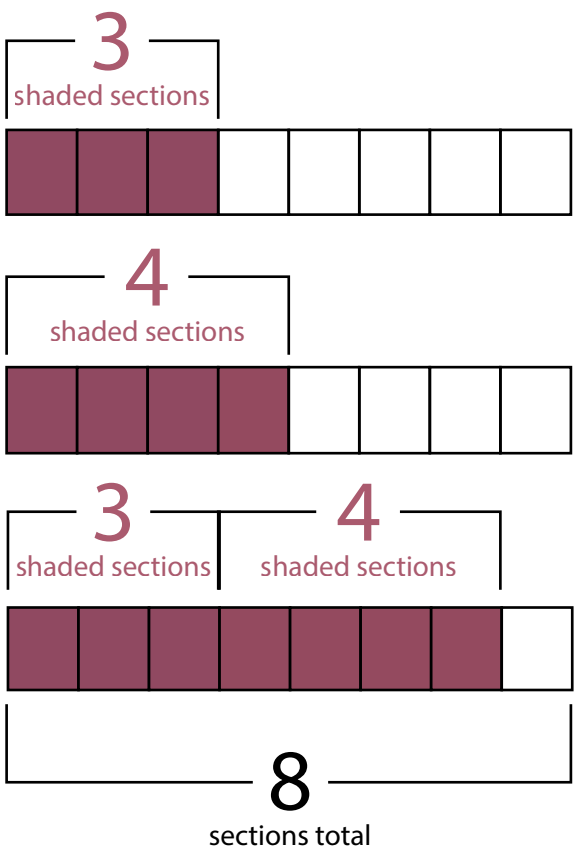
1.26 The GCF of 15 and 35 is _____ .

ADDING FRACTIONS WITH LIKE DENOMINATORS

Let's represent the addition problem $\frac{3}{8} + \frac{4}{8}$. We can use a rectangular model.

Since the addends have like denominators, or denominators that are the same number, the pieces from each rectangle will be the same size. We can add them together simply by combining the rectangles. When we combine the rectangles, in the same way as we did with mixed numbers, we end up with seven shaded pieces of a rectangle that has been evenly divided into eight pieces. So, the sum of $\frac{3}{8}$ and $\frac{4}{8}$ is $\frac{7}{8}$.

In short, to add fractions with like denominators, add the numerators and keep the denominator the same.



Example:

Find the sum.

$$\frac{1}{6} + \frac{2}{6}$$

Solution:

The denominators are the same, so we can just add the numerators and keep the denominator the same.

$$\frac{1+2}{6} + \frac{3}{6}$$

Notice that $\frac{3}{6}$ can be reduced. The GCF of 3 and 6 is 3, so divide the numerator and denominator by 3.

$$\frac{3 \div 3}{6 \div 6} + \frac{1}{2}$$

So, the sum of $\frac{1}{6}$ and $\frac{2}{6}$ is $\frac{1}{2}$.

Keep in mind...

Always reduce the sum to lowest terms.

Use Figure 1 for questions 1.9 – 1.11.

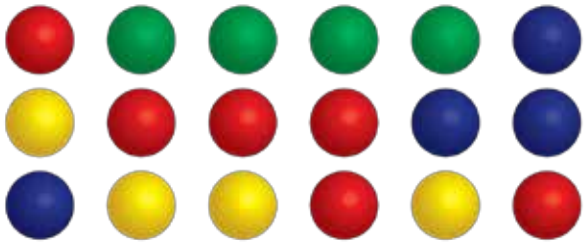


Figure 1

1.9 What is the ratio of red gumballs to yellow gumballs in lowest terms?

a. 2:3
b. 3:2
c. 2:1
d. 1:2

1.10 What is the ratio of green gumballs to blue gumballs in lowest terms?

a. 4:4
b. 2:2
c. 1:1
d. 3:2

1.11 What is the ratio of yellow gumballs to total gumballs in lowest terms?

a. $\frac{4}{18}$
b. $\frac{4}{14}$
c. $\frac{2}{7}$
d. $\frac{2}{9}$

1.12 The ratio of roses to carnations is 7 to 5. If there are 28 roses, how many carnations are there? Complete the ratio table to find the number of carnations.

Roses	7	14	28
Carnations	5		

a. 10
b. 20
c. 30
d. 40

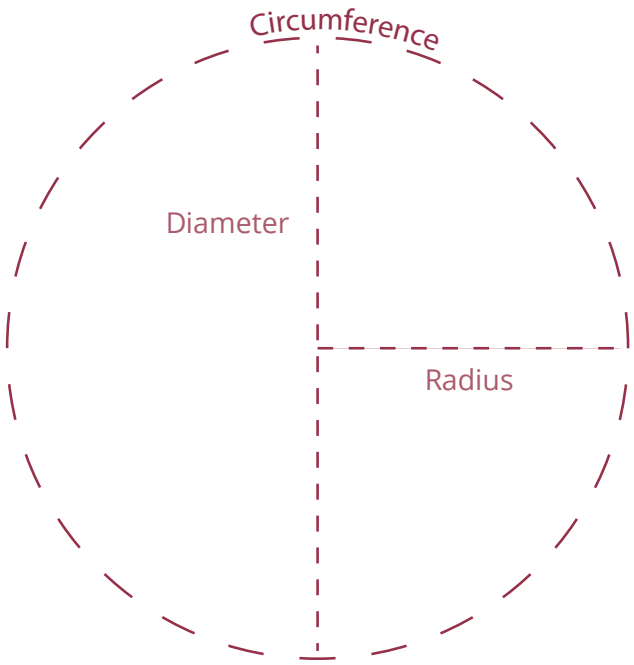
1.13 The ratio of adults to children is 16 to 10. If there are 40 adults, how many children are there? Complete the ratio table to find the number of children.

Adults	16	8	40
Children	10		

a. 25
b. 20
c. 15
d. 30

GEOMETRY: CIRCUMFERENCE

Do you know what circumference is? How about diameter? Radius? All three of these terms represent measurements on a circle. Take a look.



In this lesson, we'll explore how to find the circumference of a circle. We'll also learn what ratios have to do with geometry!

RATIO OF CIRCUMFERENCE: DIAMETER

Circles have a special property: the ratio of the circumference of a circle to its diameter is always the same. Remember that a ratio is a comparison of two numbers as a quotient. So, the quotient of the circumference and the diameter (or circumference ÷ diameter) is the same value in every circle! Let's look at two circles that you are probably very familiar with– the penny and dime.

Every type of coin has the exact same measurements. The penny always has a diameter of $\frac{3}{4}$ of an inch and a circumference of $2\frac{5}{14}$ inches.

And, the dime always has a diameter of $\frac{141}{200}$ of an inch and a circumference of $2\frac{151}{700}$ inches.

Now, let's try dividing the circumference by the diameter for both the penny and the dime. We'll have to remember what we've learned about dividing with fractions to help us!

$C = 2\frac{5}{14}$ in.

$d = \frac{3}{4}$ in.

$C = 2\frac{151}{700}$ in.

$d = \frac{141}{200}$ in.

Penny:

Divide the circumference by the diameter.

$$2\frac{5}{14} \div \frac{3}{4}$$

Rewrite the mixed number as an improper fraction.

$$\frac{33}{14} \div \frac{3}{4}$$

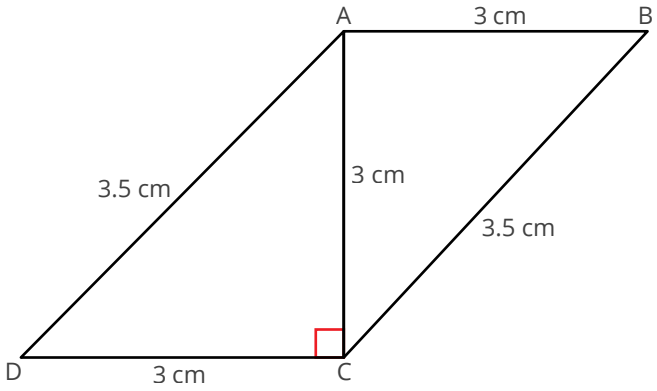
Multiply by the reciprocal of the divisor.

$$\frac{33}{14} \times \frac{4}{3}$$

SELF TEST 1: PLANE FIGURES

Circle each correct answer (each answer, 4 points).

Use parallelogram *ABCD* to answer questions 1.01 – 1.04.



1.01 What is the perimeter?

a. 14 cm b. 13 cm
c. 12 cm d. 11 cm

1.02 What is the area?

a. 7 cm^2 b. 9 cm^2
c. 12 cm^2 d. 12.25 cm^2

1.03 What is the area of triangle *ABC*?

a. 7 cm^2 b. 3.5 cm^2
c. 6 cm^2 d. 4.5 cm^2

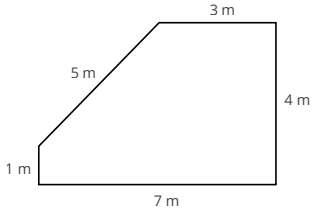
1.04 What is the perimeter of *ACD*?

a. 9.5 cm b. 7.5 cm^2
c. 8.5 cm^2 d. 5.5 cm

1.05 What is the circumference of a circle with a diameter of 5 meters? (Use 3.14 for pi.)

a. 78.5 m b. 31.4 m c. 15.7 m d. 8.14 m

1.06 What is the perimeter of the figure?



a. 28 m b. 22 m
c. 20 m d. 14 m

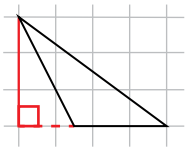
1.07 A regular pentagon has a perimeter of 60 feet. How long is each side?

a. 5 feet b. 6 feet c. 10 feet d. 12 feet

1.08 If the area of the parallelogram is 15 cm^2, what is the area of the green triangle?

a. 30 cm^2 b. 15 cm^2
c. 7.5 cm^2 d. 8 cm^2

1.09 What is the height of the triangle?



a. 2 units b. 3 square units
c. 3 units d. can't be determined

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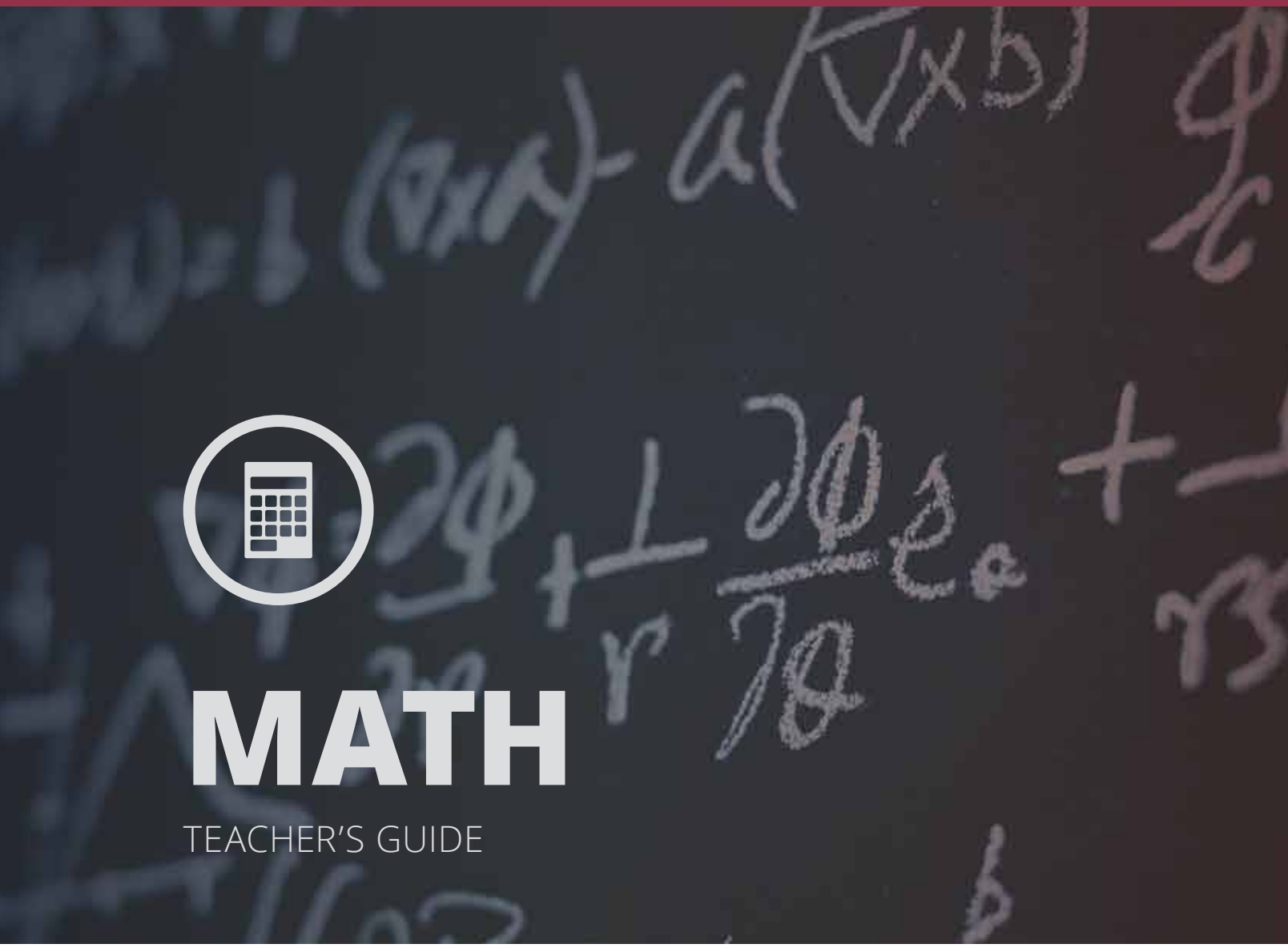

MATH

TEACHER'S GUIDE

6th Grade

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Alpha Omega PUBLICATIONS

MATH 600

Teacher's Guide

INSTRUCTIONS FOR SIXTH GRADE MATH

The LIFEPAC curriculum from grades two through twelve is structured so that the daily instructional material is written directly into the LIFEPACs. The student is encouraged to read and follow this instructional material in order to develop independent study habits. The teacher should introduce the LIFEPAC to the student, set a required completion schedule, complete teacher checks, be available for questions regarding both content and procedures, administer and grade tests, and develop additional learning activities as desired. Teachers working with several students may schedule their time so that students are assigned to a quiet work activity when it is necessary to spend instructional time with one particular student.

Math is a subject that requires skill mastery. But skill mastery needs to be applied toward active student involvement. Measurements require measuring cups, rulers, empty containers. Boxes and other similar items help the study of solid shapes. Construction paper, beads, buttons, beans are readily available and can be used for counting, base ten, fractions, sets, grouping, and sequencing. Students should be presented with problem situations and be given the opportunity to find their solutions.

Any workbook assignment that can be supported by a real world experience will enhance the student's ability for problem solving. There is an infinite challenge for the teacher to provide a meaningful environment for the study of math. It is a subject that requires constant assessment of student progress. Do not leave the study of math in the classroom.

ANSWER KEYS

SECTION 1

1.1 a, c, b
1.2 2

6,241,095: 5: ones place; 9: tens place; 0: hundreds place; 1: thousands place; 4: ten thousands place; 2: hundred thousands place; 6: millions place

1.3 thousands

993,140: 0: ones place; 4: tens place; 1: hundreds place; 3: thousands place; 9: ten thousands place; 9: hundred thousands place

1.4 6,350

Six thousand: 6,000; Three hundred: 300; Fifty: 50; 6,000+300+50=6,350

1.5 204,732

204,732: 2: ones place; 3: tens place;7: hundreds place;4: thousands place;0: ten thousands place;2: hundred thousands place

1.6 Ava rounded incorrectly; the answer should be 19,000.

When rounding 19,350 to the nearest thousand, take into consideration the number to the right of the thousands place. Because 3 is a number less than 5, it does not round 9 up to the next number. Once the rounding rules are assessed, the numbers following the 9 are turned to zeros.

1.7 13,290

9: tens place so look to the 3 for rounding rules. Since 3 is less than 5, the 9 stays the same and the 3 turns to a zero.

1.8 2,600

149: rounds down to 100; 2,470: rounds up to 2,500; 100+2,500=2,600

1.9 30

538 rounds up to 540; 509 rounds up to 510; 540-510=30

1.10 1,500

629 rounds down to 600; 215 rounds down to 200; 111 rounds down to 100; 588 rounds up to 600; 600+200+100+600=1,500

1.11 $180.00

$233.00 rounds down to $230.00; $47.00 rounds up to $50.00; Profit=$230.00-$50.00=$180.00

1.12 false

Since this is an addition problem, 12 and 58 are addends, not factors.

1.13 c, a, b
1.14 1,543

629 + 215 + 111 + 88 = 1,543

1.15 $139.00

224 - 85 = 139

1.16 192

8 × 24 = 192

1.17 16

192 ÷ 22 = 8 R16

1.18 grouping
1.19 multiplicative
1.20 order

1.012 117

9 × (4 + 9) = (9 × 4) + (9 × 9) = 36 + 81 = 117

1.013 commutative property of addition

The commutative property of addition changes the order of the addends.

1.014 (3 × 8) × 6 = 3 × (8 × 6)

The associative property of multiplication changes the grouping of the factors.

1.015 6 × (40 + 2) = (6 × 40) + (6 × 2)

42 is the same as (40 + 2). Using the distributive property, April can multiply 6 by each addend.

SECTION 2

2.1 a, b, c
2.2 c, b, a
2.3 4^5

It can be rewritten with a base of 4 and an exponent of 5.

2.4 8^3

It can be rewritten with a base of 8 and an exponent of 3.

2.5 1 × 1 × 1 × 1 × 1

1^5 is the same as multiplying five factors of 1.

2.6 7 × 7 × 7 × 7

7^4 is the same as multiplying four factors of 7.

2.7 9

Any base to the power of 1 is just that number.

2.8 2,401

7 × 7 × 7 × 7
49 × 7 × 7
343 × 7
2,401

2.9 1

1 raised to any power is just 1.

2.10 1,024

4 × 4 × 4 × 4 × 4
16 × 4 × 4 × 4
64 × 4 × 4
256 × 4
1,024

2.11 ✓ It can be read as "six cubed."
✓ It can be read as "six to the power of three."
✓ It has a base of 6.
✓ It is the same as multiplying three factors of 6.
✓ It has an exponent of 3.

2.12 ✓ It can be written as a multiplication problem with eight factors of 2.
✓ It has a power of 256.
✓ It can be read as "two raised to a power of eight."
✓ It has a base of 2.

2.13 b, a, c

2.14 81

$9^2 = 81$

2.15 225

$5^3 = 125$; $8^3 = 512$; $9^3 = 729$

2.16 64

$8^2 = 64$; $4^3 = 64$

2.17 $6^3 = 18$

$6^3 = 216$

2.18 $\sqrt{16} = 8$

$\sqrt{16} = 4$, not 8

2.19 $\sqrt[3]{64} = 8$

$\sqrt[3]{64} = 4$, not 8

2.20 c, b, d, a

2.21 2

$2^3 = 8$

2.22 12

$12^2 = 144$

2.23 2

$2^2 = 4$

2.24 3

$3^3 = 27$

2.25 12

$12^3 = 1{,}728$

2.26 10

$10^2 = 100$

2.27 b, a

2.28 true

2.29 false

Addition and subtraction are completed in the same step from left to right.

2.30 subtract inside the parentheses

The first step of the order of operations is to simplify inside any parentheses, so subtract inside the parentheses first.

2.31 simplify 2^2

The second step of the order of operations is to simplify any exponents or roots, so simplify 2^2 second.

2.32 30

$7 \times 2^2 + (8 - 2) \div 3$
$7 \times 2^2 + 6 \div 3$
$7 \times 4 + 6 \div 3$
$28 + 6 \div 3$
$28 + 2 = 30$

2.33 Her answer is incorrect. She should have multiplied before adding.

Tracy's answer is incorrect. She should have multiplied 6 and 2 *before* adding 3.

$3 + 6 \times 2$

$3 + 12$

15

2.34 0

$25 \div 5 + 7 - (4 \times 3)$
$25 \div 5 + 7 - 12$
$5 + 7 - 12$
$12 - 12 = 0$

2.012 80, 160

The sequence starts at 5 and multiplies by 2 repeatedly.
40 × 2 = 80
80 × 2 = 160

2.013 892

The sequence starts at 812 and adds 40 repeatedly.
852 + 40 = 892

2.014 true

In an arithmetic sequence, the difference between each pair of consecutive terms is the same.

2.015
- ✓ 64
- ✓ 49
- ✓ 144
- ✓ 4

$8^2 = 64$
$7^2 = 49$
$12^2 = 144$
$2^2 = 4$

SECTION 3

3.1
- ✓ A variable is represented by a letter.
- ✓ A variable represents a quantity.

3.2 a, c, d, b

3.3 Answers will vary. However, students should write out the word "four" and represent k as "a number." Some possibilities include:
- ✓ four added to a number
- ✓ the sum of four and a number
- ✓ four increased by a number
- ✓ four plus a number

3.4 Answers will vary. However, students should write out the word "eight" and represent h as "a number." Also, the order of the values is important. Some possibilities include:
- ✓ a number divided by eight
- ✓ the quotient of a number and eight

3.5 a number subtracted from five

The word "from" indicates that the values are switched. So "a number subtracted from five" translates to 5 - n.

3.6 15n

A number and variable can be written next to each other to show multiplication. The number is always written first.

3.7 y - 5

To find her age five years ago, subtract five from her current age. So, y - 5.

3.8 11 - 2x

3.9 18c

To find the total cost, multiply 18 doughnuts by the cost of each doughnut. So, 18c.

3.10 n + 21

3.11 45 ÷ z

3.12 a, b

3.13 d, c, b, a

LIFEPAC TEST

1. false

14,688 rounded to the nearest ten is 14,690.

2. true

3. square and cube

$8^2 = 64$; $4^3 = 64$

4. 2,800

1,841 rounds to 1,800
964 rounds to 1,000
1,800 + 1,000 = 2,800

5. 21

42 ÷ 2 = 21

6. 4,680

7. associative property of addition

The associative property of addition is illustrated. The grouping of addends can be changed without changing the value of the sum.

8. 11^4

9. multiply

10. The sequence starts 2 and multiplies by 3 repeatedly.

2 × 3 = 6
6 × 3 = 18
18 × 3 = 54

11. 120 cm^3

lwh
4 × 5 × 6
20 × 6
120

12. 15x + 7

Because x and 14x are like terms, they can be combined to equal 15x.

13. 14

n + 11
3 + 11
14

14. 8y + 96

8(y + 12)
8 ×· y + 8 · 12
8y + 96

15. x = 20

20 - 5 = 15, so x = 20.

16. c, a, d, b

17. 243

3 × 3 × 3 × 3 × 3 = 243

18. 12

$12^2 = 144$, so $\sqrt{144} = 12$

19. 82

7^2 - 3 + 9 × 8 ÷ 2
49 - 3 + 9 × 8 ÷ 2
49 - 3 + 72 ÷ 2
49 - 3 + 36
46 + 36
82

20. 16

The sequence starts at 28 and subtracts 3 repeatedly.
19 - 3 = 16

ALTERNATE LIFEPAC TEST

1. false

7,945 rounded to the nearest hundred is 7,900

2. true
3. cube

$2^3 = 8$

4. 470

371 rounds to 370.
97 rounds to 100.
370 + 100 = 470.

5. 6

18 ÷ 3 = 6

6. 3,540
7. commutative property of addition

With the commutative property of addition, the order of addends can be changed without changing the value of the sum.

8. 12^4
9. divide
10. Starts at 64 and divides by 2 repeatedly.

64 ÷ 2 = 32
32 ÷ 2 = 16
16 ÷ 2 = 8

11. 80 in.^{3}

lwh
2 × 8 × 5
16 × 5
80

12. 6y + 9

5y and y are like terms, so they can be combined to equal 6y.

13. 7

x - 7
14 - 7
7

14. 4n + 44

4(n + 11)
4 · n + 4 · 11
4n + 44

15. n = 5

5 + 12 = 17, so n = 5.

16. c, b, a, d
17. 256

4 × 4 × 4 × 4 = 256

18. 11

$11^2 = 121$, so $\sqrt{121} = 11$

19. 18

3^3 + 11 - 10 ÷ 2 × 4
27 + 11 - 10 ÷ 2 × 4
27 + 11 - 5 × 4
27 + 11 - 20
38 - 20
18

20. 21

The sequence starts with 5 and adds 4 repeatedly.
17 + 4 = 21

10. Describe this sequence.
64, 32, 16, 8, ...

a. Starts at 64 and subtracts 32 repeatedly.
b. Starts at 64 and divides by 2 repeatedly.
c. Starts at 8 and multiplies by 2 repeatedly.
d. Starts at 32 and subtracts 16 repeatedly.

11. The volume of a rectangular box can be found using the formula lwh where l represents the length, w represents the width, and h represents the height of the box. What is the volume of a box with the following dimensions?
l = 2 inches
w = 8 inches
h = 5 inches

a. 285 $in.^3$
b. 21 $in.^3$
c. 80 $in.^3$
d. 15 $in.^3$

12. Simplify.
(5y + 9) + y

a. 6y + 9
b. 14 + y
c. 14y
d. 15y

13. Evaluate x - 7 for x = 14.

a. 2
b. 7
c. 21
d. 147

14. Simplify using the distributive property.
4(n + 11)

a. 4n + 11
b. 15n
c. 44 + n
d. 4n + 44

15. What is the solution to the equation n + 12 = 17?

a. n = 29
b. n = 5
c. n = 3
d. n = 27

Match these items (each answer, 2 points).

16.

_____	2x	a.	two decreased by a number
_____	k + 2	b.	the sum of a number and two
_____	2 - n	c.	twice a number
_____	d ÷ 2	d.	a number divided by two

Fill in each blank with the correct answer (each answer, 2 points).

17. 4^4 = _____

18. $\sqrt{121}$ = _____

19. $3^3 + 11 - 10 \div 2 \times 4$ = _____

20. 5, 9, 13, 17, _____