



# MATH STUDENT BOOK

# 7th Grade



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# Math 701

# Integers

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**LIFEPAC Test is located in the center of the booklet**. Please remove before starting the unit.

# Fractions

# Introduction

In this unit, students will learn about the set of numbers that falls between whole numbers. These numbers are known as fractions. Students will learn about the different parts of a fraction, as well as the different types of fractions, such as proper fractions, improper fractions and mixed numbers. Once students are able to identify the different types of fractions, they will learn how to calculate equivalent fractions and simplify, or reduce, fractions. Finally, they will learn how to compute with fractions and mixed numbers.

# Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Identify parts of fractions and mixed numbers.
- Identify the different types of fractions.
- Perform operations with fractions and mixed numbers.
- Simplify fractions.
- Compare and order fractions.
- Find equivalent fractions.

# 1. Working with Fractions

## **FRACTIONS AND MIXED NUMBERS**



#### Objectives

- Identify the different parts of fractions and mixed numbers.
- Convert between mixed numbers and improper fractions.
- Round fractions and mixed numbers.

## Vocabulary

**denominator**—the number under the fraction line; tells how many equal parts the whole was broken into

fraction—a number that shows part of a whole

**improper fraction**—a fraction in which the numerator is larger than the denominator **mixed number**—a number with an integer part and a fraction part

**numerator**—the number above the fraction line; tells how many parts of the whole exist **simplified fraction**—a fraction written in lowest terms (i.e., the numerator and denominator do not have any more common factors)

## **TRANSLATING WORD SENTENCES**

In this lesson, you'll be translating word sentences into the language of math. In

order to do that, you'll have to use what you know about translating phrases.

#### **Objectives**

- Translate between word sentences and mathematical equations.
- Write an equation to represent a word problem.

#### Vocabulary

**equation**—a mathematical statement that shows two expressions are equal using an equal sign

Translating sentences is almost exactly the same as translating phrases. But there is one more symbol that you have to use: the equal sign! A word phrase translates to an expression. A word sentence translates to an *equation*. An equation is a complete thought. It tells you that two expressions are equal to each other. So an equation is two expressions joined by an equal sign.

**Vocabulary!** Remember that a variable, or letter, represents the value that is unknown.

Do you remember the three parts of mathematical language? They are numbers, symbols, and variables. The equal sign is a symbol. To determine which symbols to use in an expression or equation, look for key words. Key words can also help you determine when to use the equal sign. Figure 2 will remind you of the key words that indicate the different operations. Notice that the last column shows words that indicate the equal sign.

**Vocabulary!** Remember that a variable, or letter, represents the value that is unknown.

Now look at the difference between phrases and sentences. Remember that a phrase translates to an expression while a sentence is a complete thought and translates to an equation.

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)	Equality (=)
sum	difference	product	quotient	equals
increased by	decreased by	times	divided by	is
more than	less than	of	per	the result is
more	less	multiplied by		yields
added to	subtracted from	by		is equal to
plus	minus			the solution is
greater than	fewer than			

Figure 2 | Words and Phrases that indicate operations

the ratio of red jellybeans to blue jellybeans?

## Solution:

 You know that there are 3 red jellybeans out of every 5 jellybeans. That means that the other 2 (5 - 3 = 2) must be blue. So the ratio of red to blue is 3 to 2. This can also be written as 3:2 or 3/2.

Now take a look at another example involving a few different ratios.

Scott has a jar full of marbles. Eight of the marbles are green and sixteen are orange. What are all of the different ratios you can write for this situation?

First, you need to find the *whole*—how many total marbles Scott has: 8 + 16 = 24

So he has 24 marbles.



Remember, you can compare part-part and part-whole, and you have three different ways to name each ratio:

Comparison	Comparison Fraction Col		Words
part = green part = orange	8 16	8:16	8 to 16
$\frac{\text{part}}{\text{part}} = \frac{\text{orange}}{\text{green}}$	<u>16</u> 8	16:8	16 to 8
$\frac{\text{part}}{\text{whole}} = \frac{\text{green}}{\text{marbles}}$	8 24	8:24	8 to 24
$\frac{\text{part}}{\text{whole}} = \frac{\text{orange}}{\text{marbles}}$	16 24	16:24	16 to 24

Can you compare the whole to a part? Yes!

Comparison	Fraction Colons		Words
$\frac{\text{whole}}{\text{part}} = \frac{\text{marbles}}{\text{green}}$	<u>24</u> 8	24:8	24 to 8
$\frac{\text{whole}}{\text{part}} = \frac{\text{marbles}}{\text{orange}}$	24 16	24:16	24 to 16

The ratio of 24 to 8 means that out of 24 marbles, 8 are green. If you were talking about your marble collection, you might want to first talk about how many marbles you have and then talk about how many you have of each color: "I have 24 marbles and 16 are orange."

You've probably noticed that ratios are similar to fractions. Although they do have a lot in common with fractions, they are not fractions. Look at the similarities and differences using this example of 2 pennies and 2 nickels:



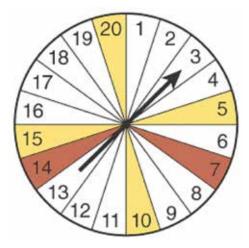
#### Similarities:

- Both fractions and ratios can compare part-whole:
  - Fraction—A fraction of the coins are pennies.
  - Ratio—The ratio of pennies to coins is  $\frac{2}{4}$ , or 2 to 4.
- Both fractions and ratios represent division:
  - Fraction—The fraction  $\frac{2}{4} = 2 \div 4$ , or 0.5., so half the coins are pennies.
  - Ratio—The ratio  $\frac{2}{4}$ , or 2 to 4, means half the coins are pennies.

#### number of favorable outcomes

$$P(\text{multiple of 7}) = \frac{2}{20}$$

- If the wheel is colored, you can see the favorable outcomes more easily.
- yellow: multiples of 5
- orange: multiples of 7



- There are no outcomes in common, so you can add the probabilities.
- P(event 1 or event 2) = P(event 1) + P(event 2)
- P(multiple of 5 or multiple of 7) =
   P(multiple of 5) + P(multiple of 7)
- P(multiple of 5 or multiple of 7) =  $\frac{4}{20}$ +  $\frac{2}{20}$
- P(multiple of 5 or multiple of 7) =  $\frac{6}{20}$
- This was the same probability as the game Ondi wanted to play, and you found that 6 out of 20 was 30%. So there is a 30% chance of spinning a multiple of 5 or 7.

#### Example:

A side game at the fair requires the game operator to guess the month of the guest's birth within 2 months. If the game operator is off by more than two months, the guest wins a prize. What is the probability that the game operator will randomly guess a person's birth month within two months of the correct month?

## Solution:

- There are 12 possible months to choose from, so there are 12 possible outcomes. Of these possible outcomes, the game operator must either guess the correct month, or one of the two months on either side of the correct month. For example, if the guest was born in July, the game operator could guess July (the correct month), August, September (the 2 months after July), June, or May (the 2 months before July). This allows 5 possible favorable outcomes.
- Compare the number of favorable outcomes with the number of possible outcomes and form a ratio.

 $\frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{5}{12}$ 

P(birth month within 2 months) =  $\frac{5}{12}$ 

- **1.5** What is the perimeter of a regular pentagon in which all sides have a length of 7 feet?
  - 🔲 49 ft

□ 28 ft □ 35 ft

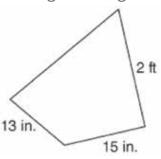
- 5 ft 🛛 🗌
  - 🔲 42 ft
- **1.6** Hans is building a fence to put around his rectangular garden. How many *yards* of fencing will Hans need to enclose the garden on all four sides?



**1.7** Two sides of a triangle measure 18 meters and 11 meters. If the perimeter of the triangle is 37 meters, what is the length of the third side?

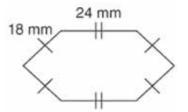
🗆 12 m	<b>8</b> m	🗖 66 m	🗖 30 m

**1.8** If the perimeter of this quadrilateral is 79 inches, what is the measure of the missing side length in inches?



77 in
52 in.
28 in.
27 in.

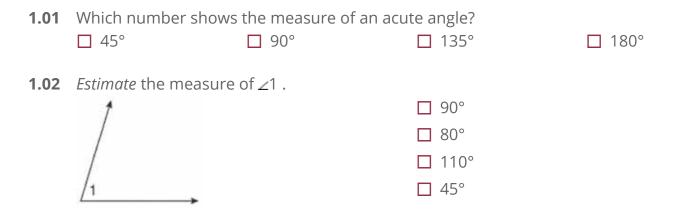
**1.9** Which of the following expressions could be used to find the perimeter of this figure?



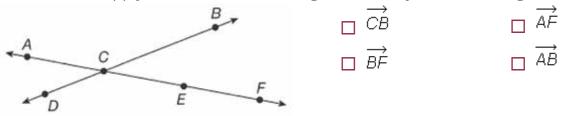
- $\square P = 4(18 \text{ mm}) + 2(24 \text{ mm})$
- □ *P* = 18 mm + 24 mm
- $\square$  *P* = 2(18 mm) + 4(24 mm)
- □ *P* = (18 mm)(24 mm)
- **1.10** The perimeter of a square is 64 millimeters. What is the length of each side? $\square$  8 mm $\square$  12 mm $\square$  16 mm $\square$  32 mm

# Self Test 1: Basic Geometry

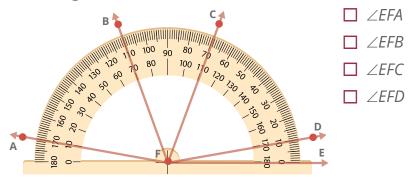
**Complete the following activities** (5 points, each numbered activity).



**1.03** Select all that apply. Which of the following names a ray in the drawing?



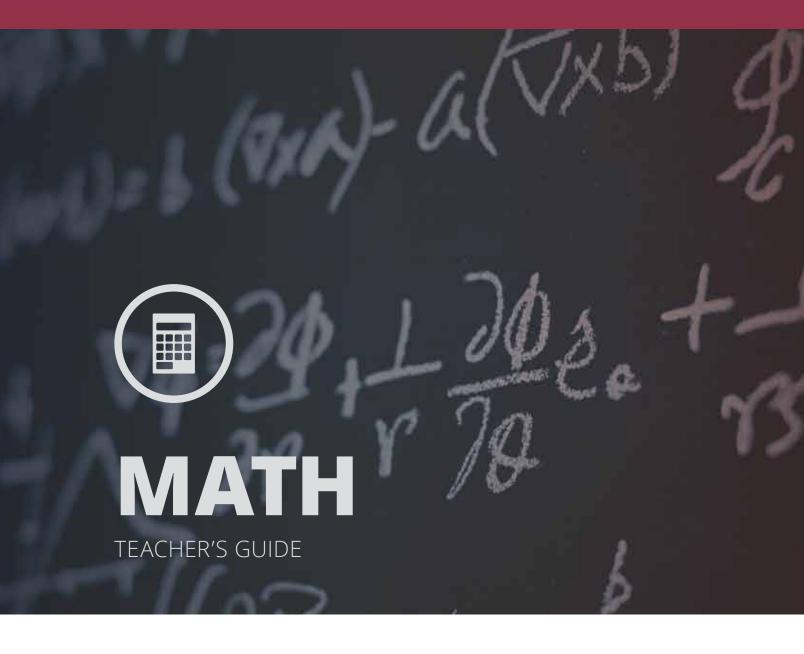
- **1.04** Select all that apply. Which of the following names an angle in the drawing used in the previous question?
  - $\Box \angle ACD \qquad \Box \angle CBE \qquad \Box \angle FBC \qquad \Box \angle DCE$
- **1.05** Which angle measures 70°?



**1.06** Select all that apply. Which pairs of angles are supplementary?

		► b		
	< <u>5 6</u> 7 8	<b>→</b> C		
1.07		hich angles are congru	ent to $\angle 4$ in the drawin	g used in the
	previous question? □ ∠1	□ ∠7	□ ∠8	□ ∠2
1.08		ementary and congruen	it. What is the measure	of each of
	these angles?	□ 45°	□ 50°	□ 180°
1.09			gles measure 37°. What	is the measure
	of the other two vertic 37°	□ 74°	□ 90°	□ 143°
1.010	What is a polygon with dodecagon	10 sides called?	□ tarragon	🗆 decagon
1.011	What is the measure c	of an angle in a regular l D 135°	hexagon? 120°	□ 108°
1.012	What is the sum of the 900°	e angle measures in a h D 540°	eptagon? 🔲 360°	□ 720°
1.013	Which polygon will hav	ve the largest angle sun D heptagon	n? 🔲 pentagon	🗌 dodecagon
1.014	A section of a circle ha circle called?	s both endpoints on th	e circle. What is the sec	tion of the









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# **MATH 700** Integers, Ratios, and Basic Geometry Teacher's Guide

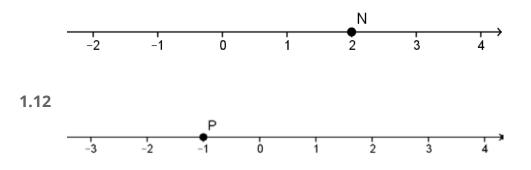
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# 1. Integers

# INTEGERS ON THE NUMBER LINE

1.1	False	1.6	True
1.2	True	1.7	eight below zero
1.3	True	1.8	six above zero
1.4	False	1.9	positive nine
1.5	False	1.10	Put a point 5 units to the left of zero.

1.11



**1.13** B corresponds with -2.

**1.14** F corresponds with 3.

**1.15** Space A represents -3.

# Inequalities and Absolute Value

1.41	True  1  = 1 1 = 1		1.49	$\begin{vmatrix} -8 \\ \geq -1 \end{vmatrix}$ $\begin{vmatrix} -8 \\ 8 > -1 \end{vmatrix}$
1.42	True  1  > 1 1 > -1		1.50	-5  ≥ -5  -5  = 5 5 > -5
1.43	False  -3  ≤ 3 3 ≤ 0		1.51	$ 4  \le  -9 $  4  = 4  and   -9  = 9 4 < 9
1.44	False  -2  ≥  - 2 ≥ 3	3		
1.45	False  -4  <  4 4 < 4	1		
1.46				
	-2	1st		
	1	2nd		
	-4	3rd		
		4th		
	11	5th		
1.47	$\begin{vmatrix} -6 \\ \ge 3 \end{vmatrix}$ $\begin{vmatrix} -6 \\ = 6 \\ 6 > 3 \end{vmatrix}$			
1.48	$ -2  \le  -5 $  -2  = 2 at $2 < 5$	and  -5  = 5		

# 2. Adding and Subtracting Integers

# Adding Integers with the Same Sign

2.1 False

This is only true if both numbers are positive.

- 2.2 False Being "in the red" represents having a negative bottom line.
- **2.3** True
- **2.4** 105

Both addends are positive, so keep the sum positive. The sum of 86 and 19 is 105.

**2.5** -22

Both addends are negative, so keep the sum negative. The sum of 18 and 4 is 22.

**2.6** -44

Both addends are negative, so keep the sum negative. The sum of 12 and 32 is 44.

**2.7** 65

Both addends are positive, so keep the sum positive. The sum of 14 and 51 is 65.

**2.8** -6

(-3) + (-3) = -6

**2.9** 14

7 + 7 = 14

## **2.10** 40

Jonathan's mother is 32. Add this to Jonathan's age: 8 + 32 = 40

- 2.11 seven red tiles plus three red tiles Red tiles represent negative numbers.
- **2.12** -13

**2.13** -11

(-5) + (-6) = -11

**2.14** 165

Both addends are positive, so keep the sum positive. The sum of 137 and 28 is 165.

**2.15** -104

Both addends are negative, so keep the sum negative. The sum of 65 and 39 is 104.

**2.16** -5

(-3) + (-2) = -5

Kaleigh was 5 spaces farther from the finish. Moving back is a negative number. Both numbers are negative, so keep the same sign and add.

## **2.27** \$16

The expression is 28 + (-12). The difference between |28| and |-12| is 16. Since |28| has the larger absolute value, the difference is positive.

## **2.28** -3 + 6

Start at zero. Move three places to the left and then six places to the right.

# ZERO PAIRS

## **2.29** 0

Opposite numbers have a sum of zero.

#### **2.30** -10

Both addends are negative, so keep the sum negative. The sum of 5 and 5 is 10.

## **2.31** 8

Both addends are positive, so keep the sum positive. The sum of 4 and 4 is 8.

## **2.32** They have a sum of -14.

Opposite numbers have a sum of zero.

## **2.33** (-8) + 15 = 7

Add 15 to -8. The difference between |15| and |-8| is 7. Since |15| has the larger absolute value, the difference is positive.

## **2.34** 13 + (-6) = 7

Add -6 to 13. The difference between |-6| and |13| is 7. Since |13| has the larger absolute value, the difference is positive.

## **2.35** (-25) + (25) = 0

Opposite numbers have a sum of zero.

# 4. The Real Number System

# THE REAL NUMBER SYSTEM

4.1	All	4.7	3
4.2	No	4.8	1.6490221
4.3	All		Irrational numbers are decimal numbers that don't end or repeat.
4.4	Some	4.9	irrational numbers
4.5	Some		Natural numbers are rational, not irrational.
4.6	irrational	4.10	real numbers rational numbers

#### 4.11-4.15

		Natural	Whole	Integer	Rational	Irrational
4.11	-2			Х	Х	
4.12	0		Х	Х	Х	
4.13	<u>5</u> 6				х	
4.14	$\pi$					Х
4.15	6	Х	Х	Х	Х	

4.016 
$$7(40+3) = 7(40) + 7(3) = 280 + 21 = 301$$
  
4.017  $35+18 \div 6 - 20 = 35 + 3 - 20 = 38 - 20 = 18$   
4.018  $-(3)^2 + 2^3 = -9 + 8 = -1$   
4.019  $(-4)^2 - (6+1) = (-4)^2 - 7 = 16 - 7 = 9$   
4.020  $(7+2)^2 - (5-2)^3 = 9^2 - 3^3 = 81 - 27 = 54$ 

# **MATH 701**

ALTERNATE LIFEPAC TEST

100 50

NAME

DATE

**Complete the following activities** (4 points, each numbered activity).

1. How would you graph the result of 4 + (-7) as a point on the number line? eleven places to the right of zero three places to the right of zero □ three places to the left of zero eleven places to the left of zero 2. Which of the following statements has a value of -8? □ the opposite of eight eight above zero □ the absolute value of eight □ the opposite of negative eight 3. Which of the following inequalities is not true? □ -5 < -8  $(-4)^2 \neq -16$  $\Box$  |-3|  $\geq$  |3|  $\Box$  |-6|  $\leq$  |-7| Use the number line to determine which of the following statements is true. Each 4. tick is 1 unit. □ The sum of J and K is positive. □ The product of K and L is negative. ☐ The absolute values of K and M □ The quotient of M and L is are the same. negative. 5. Determine which of the following lists is in order from smallest to largest. □ |-4|, 1, |3|, -2<sup>2</sup> □ 1, |3|, -2<sup>2</sup>, |-4|  $\Box$  -2<sup>2</sup>, |3|, 1, |-4|  $\Box$  -2<sup>2</sup>, 1, |3|, |-4| 6. Add (-11) + (-9).  $\square 2$ □ -2 -20 20

15.	Which of the following Some rational num irrational.			Every natural number.	per is a whole	
	Every integer is a n	atural number.		Every real number number.	is a rational	
16.	Which of the following statements demonstrates the identity property of multiplication?					
	$\square 9 \cdot (2 \cdot 7) = (9 \cdot 2) \cdot 7$			$7 \cdot -8 = -8 \cdot 7$		
	□ (-4)(1) = -4			-5(18) = -5(20 - 2)		
17.	Which of the following properties states that the way in which addends are grouped may be changed without affecting the sum?					
	distributive property			identity property of addition		
	associative property of addition		commutative property of addition			
18.	All of the following statements correctly use the distributive property <i>except</i>					
	$\Box 5(12 - 6) = 5(12) + 5(6)$			$\Box$ -3(9 - 2) = -3(9) - (-3)(2)		
	$\Box -4(17+6) = -4(17) + (-4)(6)$			7(13 + 2) = 7(13) + 7(2)		
19.	Evaluate -5(2 + 7) -  3  ÷ -3.					
	□ 16	□ 14		-46	-44	
20.	Find the value of 6 + (-2) <sup>3</sup> · (-4 + 1).					
	6	☐ -18		30	0	