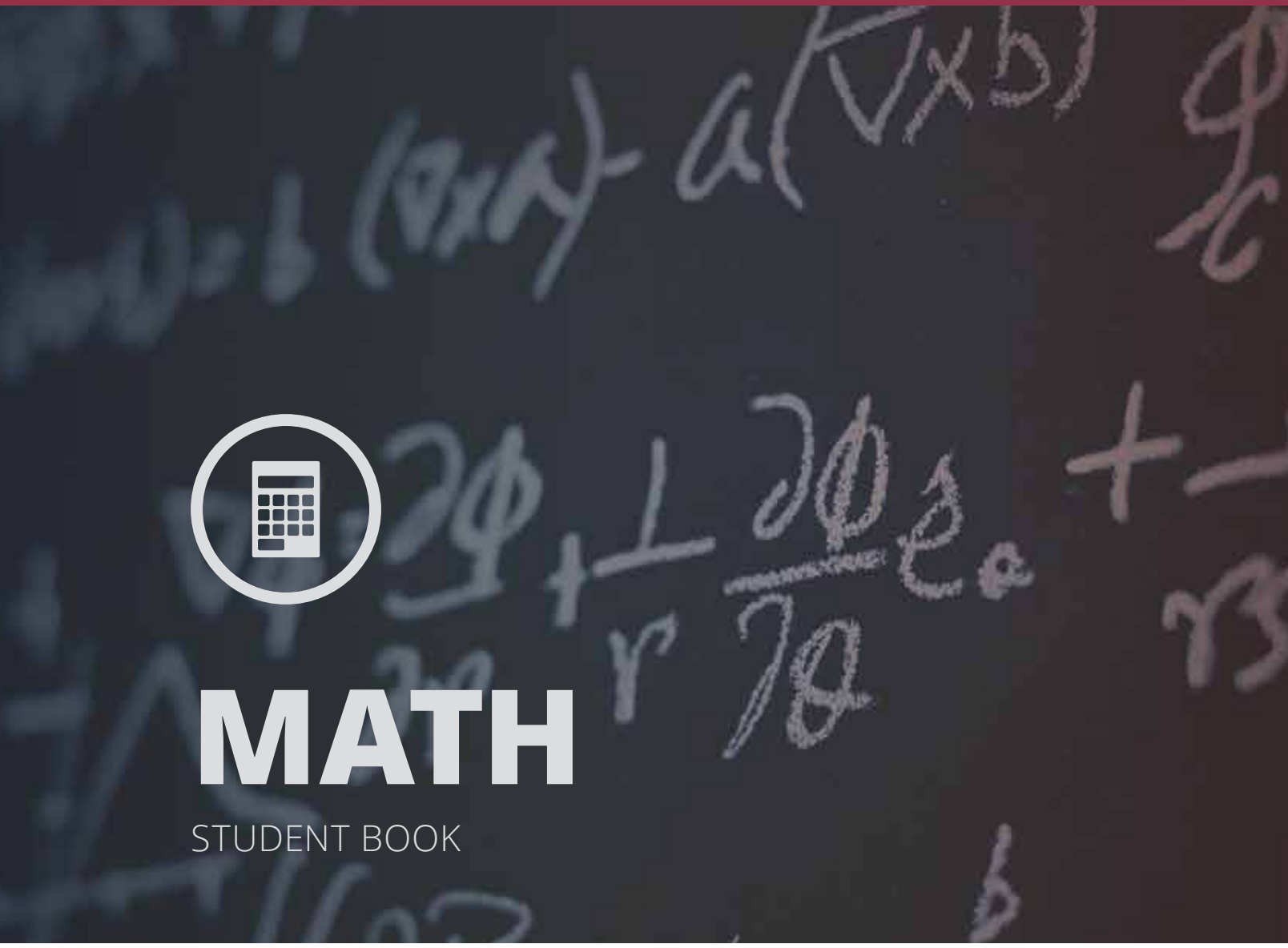
LIFEPAC®



MATH

STUDENT BOOK

8th Grade

Alpha Omega PUBLICATIONS

LIFEPAC | aop.com

Math 801

The Real Number System

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Modeling Problems in Integers

Introduction

In this unit, functions are represented in a variety of ways. Students are asked to interpret graphical models in practical situations. Variable, expression, and equation are defined. Students solve one-step and two-step equations with whole numbers first. After algebra tiles and the number line are used to establish the "rules" for adding, subtracting, multiplying, and dividing integers, students solve equations in the integers. The order of operations is revisited in this unit. Students evaluate expressions using signed numbers. Finally, students are guided through the process for using an equation.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Perform operations of integers.
- Solve one-step and two-step equations, with real numbers and integers.
- Translate contextual situations into one-step and two-step equations before solving them.
- Identify relations and functions in their many forms, including ordered pairs, mapping diagrams, t-charts, and graphing.
- Identify domains, ranges, independent variables, dependent variables, and inputs and outputs.
- Graph functions and read the graphs of functions.

1. Equations with Real Numbers

Translating Expressions and Equations

Objectives

- Translate written statements into math symbols, expressions, and equations.
- Represent a simple word problem as an equation.

Vocabulary

equation—a mathematical statement that shows two expressions are equal; it has an equal sign

Translating Expressions

Have you ever read a word problem and had to reread it a couple of times to try to figure out what it is asking? Sometimes it feels like it was written in a foreign language. In a way, it was. It was written in mathematical language. Similar to foreign languages, the mathematics language also has rules of usage and conventions. In order to understand it, all you have to do is be able to translate it into a math problem full of numbers, symbols, and variables.

To be able to read mathematical statements, you must become familiar with how to give them meaning. You can learn to translate from phrases and sentences written in English to those written in math. You will learn the language of math, just as you learned English. You started simply by learning sounds of letters. Then, you learned to use letters to make words.

Numbers are the first part of the mathematical language that you learned. You know how to translate from a number written as a word in English to its mathematical symbol, the numeral. The word “five” is the numeral “5” and you understand what it means in both languages.

Figure 1 is a table of math symbols. In the columns are some of the words that translate from English into that math symbol. You may be able to think of more words that you have used previously to represent these.

Simplifying Fractions

Fractions are used to represent parts of a whole. We can actually use shapes to represent fractions. A shape can be cut into any number of pieces that are the same size and shape. Then, some of the pieces can be shaded in. The total number of pieces represents the denominator, or the bottom number in the fraction. The number of shaded pieces represents the numerator, or the top number in the fraction. See if you can determine what fractions each of the following shapes represents.

Objectives

- Reduce positive and negative fractions.
- Reduce fractions with variables.

Vocabulary

denominator—the bottom part of a fraction; represents the whole

equivalent fractions—fractions that may be expressed differently, but still have the same value

greatest common factor—the largest number that divides evenly into two or more given numbers

improper fraction—a fraction with a numerator that is larger than or equal to the denominator

mixed number—consists of an integer and a proper fraction

numerator—the top part of a fraction; represents part of the whole

proper fraction—a fraction with a numerator that is less than the denominator

Reducing Fractions

Let's see how you did with the shapes in the introduction. In the circle, five of eight sections were shaded. So, the circle represents the fraction $\frac{5}{8}$. In the rectangle, three of six sections were shaded, or $\frac{3}{6}$. And, in the triangle, one of two sections was shaded, or $\frac{1}{2}$. Each of these fractions is a *proper fraction*, which means that the *numerator* is smaller than the *denominator*.

Vocabulary! When the numerator is larger than the denominator, the fraction is an *improper fraction*. Improper fractions can be converted to *mixed numbers* using division. For example, $\frac{3}{2}$ can also be written as $1\frac{1}{2}$.

1.6 The formula $F = \frac{9}{5}C + 32$ is used to convert Celsius to Fahrenheit temperature. If the temperature is 20°C, what is it in Fahrenheit?

- ☐ 28.8°F
- ☐ 43.1°F
- ☐ 68.0°F
- ☐ 93.6°F

1.7 Which of the following statements explains how to solve for *l* using the formula $A = lw$, when $A = 28\frac{1}{4}$ and $w = 2\frac{1}{2}$?

- ☐ Divide $28\frac{1}{4}$ by $2\frac{1}{2}$.
- ☐ Multiply $28\frac{1}{4}$ by $2\frac{1}{2}$.
- ☐ Divide $2\frac{1}{2}$ by $28\frac{1}{4}$.
- ☐ Subtract $2\frac{1}{2}$ from $28\frac{1}{4}$.

1.8 Convert 77 degrees in Fahrenheit to Celsius temperature using the formula $F = \frac{9}{5}C + 32$.

- ☐ 72°C
- ☐ 61°C
- ☐ 35°C
- ☐ 25°C

1.9 Using the formula $P = \frac{F}{A}$, find *F* if *P* = 27 and *A* = 4.

- ☐ 108
- ☐ 6.75
- ☐ 0.148
- ☐ 29

1.10 All of the following are equivalent *except* _____.

- ☐ $d = rt$
- ☐ $dt = r$
- ☐ $\frac{d}{t} = r$
- ☐ $\frac{d}{r} = t$

1.11 Solve $C = AB + D$ for *B*.

- ☐ $\frac{C+D}{A} = B$
- ☐ $AC-D = B$
- ☐ $\frac{C}{A} - D = B$
- ☐ $\frac{C-D}{A} = B$

1.12 In which of the following solutions would you multiply both sides of the equation by *n*?

- ☐ Solve $\frac{m}{n} = p$ for *m*.
- ☐ Solve $mn = p$ for *m*.
- ☐ Solve $m - n = p$ for *m*.
- ☐ Solve $m + n = p$ for *m*.

1.13 Which of the following statements explains how to solve for *w* in the equation $A = lw$?

- ☐ Multiply both sides by *l*.
- ☐ Multiply both sides by *A*.
- ☐ Divide both sides by *l*.
- ☐ Divide both sides by *A*.

1.14 Solve for b in the formula $3a + 2b = c$.

- ☐ $b = c - 3a$
- ☐ $b = \frac{c-2}{3a}$
- ☐ $b = \frac{c-3a}{2}$
- ☐ $b = \frac{2c}{3a}$

self-selected sample—members of a population that volunteer for a survey
stratified sample—a population is divided into groups and then members are selected from each group
survey—a sampling of a population used to make predictions
systematic sample—a rule is used to select members from a population

Use of Sampling

Before a company can make a claim that their product is recommended by four out of five dentists, they have to conduct a *survey* to ask dentists their opinions about the product. In this example, the dentists are the *population* for the survey.

How do you think a company is able to get every dentist in the U.S. to answer their survey? They don't. It's extremely difficult to contact every dentist and get them to give their opinions. Because of this, companies use a method called sampling. A *sample* is a small part of a population that is chosen to represent the entire group.

From the results of their samples, companies can make predictions about larger populations. A variety of types of sample groups can be used. The larger the sample used, the more reliable the results and predictions will be.

Types of Samples

Any time you choose to use a sample, you need to be careful to make a good choice of which type to use.

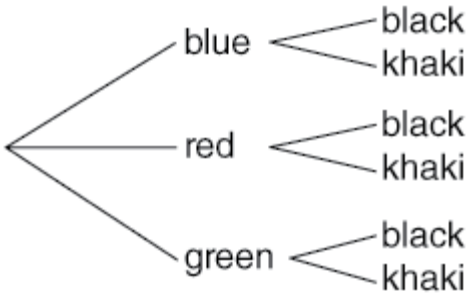
Let's take a closer look at examples of each type of sample, in Figure 1.

A *biased sample* is a sample that is not representative of the entire population. An example of a biased sample happened in the 1948 presidential election. The Chicago Tribune printed a headline that read "Dewey Defeats Truman" based on the results of a telephone survey. In fact, Truman won the election. Their survey was biased because it included only people who owned phones, which in 1948 was not very many. Those that owned phones tended to be better off financially, which was another form of bias.

A *convenience sample* is a sample where the representatives chosen are easily

Type of Sample	Definition
biased sample	sample that isn't representative of the entire population
convenience sample	sample where representatives are easily accessible
random sample (unbiased sample)	every member of the population has an equal chance of being selected
self-selected sample	sample where members of the population volunteer to take part
stratified sample	members are selected from different groups that exist in the population
systematic sample	members are selected through the use of a rule

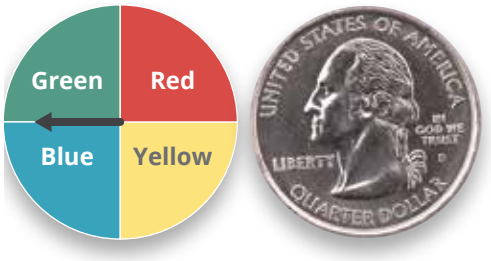
Figure 1| Types of Samples



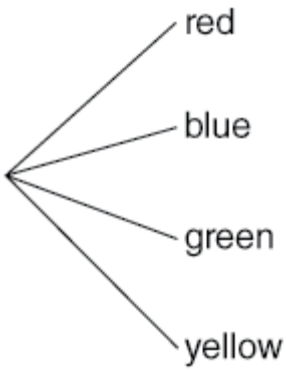
The tree diagram helps us see that there are six possible outcomes. The possible outcomes are:

- blue shirt with black pants
- blue shirt with khaki pants
- red shirt with black pants
- red shirt with khaki pants
- green shirt with black pants
- green shirt with khaki pants

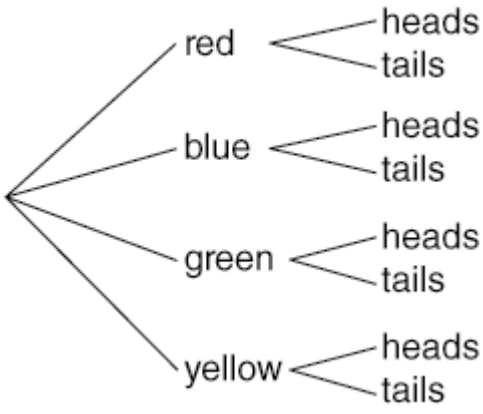
Let's take a look at another experiment. What are all the possible outcomes if you spin the following spinner and then flip the coin?



Let's make a tree diagram of the possible outcomes. The first step is to list all the possible outcomes for the spinner.



The next step is to draw two branches from each color. One branch should be labeled heads, and the other should be labeled tails. This adds eight new branches to our tree, resulting in the following diagram.



You can now see that there are eight possible outcomes. The possible outcomes are:

- red and heads
- red and tails
- blue and heads
- blue and tails
- green and heads
- green and tails
- yellow and heads
- yellow and tails

SELF TEST 1: Proportions

Complete the following activities (6 points, each numbered activity).

1.01 If x:6 as 3:9, then x is equal to _____ .

1.02 S varies directly as T. If S is 20 when T is 4, then T is _____ when S is 30.

1.03 If $\frac{9}{24} = \frac{3}{x}$, then x is _____ .

1.04 Y is directly related to X, and Y is 81 when X is 27. The constant of variation is _____ .

1.05 The ratio of boys to girls is 5 to 4. There are 80 boys. How many girls are there?

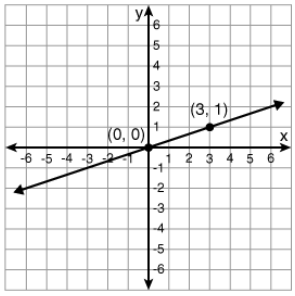
☐ 4 ☐ 16 ☐ 64 ☐ 100

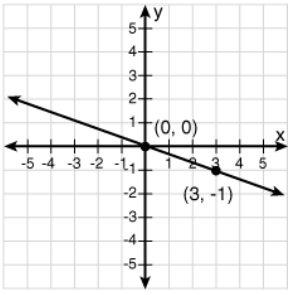
1.06 At what rate is a car traveling, if it goes 157.5 miles in 2.5 hours?

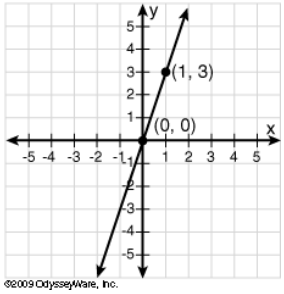
☐ 15 miles per hour ☐ 73 miles per hour

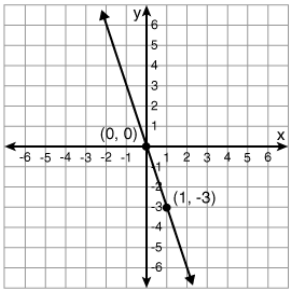
☐ 63 miles per hour ☐ 393.75 miles per hour

1.07 Which of the following direct variations has a constant of variation that is equal to -3?

☐ 

☐ 

☐ 

☐ 

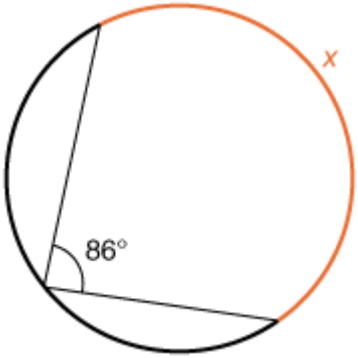
1.08 All of the following ratios are equivalent *except* _____.

☐ 2:3 ☐ $\frac{6}{9}$ ☐ $\frac{6}{4}$ ☐ 8 to 12

1.09 If two angles are complementary, then the sum of their angles is equal to _____.

☐ each other ☐ 90° ☐ 180°

1.010 What is the measure of *x*?

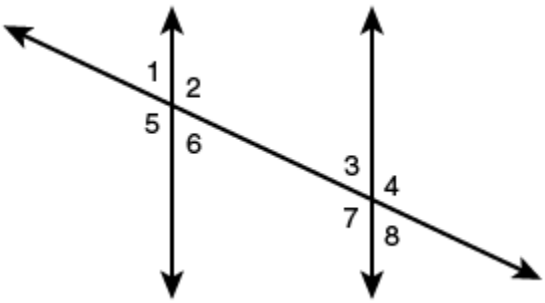


☐ 43°
☐ 86°
☐ 172°
☐ 344°

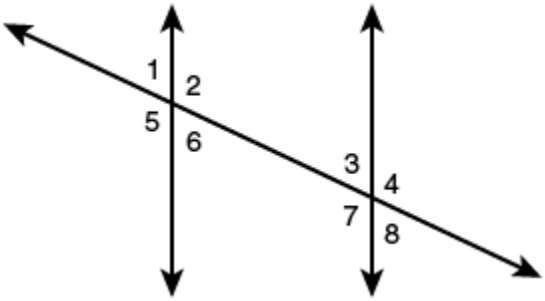
1.011 If one angle of a set of alternate interior angles on parallel lines measures 77°, then the other angle also equals 77° _____.

☐ always ☐ sometimes ☐ never

1.012 In the graphic, one pair of vertical angles is _____ .



1.013 In the image, the corresponding angle to angle 1 is angle _____ .



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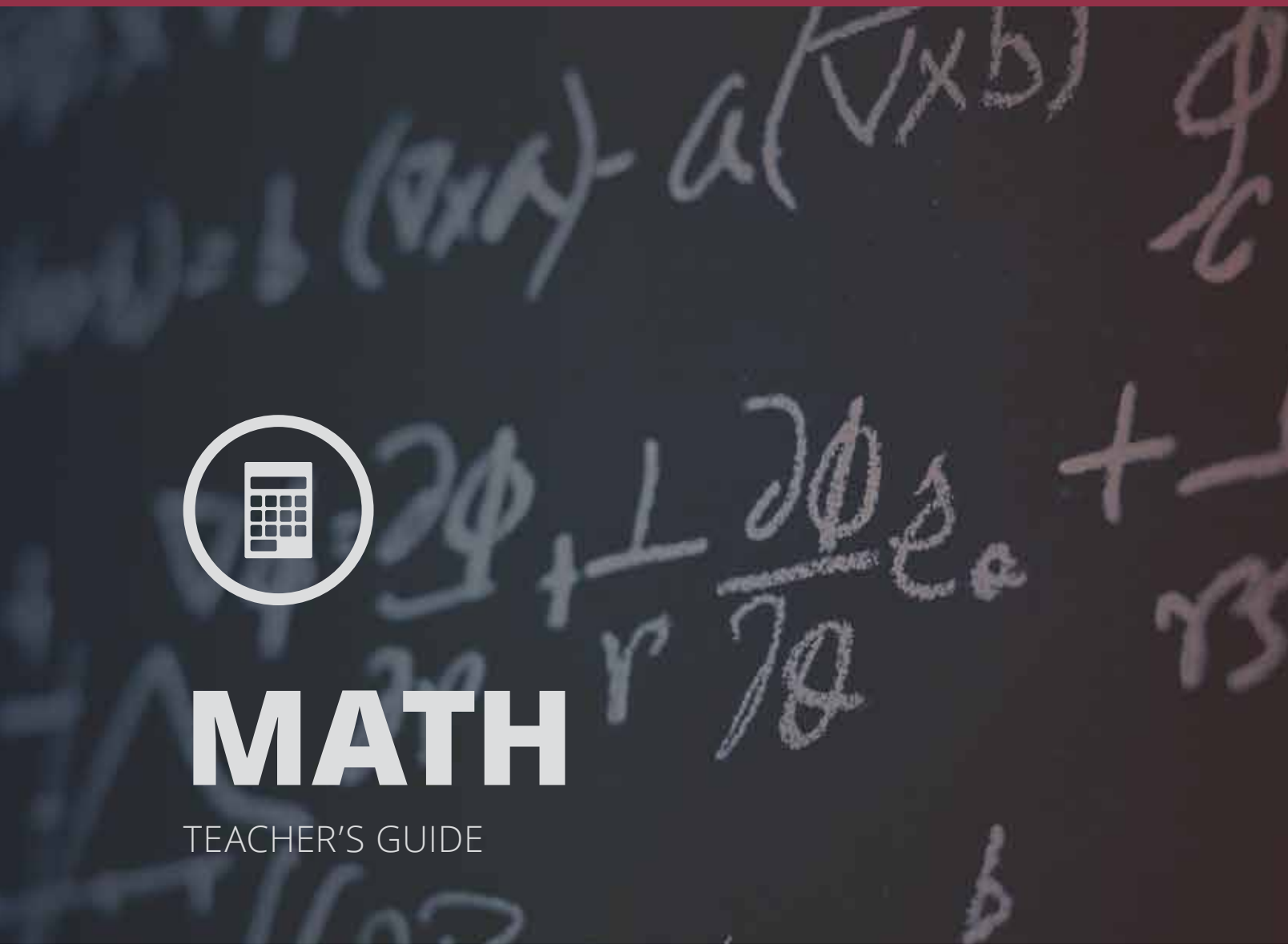


MATH

TEACHER'S GUIDE

8th Grade

Alpha Omega PUBLICATIONS

MATH 800
Pre-Algebra
Teacher's Guide

1. Relationships

Subsets of the Real Number System

1.1 π

1.2 an irrational number

1.3 a whole number

1.4 an irrational number

1.5 rational

1.6 3.14159...

1.7

rational Numbers	1/3
integers	5
whole numbers	0
natural numbers	56
irrational numbers	1.17513698

Using Variables

1.8 an expression

1.9 a variable

1.10 a variable

1.11 a formula

1.12

18	ab
6	$c + 4$
2	$\frac{a}{b}$
3	$a - b$
4	$2c$
1	$b \div 3$
5	$a - 1$
8	$6 + c$

$ab = (6)(3) = 18$

$c + 4 = 2 + 4 = 6$

$\frac{a}{b} = \frac{6}{3} = 2$

$a - b = 6 - 3 = 3$

$2c = 2 \cdot 2 = 4$

$b \div 3 = 3 \div 3 = 1$

$a - 1 = 6 - 1 = 5$

$6 + c = 6 + 2 = 8$

1.13 12 in.^{2}

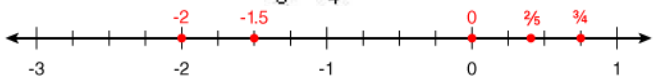
$$A = \frac{1}{2}bh = \frac{1}{2}(3)(8) = 12$$

Comparing Rational Numbers

1.31 $r \geq s$

$|-13| = 13$, so $s = 13$. Substitute the values in for r and s. 12.5 is not greater than or equal to 13.

1.32 $-2, -1.5, 0, \frac{2}{5}, \frac{3}{4}$



1.33 1.01

1.34 −0.1

In fraction form, −0.1 is $-\frac{1}{10}$. When comparing negative numbers, the number with the larger absolute value is actually smaller because it is further to the left of zero. $-\frac{1}{7} < -\frac{1}{10}$ because $-\frac{1}{7}$ has the larger absolute value.

1.35 $2 \geq 8$

1.36 Buffalo

−8 is the furthest to the left on the number line, so it has the smallest value, and Buffalo has the coldest temperature.

1.37 Tyrell

When comparing negative numbers, the number with the largest absolute value is the smallest number because it is the furthest to the left from zero. −18 has the largest absolute value, so Tyrell won.

1.38 $\frac{3}{2}$ and $1\frac{9}{10}$

Written as decimals, $\frac{3}{2}$ and $1\frac{9}{10}$ are 1.5 and 1.9, respectively. $1.5 < 1.75 < 1.9$.

1.39 2.6

The number line is divided into fifths.

Point A is located at $2\frac{3}{5}$, or 2.6.

1.40 $P < -1$

Point P is between −1 and −2, so it is greater than −2, but less than −1.

1.41 $A > D$

Point A is to the right of Point D on the number line, so Point A is greater than Point D.

2.09 2^4

Monday: $2 = 2^1$
Tuesday: $4 = 2^2$
Wednesday: $8 = 2^3$
Thursday: $16 = 2^4$

2.010 He should have written 1×10^6.

One million equals 1,000,000. To get a number between 1 and 10, move the decimal 6 places to the left. Written in scientific notation, the number should be in the form: a number greater than or equal to 1 but less than 10 × a power of 10.

2.011 He is incorrect because he should have only 5 factors of 3.

By the multiplication rule, add the exponents. So, $3^3 \cdot 3^2 = 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$.

2.012 16,300

Move the decimal 4 places to the right.

2.013 distributive property

$3 \cdot 0.40 + 5 \cdot 0.40 = (3 + 5) \cdot 0.40$

2.014 distributive property

$4 \cdot 7 \cdot \frac{1}{4}$

$4 \cdot \frac{1}{4} \cdot 7$ commutative property of multiplication
$1 \cdot 7$ multiplicative inverse
7 multiplicative identity

2.015 16

$$\frac{1}{n^2} = \frac{1}{4^2} = \frac{1}{16}$$

$$n^{-2} = \frac{1}{n^2} = \frac{1}{16}$$

2.016 Answers may vary. One possible answer could look like this:

$-5 + (5 + 3)$
$(-5 + 5) + 3$ associative property of addition
$0 + 3$ additive inverse
3 additive identity

3. Simplifying

Square Roots

3.1 False

50 is not a perfect square.

3.2 False

8 is not between 7^2 and 9^2. It is between 2^2 and 3^2.

3.3 True

$4^2 = 16$, and $\sqrt{16} = 4$.

3.4 8

3.5 3.1

10 is between 9 and 16, so $\sqrt{10}$ is between $\sqrt{9}$ and $\sqrt{16}$.

3.6 16 and 25

If $\sqrt{n} = 4.2$, then $n = 4.2^2$, or 17.64.

3.7 3 and 4

12 is between 9 and 16, so $\sqrt{12}$ is between $\sqrt{9}$ and $\sqrt{16}$.

3.8 49

If $\sqrt{x} = 7$, then $x = 7^2$, or 49.

3.9 $\sqrt{2}$

Two is not a perfect square, so its square root must be irrational.

3.10 $\sqrt{19}$

Nineteen is not a perfect square, so its square root cannot be rational.

3.11 0.6

$(0.6)^2 = 0.36$

3.12 1.5

$(1.5)^2 = 2.25$

3.13 $\sqrt{38}$

No perfect square goes into 38.

3.14 $5\sqrt{3}$

$\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

3.15 He is incorrect because he did not use the largest perfect square.

$\sqrt{128} = \sqrt{64} \cdot \sqrt{2} = 8\sqrt{2}$

3.16 $3\sqrt{6}$

$\sqrt{54} = \sqrt{9} \cdot \sqrt{6} = 3\sqrt{6}$

Self Test 3: Simplifying

3.01

4	$\sqrt{16}$
15	$\sqrt{2.25}$
8	$6^2 \div 9 \cdot 2$
1	$\frac{12-2}{6+4}$
5	$\sqrt{16+9}$
9	$63 \div 3^2 + \lvert 2 \rvert$

$\sqrt{16} = 4$, because $4^2 = 16$.

$\sqrt{2.25} = 1.5$, because $1.5^2 = 2.25$

$6^2 \div 9 \cdot 2 = 36 \div 9 \cdot 2 = 4 \cdot 2 = 8$

$\frac{12-2}{6+4} = \frac{10}{10} = 1$

$\sqrt{16+9} = \sqrt{25} = 5$

$63 \div 3^2 + \lvert 2 \rvert = 63 \div 9 + \lvert 2 \rvert = 7 + 2 = 9$

3.02 13

$36 \div 6 \cdot 2 + 1$
$6 \cdot 2 + 1$
$12 + 1$
13

3.03 $\sqrt{6}$

Six is not a perfect square.

3.04 exponent

1. Multiply inside the parentheses.
2. Exponent.

3.05 $\sqrt{9}$

Nine is a perfect square.

3.06 She needs to subtract 5 - 3 first.

Following the order of operations, parentheses come before exponents.

3.07 3 and 4

Thirteen is between 9 and 16, so $\sqrt{13}$ is between $\sqrt{9}$ and $\sqrt{16}$.

3.08 $\sqrt{20}$

The number is irrational, so it can't be 4.5. 4^2 is 16, and 5^2 is 25, so the number must be between $\sqrt{16}$ and $\sqrt{25}$

3.09 $\sqrt{17}$

Seventeen is not a perfect square.

3.010 9

$2^3 + \sqrt{64} \div 8$
$8 + 8 \div 8$
$8 + 1 = 9$

3.011 80 and 90

If $\sqrt{x} = 9$, then $x = 9^2 = 81$.

3.012 $\sqrt{30}$ can't be simplified.

No perfect squares go into 30.

3.013 0.4

$0.4^2 = 0.16$

3.014 division

1. Subtract inside the parentheses.
2. Exponent.
3. Divide

3.015 5

$3 + (4)^2 \div 8$
$3 + 16 \div 8$
$3 + 2$

28. Answers will vary, but should include the following points.

When you write a number in scientific notation, the first part is a number greater than or equal to 1 and less than 10, while the second part is a power of 10.

When you convert a number from standard form to scientific notation, the number of places the decimal moves determines the number of the exponent. If the decimal is moved to the left, then the exponent is positive. If the decimal is moved to the right, then the exponent is negative.

Alternative description: If in standard form the number is less than 1, the exponent of 10 is negative. If in standard form the number is greater than 1, the exponent of 10 is positive.

MATH 801

ALTERNATE LIFEPAC TEST

90
112
Score _____

NAME _____ **DATE** _____

Complete the following activities (4 points, each numbered activity).

1. $|-13| \geq -(-13)$
- ○ True
- ○ False

2. $3 < 4^0$
- ○ True
- ○ False

3. $0.012 = 1.2 \times 10^{-3}$
- ○ True
- ○ False

4. $\sqrt{49} > 3 + 32 \div 2^3$
- ○ True
- ○ False

5. $2 < \sqrt{6}$
- ○ True
- ○ False

---

6. Match the name of the number property used to get to each step from the previous step.

$[-5 + (\frac{1}{2} + 5)] \cdot 2$

_____ $[-5 + (5 + \frac{1}{2})] \cdot 2$

_____ $[(-5 + 5) + \frac{1}{2}] \cdot 2$

_____ $[0 + \frac{1}{2}] \cdot 2$

_____ $\frac{1}{2} \cdot 2$

_____ 1

commutative property

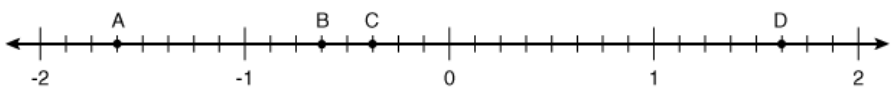
associative property

inverse property of multiplication

identity property of addition

inverse property of addition

7. Which of the points best represents the location of $-\frac{5}{8}$ on the number line?



☐ A ☐ B ☐ C ☐ D

8. $2^3 \cdot 2^4$ is equal to _____.

- ☐ seven factors of two
- ☐ 12 factors of two
- ☐ two factors of seven
- ☐ two times 12

9. Which of the following statements is true?

- ☐ $1.7 < 1\frac{1}{2}$
- ☐ $\frac{3}{4} \leq \frac{7}{8}$
- ☐ $-1\frac{1}{2} < -1.5$
- ☐ $0.01 > 0.1$

10. If $x = 45$, then $\sqrt{x}$ is between _____.

- ☐ 4 and 5
- ☐ 6 and 7
- ☐ 22 and 23
- ☐ 44 and 46

11. All of the following are irrational *except* _____.

- ☐ $\sqrt{15}$
- ☐ 3.14829...
- ☐ $\sqrt{18}$
- ☐ $1.\overline{45}$

12. Simplify $|-3| + 6 \div (4 - 1) \cdot 8$.

- ☐ 8
- ☐ 13
- ☐ 19
- ☐ 24

13. Written in standard form, a number is 10,250. In scientific notation, the number is _____.

- ☐ 1.025×10^3
- ☐ 1.025×10^{-4}
- ☐ 10.25×10^3
- ☐ 1.025×10^4

14. The following score card shows the scores for a golf game. If the lowest score wins, who came in last place?

Player	Score
Wei	+1
Mary	-5
Ling	-3
Mia	+2

- ☐ Wei
- ☐ Mary
- ☐ Ling
- ☐ Mia

15. If $3 \cdot N = 1$, then N is the _____.

- ☐ additive inverse
- ☐ multiplicative inverse
- ☐ additive identity
- ☐ multiplicative identity

16. If $m = 3$ and $n = 2$, then $m^2 - n^2$ is equal to _____.

- ☐ 1
- ☐ 2
- ☐ 5
- ☐ 13

17. Which of the following lists is in order from least to greatest?

- ☐ $-3, -1, \sqrt{10}, 2^2$
- ☐ $-3, -1, 2^2, \sqrt{10}$
- ☐ $-1, -3, 2^2, \sqrt{10}$
- ☐ $-1, -3, \sqrt{10}, 2^2$